Model independent determination of the axial mass in quasielastic neutrino-nucleon scattering.

Gil Paz

Department of Physics and Astronomy, Wayne State University

Richard. J. Hill, GP	PRD	**82**	113005 (2010)	arXiv:1008.4619
Bhubanjyoti Bhattacharya, Richard J. Hill, GP	PRD	**84**	073006 (2011)	arXiv:1108.0423

Outline

- Introduction: The axial mass problem
- Model independent extraction of the proton charge radius from electron scattering
- Model independent determination of the axial mass parameter in quasielastic neutrino-nucleon scattering
- Conclusions and outlook

Introduction: The axial mass problem

Quasiealstic $\nu - N$ scattering

- Quasiealstic $\nu - N$ scattering: $\nu_\ell + n \to \ell^- + p$
  basic signal for ν oscillation experiment

- At the quark level: $\nu_\ell + d \to \ell^- + u$

- Process "folded" twice

Quark: $\nu_\ell + d \to \ell^- + u$

$\Downarrow$ Form factor

Nucleon $\nu_\ell + n \to \ell^- + p$

$\Downarrow$ Nuclear model

Nucleus: $\nu_\ell + \text{nucleus} \to \ell^- + \dots$

Quark $\rightarrow$ Nucleon

- The interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{ud}^{*} \, \bar{l}\gamma^{\alpha}(1-\gamma^5)\nu \, \bar{u}\gamma_{\alpha}(1-\gamma^5)d.$$

- We know the current $\bar{u}\gamma_{\alpha}(1-\gamma^5)d$

  We cannot calculate its matrix elements from first principles

  Solution: parametrize ignorance by form factors

Quark $\to$ Nucleon

- The interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{ud}^{*}\, \bar{l}\gamma^{\alpha}(1-\gamma^5)\nu\, \bar{u}\gamma_{\alpha}(1-\gamma^5)d.$$

- We know the current $\bar{u}\gamma_{\alpha}(1-\gamma^5)d$

  We cannot calculate its matrix elements from first principles

  Solution: parametrize ignorance by form factors

$$\langle p(p')|\bar{u}\gamma_{\mu}(1-\gamma_5)d|n(p)\rangle = \bar{u}^{(p)}(p')\Gamma_{\mu}(q)u^{(n)}(p)\,,$$

where $q = k - k' = p' - p$ and

$$\begin{aligned}\Gamma_{\mu}(q) &= \gamma_{\mu}F_1(q^2) + \frac{i}{2m_N}\sigma_{\mu\nu}q^{\nu}F_2(q^2) + \frac{q_{\mu}}{m_N}F_S(q^2)\\ &+ \gamma_{\mu}\gamma_5 F_A(q^2) + \frac{p_{\mu}+p'_{\mu}}{m_N}\gamma_5 F_T(q^2) + \frac{q_{\mu}}{m_N}\gamma_5 F_P(q^2)\end{aligned}$$

Form Factors

$$\begin{aligned}\Gamma_\mu(q) &= \gamma_\mu F_1(q^2) + \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2(q^2) + \frac{q_\mu}{m_N}F_S(q^2)\\ &+ \gamma_\mu\gamma_5 F_A(q^2) + \frac{p_\mu + p'_\mu}{m_N}\gamma_5 F_T(q^2) + \frac{q_\mu}{m_N}\gamma_5 F_P(q^2)\end{aligned}$$

- Constrained by symmetries of the strong interactions:
- Time reversal $\Rightarrow$ F_i are real
- Time reversal & isospin $\Rightarrow$ $F_T, F_S = 0$

Form Factors

$$\begin{aligned}\Gamma_\mu(q) &= \gamma_\mu F_1(q^2) + \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2(q^2) \\ &+ \gamma_\mu\gamma_5 F_A(q^2) \qquad\qquad + \frac{q_\mu}{m_N}\gamma_5 F_P(q^2)\end{aligned}$$

- Constrained by symmetries of the strong interactions:
- Time reversal $\Rightarrow \quad F_i$ are real
- Time reversal & isospin $\Rightarrow \quad F_T, F_S = 0$
- Isopsin $\Rightarrow \quad F_1, F_2$ related to EM form factors: $F_{1,2} = F^p_{1,2} - F^n_{1,2}$

Form Factors

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2(q^2)$$
$$+ \gamma_\mu\gamma_5 F_A(q^2) + \frac{q_\mu}{m_N}\gamma_5 F_P(q^2)$$

- Constrained by symmetries of the strong interactions:
- Time reversal $\Rightarrow$ F_i are real
- Time reversal & isospin $\Rightarrow$ $F_T, F_S = 0$
- Isopsin $\Rightarrow$ F_1, F_2 related to EM form factors: $F_{1,2} = F^p_{1,2} - F^n_{1,2}$
- Axial current is conserved in the $m_\pi \to 0$ limit (PCAC):

$$F_P(q^2) = -\frac{2m_N^2}{q^2}\, F_A(q^2)\Big|_{m_\pi\to 0}$$

F_P contribution suppressed by m_ℓ^2/m_N^2

Form Factors

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2(q^2)$$
$$+ \gamma_\mu\gamma_5 F_A(q^2) + \frac{q_\mu}{m_N}\gamma_5 F_P(q^2)$$

- Constrained by symmetries of the strong interactions:
- Time reversal $\Rightarrow$ F_i are real
- Time reversal & isospin $\Rightarrow$ $F_T, F_S = 0$
- Isopsin $\Rightarrow$ F_1, F_2 related to EM form factors: $F_{1,2} = F_{1,2}^p - F_{1,2}^n$
- Axial current is conserved in the $m_\pi \to 0$ limit (PCAC):

$$F_P(q^2) = -\frac{2m_N^2}{q^2} F_A(q^2)\Big|_{m_\pi \to 0}$$

F_P contribution suppressed by m_ℓ^2/m_N^2

- Only F_A is not constrained

The Axial Mass

- What do we know about $F_A(q^2)$?

- Consider a small q^2 expansion of $F_A(q^2)$

- $F_A(0) = -1.269$ is known from neutron decay

- Define the axial mass m_A as

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2}q^2 + \ldots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Keep in mind: to fully describe F_A need more than one parameter!

Dipole model for F_A

- Common **model** for F_A: the dipole model

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\mathrm{dipole}})^2]^{-2}$$

- One parameter **model** for F_A

  Known to be inadequate for EM form factors

Nucleon $\rightarrow$ Nucleus

- Experiments usually scatter ν off nuclei
  Need a nuclear model: how do nucleons behave in the nucleus
- Popular model: "Relativistic Fermi Gas" (RFG)
  [Smith, Moniz, NPB **43**, 605 (1972)]

Nucleon $\to$ Nucleus

- Experiments usually scatter ν off nuclei
  Need a nuclear model: how do nucleons behave in the nucleus
- Popular model: "Relativistic Fermi Gas" (RFG)
  [Smith, Moniz, NPB **43**, 605 (1972)]
- Nuclear cross section

$$\sigma_{\rm nuclear} = n_i(\mathbf{p}) \otimes \sigma_{\rm free}(\mathbf{p} \to \mathbf{p}') \otimes [1 - n_f(\mathbf{p}')]$$

  neutron 3-momentum distribution:
  $n_i(\mathbf{p}) = \theta(p_F - |\mathbf{p}|)$, $n_f(\mathbf{p}') = \theta(p_F - |\mathbf{p}'|)$
- Two modifications:

- $k \cdot p \to E_{\mathbf{k}} E_{\mathbf{p}}$: ignores nonzero velocity of initial state nucleon.
- $p^0 \to \epsilon_{\mathbf{p}} \equiv \sqrt{m_N^2 + |\mathbf{p}|^2} - \epsilon_b$: introduces binding energy

Nucleon $\rightarrow$ Nucleus

- Experiments usually scatter ν off nuclei
  Need a nuclear model: how do nucleons behave in the nucleus
- Popular model: "Relativistic Fermi Gas" (RFG)
  [Smith, Moniz, NPB **43**, 605 (1972)]
- Nuclear cross section

$$\sigma_{\text{nuclear}} = n_i(\mathbf{p}) \otimes \sigma_{\text{free}}(\mathbf{p} \to \mathbf{p}') \otimes [1 - n_f(\mathbf{p}')]$$

  neutron 3-momentum distribution:
  $n_i(\mathbf{p}) = \theta(p_F - |\mathbf{p}|)$, $n_f(\mathbf{p}') = \theta(p_F - |\mathbf{p}'|)$
- Two modifications:

- $k \cdot p \to E_{\mathbf{k}} E_{\mathbf{p}}$: ignores nonzero velocity of initial state nucleon.
- $p^0 \to \epsilon_{\mathbf{p}} \equiv \sqrt{m_N^2 + |\mathbf{p}|^2} - \epsilon_b$: introduces binding energy

- RFG model has two parameters: ϵ_b and p_F
  Model validity and parameters from quasielastic e-nuclei scattering
  Moniz, Sick, Whitney, Ficenec, Kephart, Trower, PRL **26**, 445 (1971)

Reminder

- Quasielastic scattering "folded" twice

$$\begin{array}{lcc}
\text{Quark:} & \nu_\ell + d \to \ell^- + u & \\
 & \Downarrow & \text{Form factor} \\
\text{Nucleon} & \nu_\ell + n \to \ell^- + p & \\
 & \Downarrow & \text{Nuclear model} \\
\text{Nucleus:} & \nu_\ell + \text{nucleus} \to \ell^- + \ldots &
\end{array}$$

- Experimental measurement are sensitive to **both** form factor uncertainty and nuclear modeling

The axial mass problem

- Neutrino scattering:

  $m_A^{\text{dipole}} = 1.35 \pm 0.17$ GeV

  MiniBooNE Collaboration

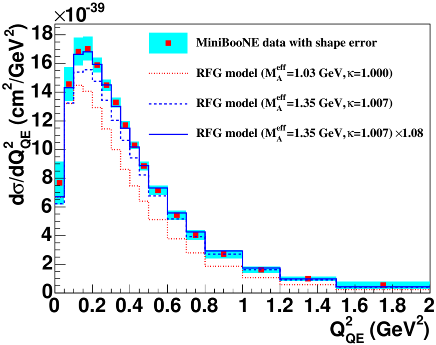
  PRD **81** (2010) 092005

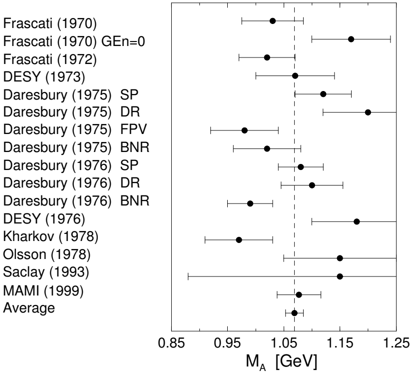
- Pion electro-prodcution:

  $m_A^{\text{dipole}} = 1.07 \pm 0.02$ GeV

  Bernard, Elouadrhiri, Meissner

  J. Phys. G 28, R1 (2002)





Both use dipole ansatz for axial form factor

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\text{dipole}})^2]^{-2}$$

The axial mass problem

- Axial mass $m_A^{\rm dipole} = 1.35 \pm 0.17$ GeV
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
- Similar result from other recent ν experiments
- K2K SciFi: $m_A^{\rm dipole} = 1.20 \pm 0.12$ GeV
  [K2K Collaboration, PRD **74** 052002 (2006)]
- K2K SciBar $m_A^{\rm dipole} = 1.144 \pm 0.077({\rm fit})^{+0.078}_{-0.072}({\rm syst})$ GeV
  Espinal, Sanchez, AIP Conf. Proc. **967**, 117 (2007)
- Minos $m_A^{\rm dipole} = 1.19^{+0.09}_{-0.1}({\rm fit})^{+0.12}_{-0.14}({\rm syst})$ GeV
  [MINOS Collaboration, AIP Conf. Proc. **1189**, 133 (2009)]

- Nomad: $m_A^{\rm dipole} = 1.05 \pm 0.02 \pm 0.06$ GeV
  [NOMAD Collaboration, EPJ C **63**, 355 (2009)]
- Pion electro-prodcution: $m_A^{\rm dipole} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)
- ν experiments before 1990: $m_A^{\rm dipole} = 1.026 \pm 0.021$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)

The axial mass problem

- Axial mass $m_A^{\text{dipole}} = 1.35 \pm 0.17$ GeV
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
- Similar result from other recent ν experiments
- K2K SciFi: $m_A^{\text{dipole}} = 1.20 \pm 0.12$ GeV
  [K2K Collaboration, PRD **74** 052002 (2006)]
- K2K SciBar $m_A^{\text{dipole}} = 1.144 \pm 0.077(\text{fit})^{+0.078}_{-0.072}(\text{syst})$ GeV
  Espinal, Sanchez, AIP Conf. Proc. **967**, 117 (2007)
- Minos $m_A^{\text{dipole}} = 1.19^{+0.09}_{-0.1}(\text{fit})^{+0.12}_{-0.14}(\text{syst})$ GeV
  [MINOS Collaboration, AIP Conf. Proc. **1189**, 133 (2009)]

- Nomad: $m_A^{\text{dipole}} = 1.05 \pm 0.02 \pm 0.06$ GeV
  [NOMAD Collaboration, EPJ C **63**, 355 (2009)]
- Pion electro-prodcution: $m_A^{\text{dipole}} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)
- ν experiments before 1990: $m_A^{\text{dipole}} = 1.026 \pm 0.021$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)
- **What could be the source of the discrepancy?**

Theoretical studies focus on nuclear modeling

- Modify nuclear model
  [Butkevich, PRC **82**, 055501 (2010); Benhar, Coletti, Meloni, PRL **105**, 132301 (2010); Juszczak, Sobczyk, Zmuda, PRC **82**, 045502 (2010)]

- Include multi-nucleon emission
  [Martini, Ericson, Chanfray, Marteau
  PRC **80**, 065501 (2009), PRC **81**, 045502 (2010);
  Amaro, Barbaro, Caballero, Donnelly, Williamson
  PLB **696**, 151 (2011), PRD **84**, 033004 (2011);
  Nieves, Ruiz Simo, Vicente Vacas
  PRC **83**, 045501 (2011), arXiv:1106.5374]

- Modify G_M for bound nucleons but not G_E or F_A
  [Bodek, Budd, EPJ C **71**, 1726 (2011)]

- **All** use dipole form factor

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\rm dipole})^2]^{-2}$$

What is the axial mass?

- The axial mass is defined as

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2} q^2 + \dots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Everyone extracts $m_A^{\rm dipole}$ from

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\rm dipole})^2]^{-2}$$

What is the axial mass?

- The axial mass is defined as

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2} q^2 + \dots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Everyone extracts m_A^{dipole} from

$$F_A = F_A(0)\left[1 - q^2/(m_A^{\mathrm{dipole}})^2\right]^{-2}$$

- m_A^{dipole} is not m_A!

What is the axial mass?

- The axial mass is defined as

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2} q^2 + \ldots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Everyone extracts m_A^{dipole} from

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\mathrm{dipole}})^2]^{-2}$$

- m_A^{dipole} is not m_A!

- When extractions of m_A^{dipole} disagree is it
  - A problem of the use of the dipole model?
  - Real disagreement between experiments?

- Need to extract m_A in a model independent way!

Model independent extraction of m_A

- Need to extract m_A in a model independent way!
- How to do that?

Model independent extraction of m_A

- Need to extract m_A in a model independent way!
- How to do that?
- Let's look at a simpler problem: The charge radius of the proton

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p' - p$)

$$\langle N(p')|\sum_q e_q\,\bar{q}\gamma^\mu q|N(p)\rangle = \bar{u}(p')\left[\gamma^\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^N(q^2)q^\nu\right]u(p)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\langle r^2\rangle_E^p = 6\frac{dG_E^p}{dq^2}\bigg|_{q^2=0}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

Charge radius from atomic physics

$$\langle N(p')|\sum_q e_q\,\bar{q}\gamma^\mu q|N(p)\rangle = \bar{u}(p')\left[\gamma^\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^N(q^2)q^\nu\right]u(p)$$

- For a point particle amplitude for $p+\ell \to p+\ell$

$$\mathcal{M} \propto \frac{1}{q^2} \quad \Rightarrow \quad U(r) = -\frac{Z\alpha}{r}$$

- Including q^2 corrections from proton structure

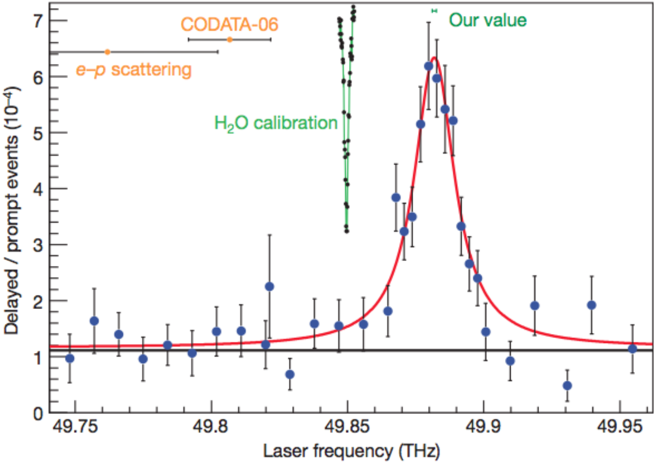
$$\mathcal{M} \propto \frac{1}{q^2}q^2 = 1 \quad \Rightarrow \quad U(r) = \frac{4\pi Z\alpha}{6}\delta^3(r)(r_E^p)^2$$

- Proton structure corrections $\left(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell\right)$

$$\Delta E_{r_E^p} \;=\; \frac{2(Z\alpha)^4}{3n^3}m_r^3(r_E^p)^2\delta_{\ell 0}$$

- **Muonic hydrogen can give the best measurement of $\mathbf{r_E^p}$!**

Charge radius from Muonic Hydrogen



- CREMA Collaboration measured for the **first time** $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ transition in Muonic Hydrogen [Pohl et al. Nature **466**, 213 (2010)]

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**
- We can also extract it from electron-proton scattering data
  What does the PDG say?

What does the PDG say?

K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010)

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$ep \to ep$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$ep \to ep$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR...	00		ep + Coul. corrections
0.847 ±0.008	MERGELL	96		ep + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz ep data
0.865 ±0.020	MCCORD	91		$ep \to ep$
0.862 ±0.012	SIMON	80		$ep \to ep$
0.880 ±0.030	BORKOWSKI	74		$ep \to ep$
0.810 ±0.020	AKIMOV	72		$ep \to ep$
0.800 ±0.025	FREREJACQ...	66		$ep \to ep$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$ep \to ep$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

[24] ESCHRICH 01 actually gives $\langle r^2 \rangle = (0.69 \pm 0.06 \pm 0.06)$ fm^2.

What does PDG say?

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	DOCUMENT ID		TECN	COMMENT
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$ep \to ep$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$ep \to ep$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR.	00		ep + Coul. corrections
0.847 ±0.008	MERGELL	96		ep + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz ep data
0.865 ±0.020	MCCORD	91		$ep \to ep$
0.862 ±0.012	SIMON	80		$ep \to ep$
0.880 ±0.030	BORKOWSKI	74		$ep \to ep$
0.810 ±0.020	AKIMOV	72		$ep \to ep$
0.800 ±0.025	FREREJACQ...	66		$ep \to ep$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$ep \to ep$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

[24] ESCHRICH 01 actually gives $\langle r^2 \rangle = (0.69 \pm 0.06 \pm 0.06)$ fm^2.

- What does PDG say?
  - $\approx$ 50 years of $e - p$ scattering data
  - r_E^p between 0.8 – 0.9 fm
  - Different data sets
  - Different extraction methods

"We do not use the following data for averages, fits, limits, etc."

- PDG refuses to say anything...
- What does the Data say?

Model independent extraction of the proton charge radius from electron scattering

Richard J. Hill, GP

PRD **82** 113005 (2010) [arXiv:1008.4619]

What does the Data say?

- First problem: no agreed data set
  Some work in recent years on combining data sets
  [Arrington et al. PRC **76**, 035205 (2007)]

What does the Data say?

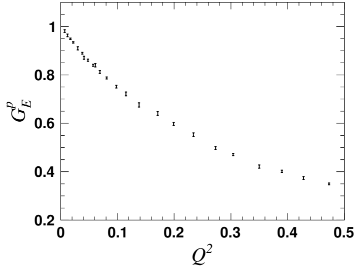
- First problem: no agreed data set
  Some work in recent years on combining data sets
  [Arrington et al. PRC **76**, 035205 (2007)]
- Second problem: How to extract r_E^p?

What does the Data say?

- First problem: no agreed data set
  Some work in recent years on combining data sets
  [Arrington et al. PRC **76**, 035205 (2007)]
- Second problem: How to extract r_E^p?
  Is this a problem? why not fit a straight line?

What does the Data say?

- First problem: no agreed data set
  Some work in recent years on combining data sets
  [Arrington et al. PRC **76**, 035205 (2007)]
- Second problem: How to extract r_E^p?
  Is this a problem? why not fit a straight line?



Data from [Arrington et al. PRC **76**, 035205 (2007)]

- We don't know the functional form of G_E^p

How to extract r_E^p?

- How to extract r_E^p from G_E^p? Usually use either

  1) model dependent form for G_E^p, e.g. poles+continuum form

     problem: how to estimate model dependence?

  2) A series expansion

How to extract r_E^p?

- How to extract r_E^p from G_E^p? Usually use either

  1) model dependent form for G_E^p, e.g. poles+continuum form

     problem: how to estimate model dependence?

  2) A series expansion

- There are several possibilities of series expansion

  1) Taylor series

$$G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \ldots,$$

How to extract r_E^p?

- How to extract r_E^p from G_E^p? Usually use either
  1) model dependent form for G_E^p, e.g. poles+continuum form

     problem: how to estimate model dependence?
  2) A series expansion
- There are several possibilities of series expansion
  1) Taylor series
  $$G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \ldots,$$
  2) Continued fraction [Sick PLB **576**, 62 (2003)]
  $$G_E^p(q^2) = \frac{1}{1 + \frac{a_1\, q^2}{1+\frac{a_2\, q^2}{1+\ldots}}}$$

How to extract r_E^p?

- How to extract r_E^p from G_E^p? Usually use either

  1) model dependent form for G_E^p, e.g. poles+continuum form

     problem: how to estimate model dependence?

  2) A series expansion

- There are several possibilities of series expansion

  1) Taylor series

$$G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \ldots,$$

  2) Continued fraction [Sick PLB **576**, 62 (2003)]

$$G_E^p(q^2) = \frac{1}{1 + \frac{a_1\, q^2}{1+\frac{a_2\, q^2}{1+\ldots}}}$$

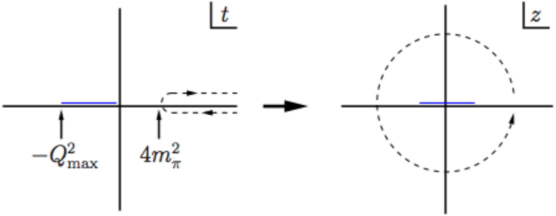
  3) z expansion

z expansion

- Analytic properties of $G_E^p(t)$ are known
  $G_E^p(t)$ is analytic outside a cut $t \in [4m_\pi^2, \infty]$
  $e - p$ scattering data is in $t < 0$ region
- We can map the domain of analyticity onto the unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



- Expand G_E^p in a Taylor series in z: $G_E^p(q^2) = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$

z expansion

- Standard tool in analyzing **meson** transition form factors
  - Bourrely et al. NPB **189**, 157 (1981)
  - Boyd et al. arXiv:hep-ph/9412324
  - Boyd et al. arXiv:hep-ph/9508211
  - Lellouch arXiv:hep-ph/9509358
  - Caprini et al. arXiv:hep-ph/9712417
  - Arnesen et al. arXiv:hep-ph/0504209
  - Becher et al. arXiv:hep-ph/0509090
  - Hill arXiv:hep-ph/0607108
  - Bourrely et al. arXiv:0807.2722 [hep-ph]
  - Bharucha et al. arXiv:1004.3249 [hep-ph]
  - ...
- Not applied to **nucleon** form factors before

Comparison of series expansions

- Does it matter which expansion we use? Let's compare!

Comparison of series expansions

- Does it matter which expansion we use? Let's compare!
- Use data sets tabulated by Rosenfelder [arXiv:nucl-th/9912031] with $Q^2 < 0.04\,\mathrm{GeV}^2$, fit the following ($t_{\rm cut} = 4m_\pi^2$)

Comparison of series expansions

- Does it matter which expansion we use? Let's compare!
- Use data sets tabulated by Rosenfelder [arXiv:nucl-th/9912031] with $Q^2 < 0.04\,\mathrm{GeV}^2$, fit the following ($t_{\rm cut} = 4m_\pi^2$)

  1) Taylor

$$G_E^p(q^2) = 1 + a_1 \frac{q^2}{t_{\rm cut}} + a_2 \left(\frac{q^2}{t_{\rm cut}}\right)^2 + \dots$$

Comparison of series expansions

- Does it matter which expansion we use? Let's compare!
- Use data sets tabulated by Rosenfelder [arXiv:nucl-th/9912031] with $Q^2 < 0.04\,\mathrm{GeV}^2$, fit the following ($t_{\rm cut} = 4m_\pi^2$)

  1) Taylor

$$G_E^p(q^2) = 1 + a_1 \frac{q^2}{t_{\rm cut}} + a_2 \left(\frac{q^2}{t_{\rm cut}}\right)^2 + \ldots$$

  2) Continued fraction

$$G_E^p(q^2) = \frac{1}{1 + a_1 \frac{q^2/t_{\rm cut}}{1 + a_2 \frac{q^2/t_{\rm cut}}{1+\ldots}}}$$

Comparison of series expansions

- Does it matter which expansion we use? Let's compare!
- Use data sets tabulated by Rosenfelder [arXiv:nucl-th/9912031] with $Q^2 < 0.04\,\mathrm{GeV}^2$, fit the following ($t_{\rm cut} = 4m_\pi^2$)
  1) Taylor
  $$G_E^p(q^2) = 1 + a_1\frac{q^2}{t_{\rm cut}} + a_2\left(\frac{q^2}{t_{\rm cut}}\right)^2 + \ldots$$
  2) Continued fraction
  $$G_E^p(q^2) = \frac{1}{1 + a_1\frac{q^2/t_{\rm cut}}{1+a_2\frac{q^2/t_{\rm cut}}{1+\ldots}}}$$
  3) z expansion
  $$G_E^p(q^2) = 1 + a_1 z(q^2) + a_2 z^2(q^2) + \ldots$$

Comparison of series expansions

- Does it matter which expansion we use? Let's compare!
- Use data sets tabulated by Rosenfelder [arXiv:nucl-th/9912031] with $Q^2 < 0.04\,\mathrm{GeV}^2$, fit the following ($t_{\rm cut} = 4m_\pi^2$)

1) Taylor

$$G_E^p(q^2) = 1 + a_1 \frac{q^2}{t_{\rm cut}} + a_2 \left(\frac{q^2}{t_{\rm cut}}\right)^2 + \ldots$$

2) Continued fraction

$$G_E^p(q^2) = \frac{1}{1 + a_1 \frac{q^2/t_{\rm cut}}{1 + a_2 \frac{q^2/t_{\rm cut}}{1+\ldots}}}$$

3) z expansion

$$G_E^p(q^2) = 1 + a_1 z(q^2) + a_2 z^2(q^2) + \ldots$$

4) z expansion with a constraint on a_k: $|a_k| \le 10$

Comparison of series expansions

r_E^p in $10^{-18}m$

polynomial

continued fraction

z expansion (no bound)

z expansion ($|a_k| \leq 10$)

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\max} = 1$
polynomial	836^{+8}_{-9}
continued fraction	882^{+10}_{-10}
z expansion (no bound)	918^{+9}_{-9}
z expansion ($\lvert a_k \rvert \leq 10$)	918^{+9}_{-9}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\rm max} = 1$	2
polynomial	836^{+8}_{-9}	867^{+23}_{-24}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}
z expansion ($\lvert a_k\rvert \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\max} = 1$	2	3
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	$-$
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}
z expansion ($\lvert a_k\rvert \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\max} = 1$	2	3	4
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}	959^{+85}_{-93}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}	1022^{+102}_{-114}
z expansion ($\|a_k\| \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}	880^{+39}_{-61}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\max} = 1$	2	3	4	5
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}	959^{+85}_{-93}	1122^{+122}_{-137}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–	–	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}	1022^{+102}_{-114}	1193^{+152}_{-174}
z expansion ($\lvert a_k \rvert \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}	880^{+39}_{-61}	880^{+39}_{-62}

Comparison of series expansions

r_E^p in $10^{-18} m$

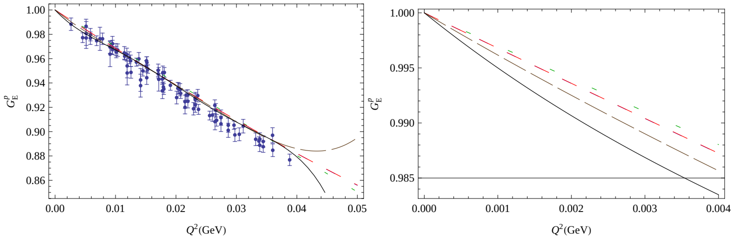
	$k_{\rm max} = 1$	2	3	4	5
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}	959^{+85}_{-93}	1122^{+122}_{-137}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–	–	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}	1022^{+102}_{-114}	1193^{+152}_{-174}
z expansion ($\lvert a_k \rvert \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}	880^{+39}_{-61}	880^{+39}_{-62}

Conclusions:

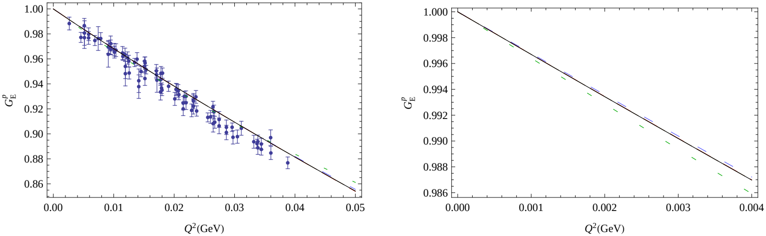
- Fit with two parameters agree well
- As we increase $k_{\rm max}$ the errors for the first three fits grow
- For the continued fraction fit for $k_{\rm max} > 3$ the slope is not positive
- To get a meaningful answer we must constrain a_k. How?

Comparison of Taylor and constrained z fits

- Taylor fit



- Constrained z fit

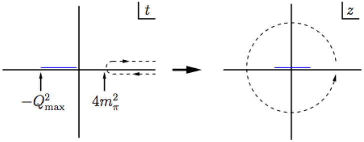


See also:

"Constrained curve fitting" : Lepage et al. Nucl.Phys.Proc.Suppl. 106 (2002) 12-20

Analytic structure and a_k

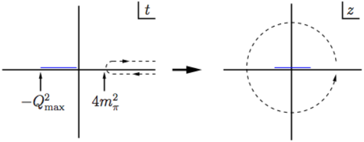
$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$



- Analytic structure implies:
  Information about $\mathrm{Im}\, G_E^p(t + i0) \Rightarrow$ information about a_k

Analytic structure and a_k

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$



- Analytic structure implies:
  Information about $\mathrm{Im}\, G_E^p(t + i0) \Rightarrow$ information about a_k
- $G(t) = \sum_{k=0}^{\infty} a_k\, z(t)^k$, z^k are orthogonal over $|z| = 1$

$$\begin{aligned} a_0 &= G(t_0) \\ a_k &= \frac{2}{\pi} \int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0} \sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}}\, \mathrm{Im}\, G(t) \sin[k\theta(t)]\,, \quad k \geq 1 \\ \sum_k a_k^2 &= \frac{1}{\pi} \int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0} \sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}} |G|^2 \end{aligned}$$

- How to constrain $\mathrm{Im}\, G(t)$?

Size of a_k: Summary

- We study the size of a_k using
  - vector dominance ansatz
  - $\pi\pi$ continuum
  - $e^+e^- \to N\bar{N}$ data

Size of a_k: Summary

- We study the size of a_k using
  - vector dominance ansatz
  - $\pi\pi$ continuum
  - $e^+e^- \to N\bar{N}$ data

- In all of the above $|a_k| \leq 10$ appears very conservative

Size of a_k: Summary

- We study the size of a_k using
  - vector dominance ansatz
  - $\pi\pi$ continuum
  - $e^+e^- \to N\bar{N}$ data

- In all of the above $|a_k| \leq 10$ appears very conservative

  Final results are presented for both $|a_k| \leq 5$ and $|a_k| \leq 10$

Size of a_k: Summary

- We study the size of a_k using
  - vector dominance ansatz
  - $\pi\pi$ continuum
  - $e^+e^- \to N\bar{N}$ data

- In all of the above $|a_k| \leq 10$ appears very conservative

  Final results are presented for both $|a_k| \leq 5$ and $|a_k| \leq 10$

- We extract r_E^p using
  - Low Q^2 proton data
  - Low + High Q^2 proton data
  - proton and neutron data
  - proton, neutron and $\pi\,\pi$ data

Results

- Using proton low: $Q^2 < 0.04\,\mathrm{GeV}^2$ scattering data from Rosenfelder [arXiv:nucl-th/9912031], we find

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

- Rosenfelder gets

$$r_E^p = 0.880 \pm 0.015\,\mathrm{fm}$$

  from the same data!

- Conclusion: not using model independent approach underestimates the error by a factor of two!

Results

- Proton low: $Q^2 < 0.04\,\mathrm{GeV}^2$

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

- Proton high: $Q^2 < 0.5\,\mathrm{GeV}^2$

$$r_E^p = 0.870 \pm 0.023 \pm 0.012\,\mathrm{fm}$$

- Proton and neutron data

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007\,\mathrm{fm}$$

- Proton, neutron and $\pi\,\pi$ data

$$r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$$

The recent discrepancy

- Based on a model-independent approach
  using scattering data from proton, neutron and $\pi\pi$
  [Hill, GP PRD **82** 113005 (2010)]
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- CODATA value (extracted mainly from electronic hydrogen)
  [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm

- Lamb shift in muonic hydrogen
  [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm

The recent discrepancy

- Based on a model-independent approach
  using scattering data from proton, neutron and $\pi\,\pi$
  [Hill, GP PRD **82** 113005 (2010)]
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- CODATA value (extracted mainly from electronic hydrogen)
  [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm

- Lamb shift in muonic hydrogen
  [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm

- Theoretical treatment for muonic hydrogen is lacking
  [Richard J. Hill, GP PRL **107** 160402 (2011)]

Model independent determination of the axial mass parameter in quasielastic neutrino-nucleon scattering

Bhubanjyoti Bhattacharya, Richard J. Hill, GP

PRD **84** 073006 (2011) [arXiv:1108.0423]

Goal

- Extract m_A in a model independent way: z expansion
- Following the charge radius analysis: $|a_k| \leq 5$ and $|a_k| \leq 10$
- Extract m_A from fit to MiniBooNE data for $d\sigma/dE_\mu d\cos\theta_\mu$
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
- Mostly follow MiniBooNE's analysis: use RFG as nuclear model

Fit Details

- Error matrix: $E_{ij} = (\delta\sigma_i)^2\delta_{ij} + (\delta N)^2\sigma_i\sigma_j$
- $\sigma_i = (d\sigma/dE_\mu d\cos\theta_\mu)\Delta E_\mu \Delta\cos\theta_\mu$
- $\delta\sigma_i$ shape uncertainty
- $\delta N = 0.107$ normalization error

- Minimize $\chi^2 = \sum_{ij}(\sigma_i^{\rm expt.} - \sigma_i^{\rm theory})E_{ij}^{-1}(\sigma_j^{\rm expt.} - \sigma_j^{\rm theory})$ to find best fit for m_A, error from $\Delta\chi^2 = 1$

- Use BBA2003 parametrization of F_1 and F_2 [Budd, Bodek, Arrington, arXiv:hep-ex/0308005]

Binding energy

- We use $\epsilon_b = 25$ MeV, as extracted from e-nuclei scattering data
  Moniz, Sick, Whitney, Ficenec, Kephart, Trower, PRL **26**, 445 (1971)

- Value different from MiniBooNE analysis: $\epsilon_b = 34 \pm 9$ MeV

- Fitting ϵ_b to MiniBooNE data we find $\epsilon_b = 28 \pm 3$ MeV

Q^2 cut

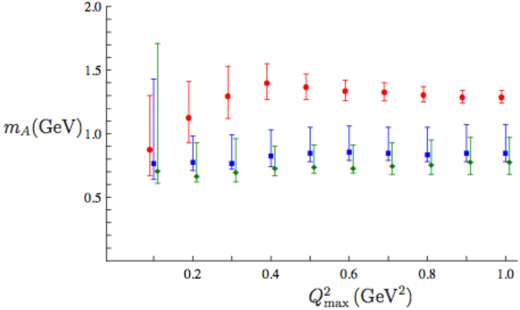
- The slope at $q^2 = 0$ is mostly sensitive to low-Q^2 data
- Assuming free nucleon can determine Q^2 from E_μ and $\cos\theta_\mu$
  No longer true when including nuclear effects
- As a proxy to Q^2 we use

$$
\begin{aligned}
Q^2_{\rm rec} &= 2E_\nu^{\rm rec} E_\mu - 2E_\nu^{\rm rec}\sqrt{E_\mu^2 - m_\mu^2}\cos\theta_\mu - m_\mu^2 \\
E_\nu^{\rm rec} &= \frac{m_N E_\mu - m_\mu^2/2}{m_N - E_\mu + \sqrt{E_\mu^2 - m_\mu^2}\cos\theta_\mu}
\end{aligned}
$$

- $Q^2_{\rm rec}$ coincides with K2K's $Q^2_{\rm rec}$ for $\epsilon_b \to 0$
  K2K Collaboration PRD **74**, 052002 (2006)
  and MiniBooNE's $Q^2_{\rm QE}$ for $\epsilon_b \to 0$ and $m_p = m_n = m_N$

Neutrino: Model independent approach

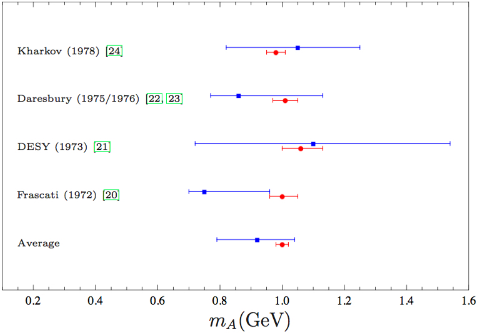
- Our z expansion fit to MiniBooNE data (Assuming RFG):
  Red: dipole, Blue: z, $|a_k| \leq 5$, Green: z, $|a_k| \leq 10$



- Our fit using z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV
  Our fit using dipole model: $m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV
  MiniBooNE's fit: $m_A^{\rm dipole} = 1.35 \pm 0.17$ GeV

Pion Electro-production: Model independent approach

- Is there a discrepancy with pion electro-production data?
  Red: dipole, Blue: z, $|a_k| \leq 5$



- Our fit using z expansion: $m_A = 0.92^{+0.12}_{-0.13} \pm 0.08$ GeV
  Our fit using dipole model: $m_A^{\text{dipole}} = 1.00 \pm 0.02$ GeV
  Bernard et. al. fit using dipole model: $m_A^{\text{dipole}} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)

Model independent approach

- MiniBooNE (Assuming RFG):

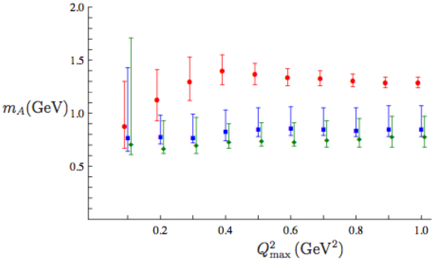
  $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV

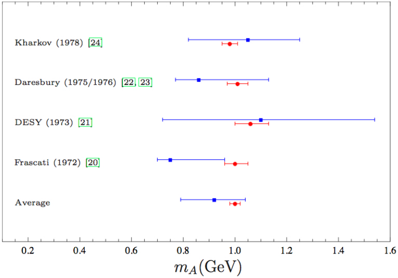
  $m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV

- Pion electro-prodcution:

  $m_A = 0.92^{+0.12}_{-0.13} \pm 0.08$ GeV

  $m_A^{\rm dipole} = 1.00 \pm 0.02$ GeV

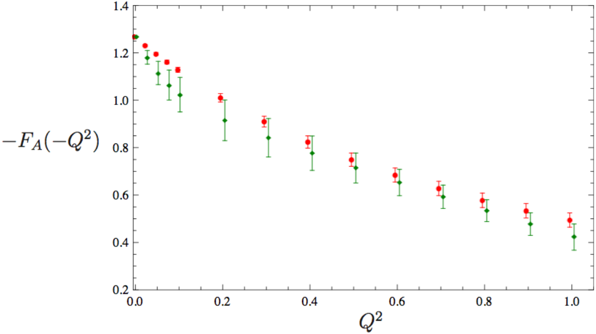




Discrepancy is an artifact of the use of the dipole form factor!

Going beyond m_A

- We can also extract F_A directly from MiniBooNE data
  Red: dipole, Green : z, $|a_k| \leq 10$



Error on F_A underestimated in the dipole model

Conclusions and Outlook

Conclusions

- Recent m_A^{dipole} extractions from quasielastic $\nu - N$ scattering are typically higher than pre-1990 ν experiments and pion electro-production data

- We presented model-independent extraction of the axial mass from quasielastic $\nu - N$ scattering data using the z expansion

- MiniBooNE (Assuming RFG):
  $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV $\quad m_A^{\mathrm{dipole}} = 1.29 \pm 0.05$ GeV

- Pion electro-prodcution:
  $m_A = 0.92^{+0.12}_{-0.13} \pm 0.08$ GeV $\quad m_A^{\mathrm{dipole}} = 1.00 \pm 0.02$ GeV

- As far as M_A is concerned:
  discrepancy is an artifact of the use of the dipole form factor!

Future directions

- Extract m_A from other ν experiments, e.g. Minerνa
- Is m_A consistent between experiments?
- m_A from pion electro-production data, extrapolated from soft π limit

  Extract m_A in a model-independent way
- ν experiments need F_A, extract it from another source
- After F_A is under control, discuss nuclear models