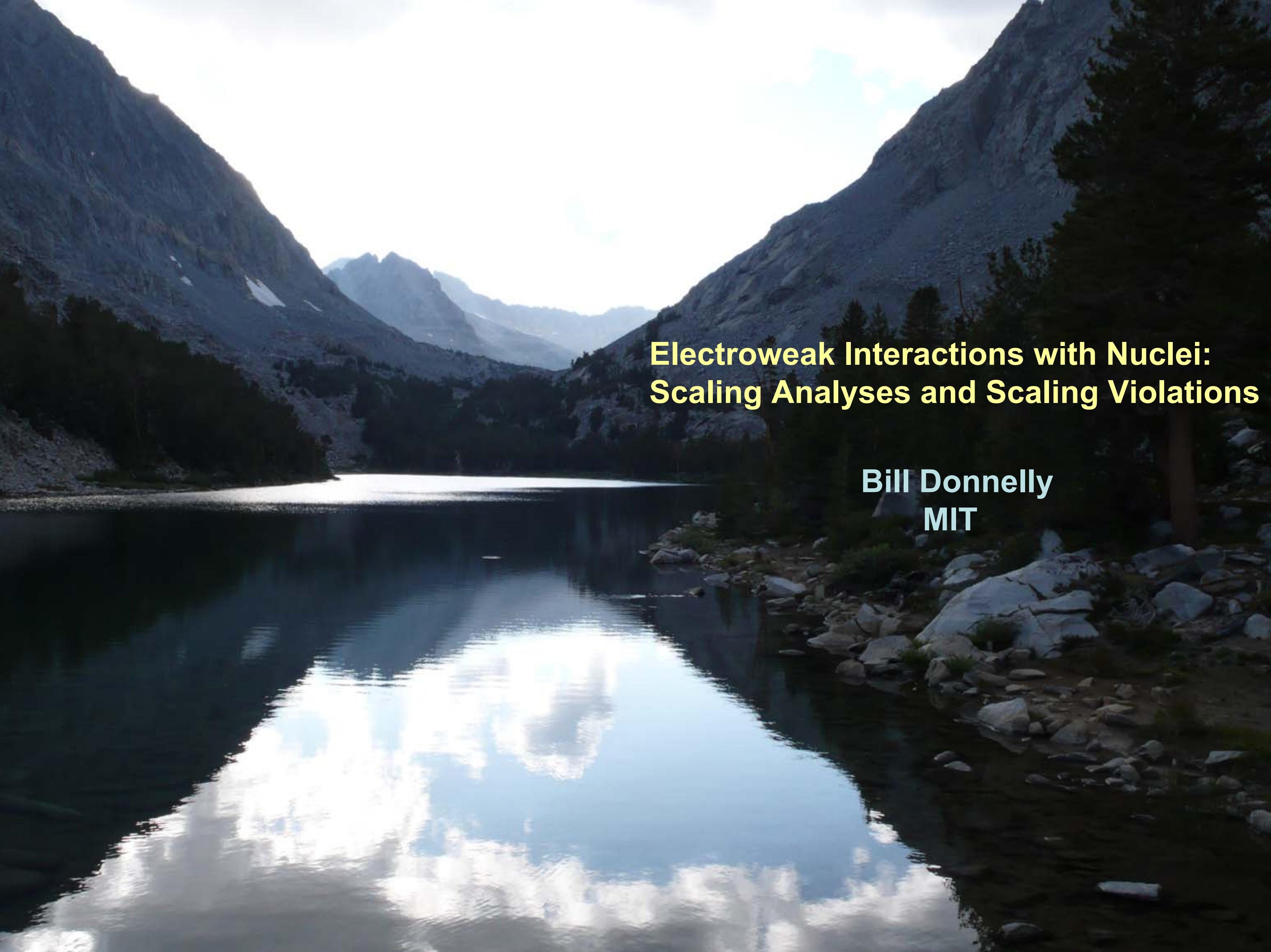

Electroweak Interactions with Nuclei: Scaling Analyses and Scaling Violations

Bill Donnelly
MIT

Outline:

- Introduction
- Scaling of the 1^{st} kind (y-scaling)
- Scaling of the 2^{nd} kind
- Scaling of the 0^{th} kind
  and Superscaling
- Non-QE scaling
  - Inelastic scattering
  - 2p-2h MEC effects
- Predicting ν cross sections using scaling
  and scaling of the 3^{rd} kind

Scaling phenomena are seen in many quantum many-body systems:

- Condensed matter physics (electron scattering, neutron scattering)
- Nuclear physics (lepton scattering, hadron scattering from nucleons)
- Particle physics (lepton-parton scattering)

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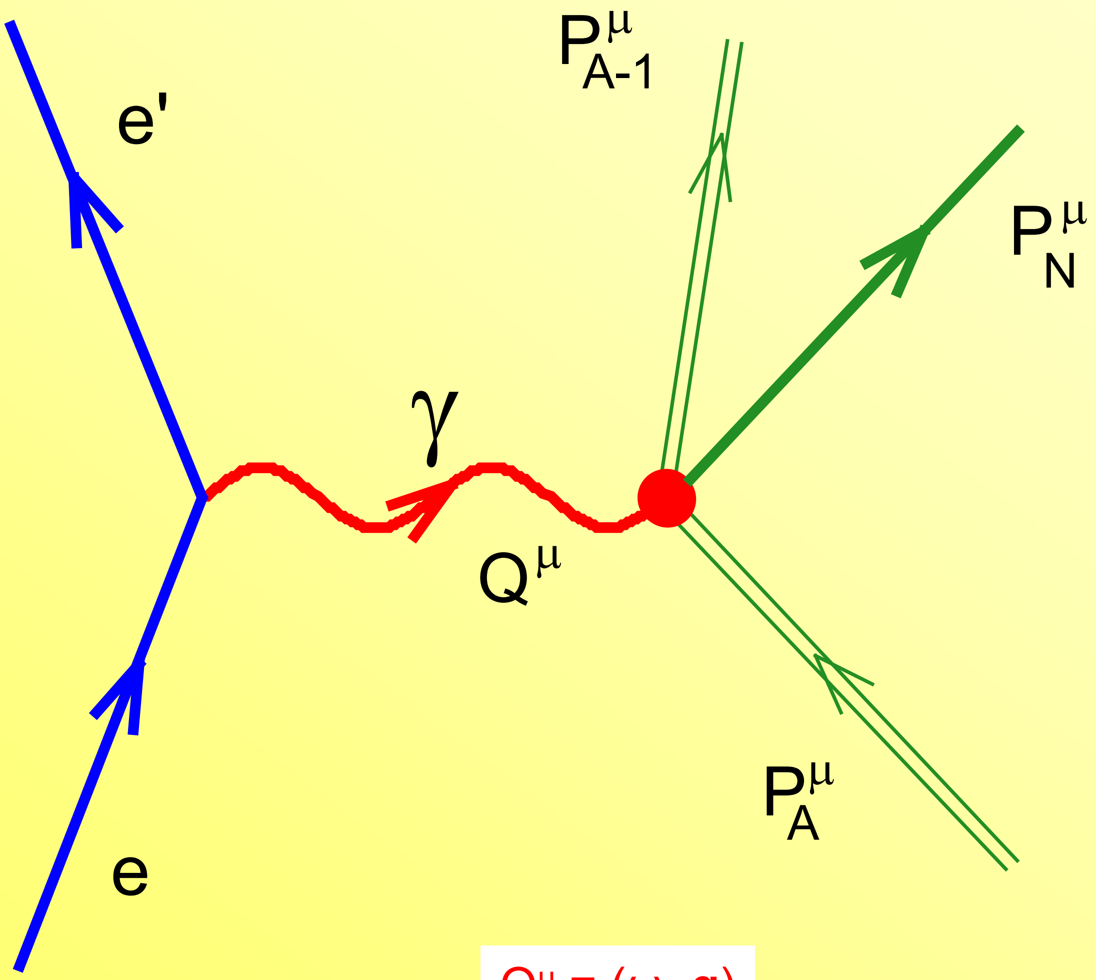
… in this talk I will focus on lepton scattering from nuclei

Begin by assuming that QE scattering is dominated by (e,e´N):



$Q^{\mu} = (\omega, \mathbf{q})$

The daughter nucleus has 4-momentum

$$P^{\mu}_{A-1} = (E_{A-1}, \mathbf{p}_{A-1}) = Q^{\mu} + P^{\mu}_{A} - P^{\mu}_{N}$$

In the lab. system we define the **missing momentum**

$$p = |\mathbf{p}| \equiv |\mathbf{p}_N - \mathbf{q}| = |\mathbf{p}_{A-1}|$$

and an “excitation energy” (essentially **missing energy** – separation energy)

$$\mathcal{E}(p) \equiv \sqrt{(M_{A-1})^2 + p^2} - \sqrt{(M^0_{A-1})^2 + p^2}$$

where

$$M^0_{A-1} = M^0_A - m_N + E_s$$

with E_s the separation energy and M^0_{A-1} the daughter rest mass

Energy conservation gives

$$M_A^0 + \omega = E_N + E_{A-1}$$
$$= \sqrt{m_N{}^2 + p_N{}^2} + E_{A-1}^0 + \mathcal{E}$$
$$= \sqrt{m_N{}^2 + (\mathbf{q} + \mathbf{p})^2} + \sqrt{(M_{A-1}^0)^2 + p^2} + \mathcal{E}$$

which can be turned around to yield an expression for the excitation energy:

$$\mathcal{E} = M_A^0 + \omega - \sqrt{(M_{A-1}^0)^2 + p^2} - \sqrt{m_N{}^2 + q^2 + p^2 + 2pq\cos\theta}$$

One can let the angle between *p* and *q* vary over all values and impose the constraints

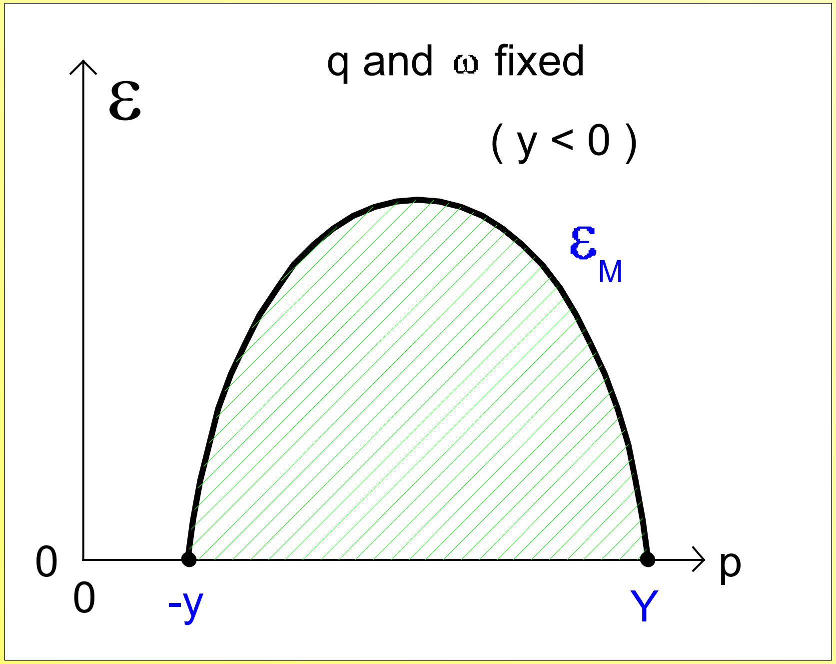
$$p \geq 0$$

$$\varepsilon \geq 0$$

to find the allowed region in the missing-energy, missing-momentum plane. When

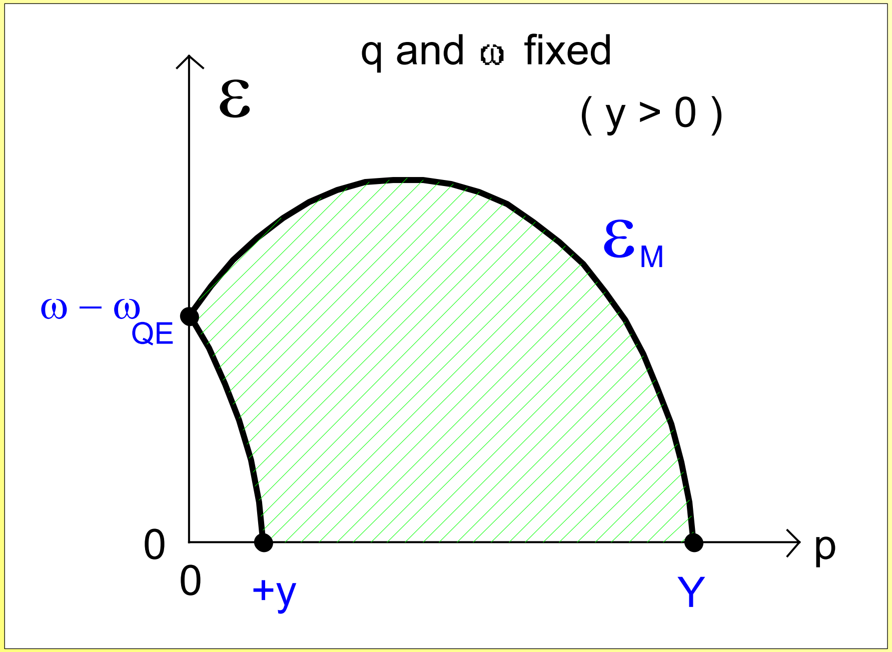
$$\omega < \omega_{QE} = \left|Q^2\right| / 2m_N$$

one finds



… and when

$$\omega > \omega_{QE} = \left|Q^2\right| / 2m_N \qquad \text{one has}$$



where one has the smallest and largest values of the missing momentum at zero excitation energy occurring at

$$y = \frac{1}{2W^2}\left[\alpha - \beta\right]$$

$$Y = \frac{1}{2W^2}\left[\alpha + \beta\right]$$

with

$$W = \sqrt{(M_A^0 + \omega)^2 - q^2} \geq W_T = M_{A-1}^0 + m_N$$

$$\alpha = (M_A^0 + \omega)\sqrt{W^2 - W_T^2}\sqrt{W^2 - (W_T - 2m_N)^2}$$

$$\beta = q\left[W^2 + W_T\left(W_T - 2m_N\right)\right]$$

The so-called **y-scaling variable** is approximately given by

$$y \cong \sqrt{\nu(2m_N + \nu)} - q$$

$$\nu \equiv \omega - E_s$$

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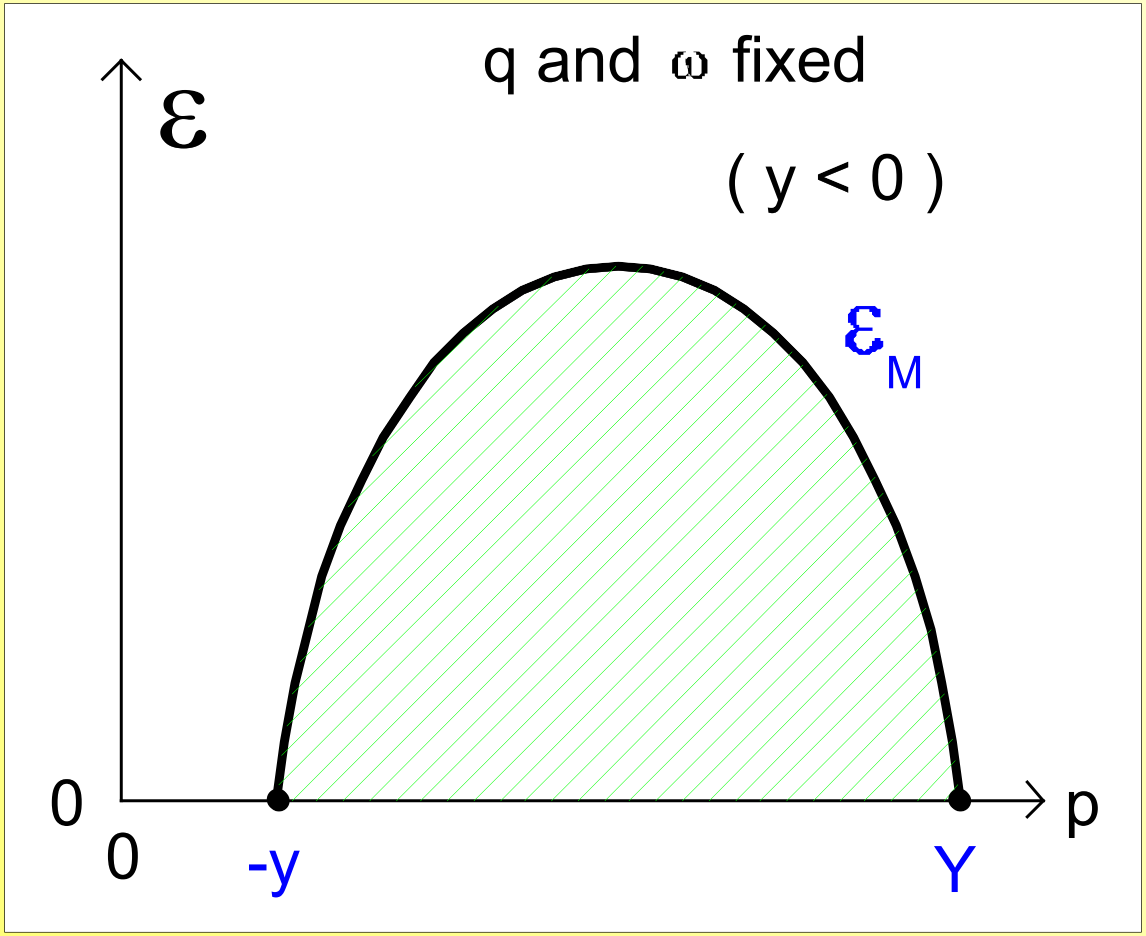
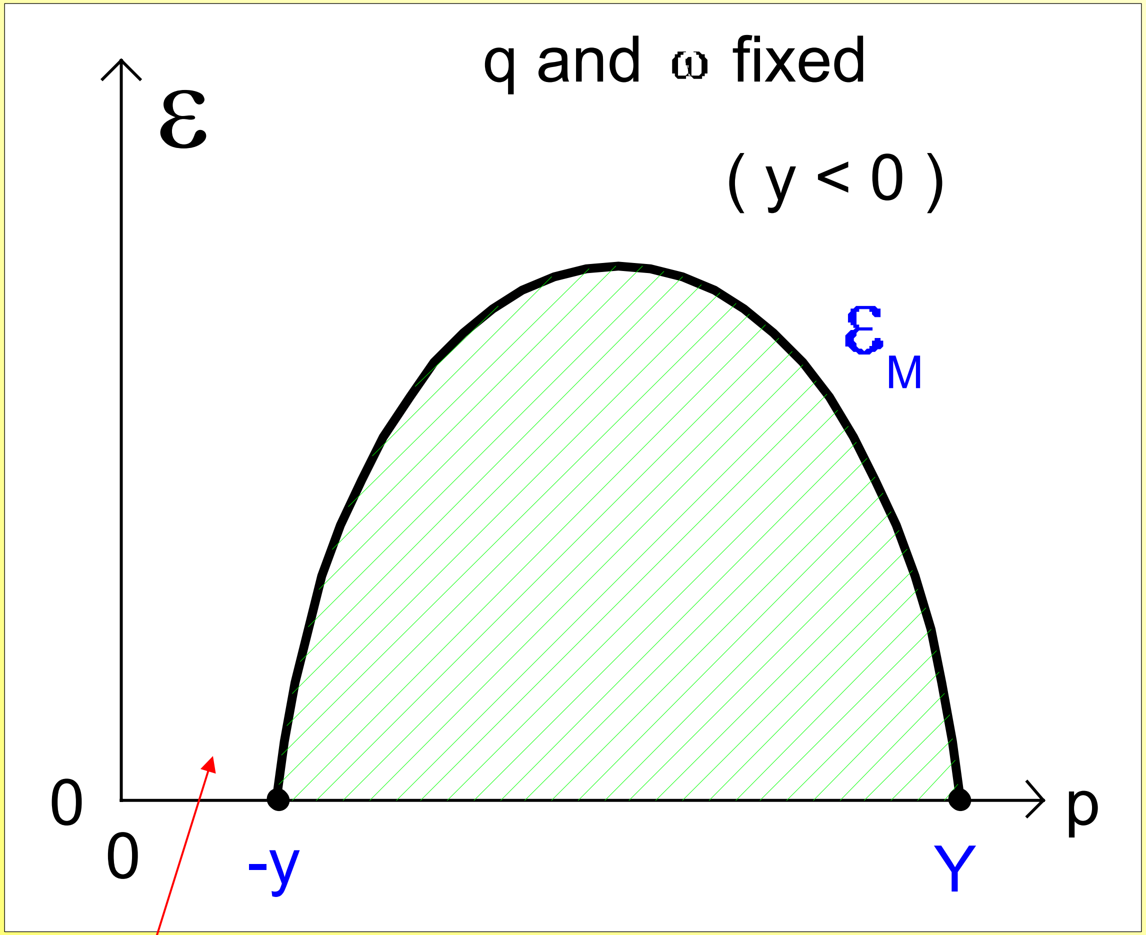
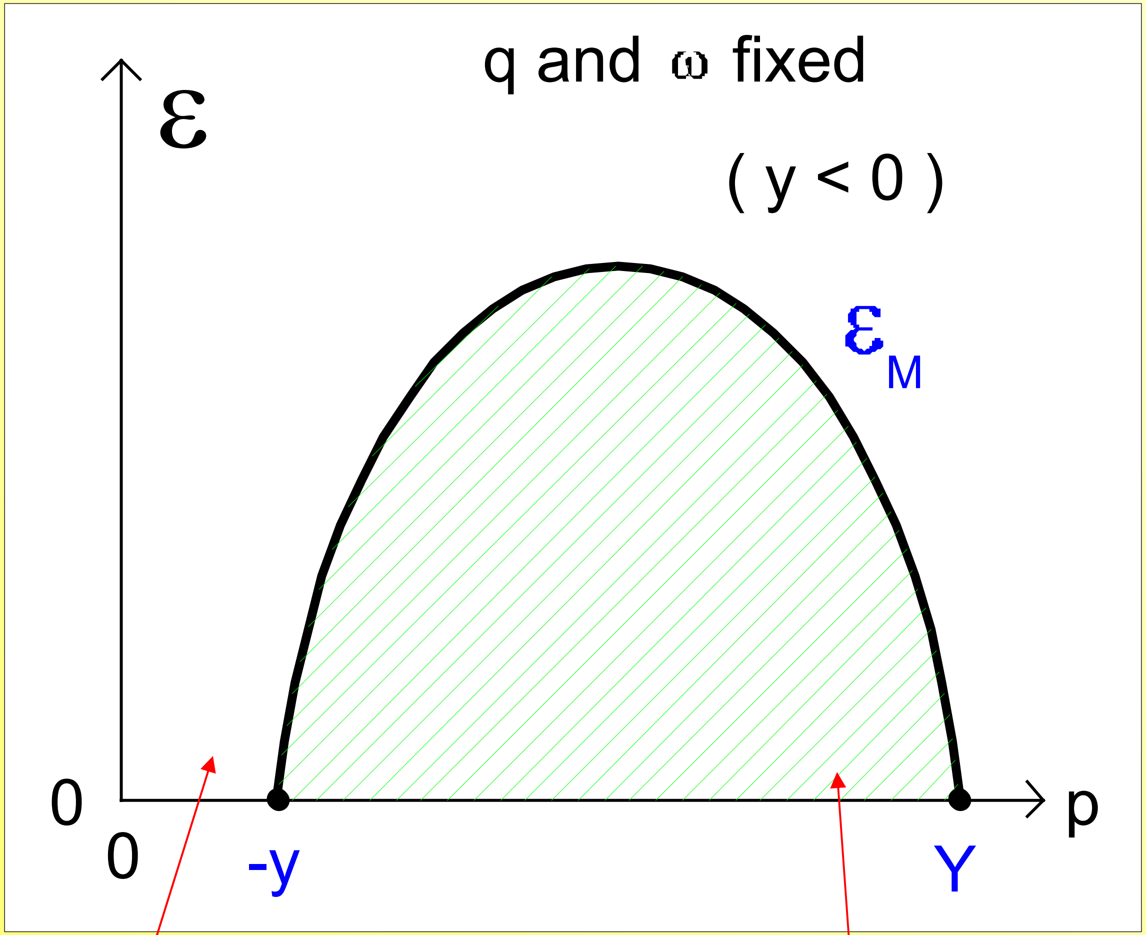
- First, one uses ***(q,y)*** rather than ***(q,ω)*** for the functional dependence of the inclusive cross section. The inclusive cross section is assumed to be the sum of the integrals over the semi-inclusive (e,e'p) and (e,e'n) cross sections, *i.e.,* over the momentum of the ejected nucleon $\mathbf{p}_N$. These can be turned into integrals over p and ε covering the regions discussed above.


q and ω fixed
ε
(y < 0)
ε_M
0
0
-y
Y
p



The semi-inclusive cross section is

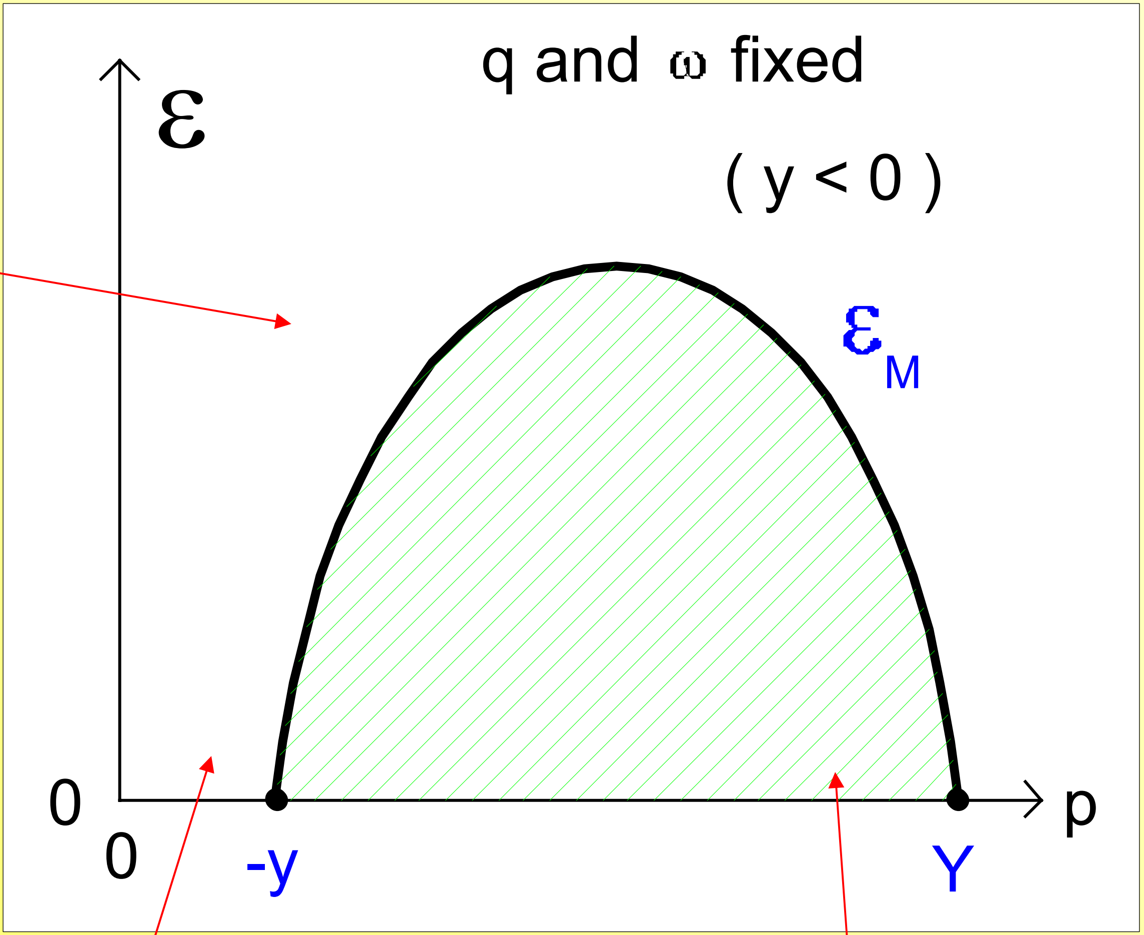
typically largest at small **p** and ε



The semi-inclusive cross section is typically largest at small **p** and ε

… and is very small at large **p** and small ε

For given **y<0** the region at small **p**, but high $\mathcal{E}$ is inaccessible



The semi-inclusive cross section is typically largest at small **p** and $\mathcal{E}$

… and is very small at large **p** and small $\mathcal{E}$

- First, one uses ***(q,y)*** rather than ***(q,ω)***

- Second, one notes that the typical parametrizations for the off-shell single-nucleon cross sections (functions of q, ω, p, ε, and ϕ_N) vary rather slowly as functions of (p, ε) for fixed (q, ω, ϕ_N). This suggests integrating over ϕ_N (leaving only L and T responses) and then removing the result evaluated at an "optimal" choice of p and ε.

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What is optimal?

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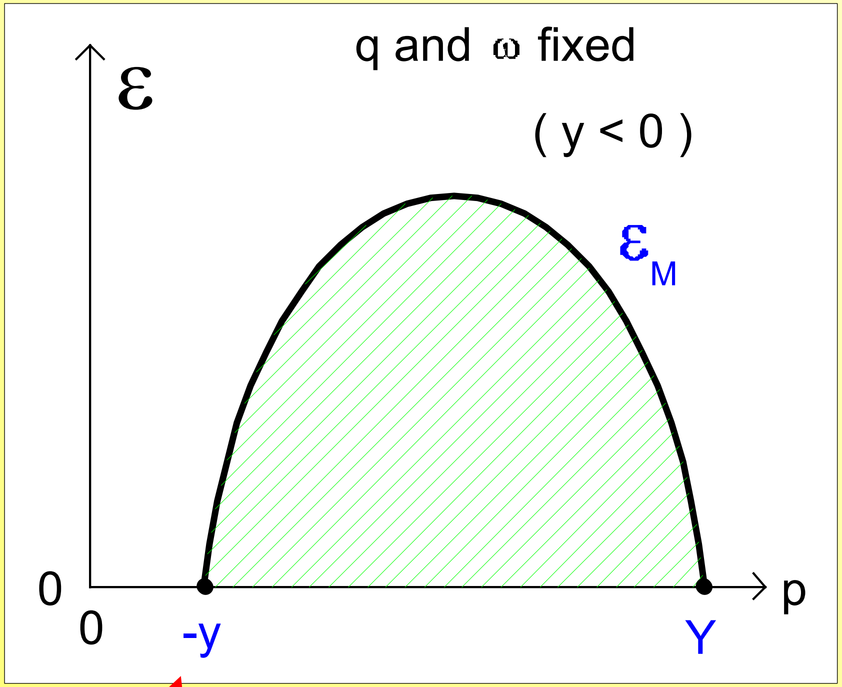
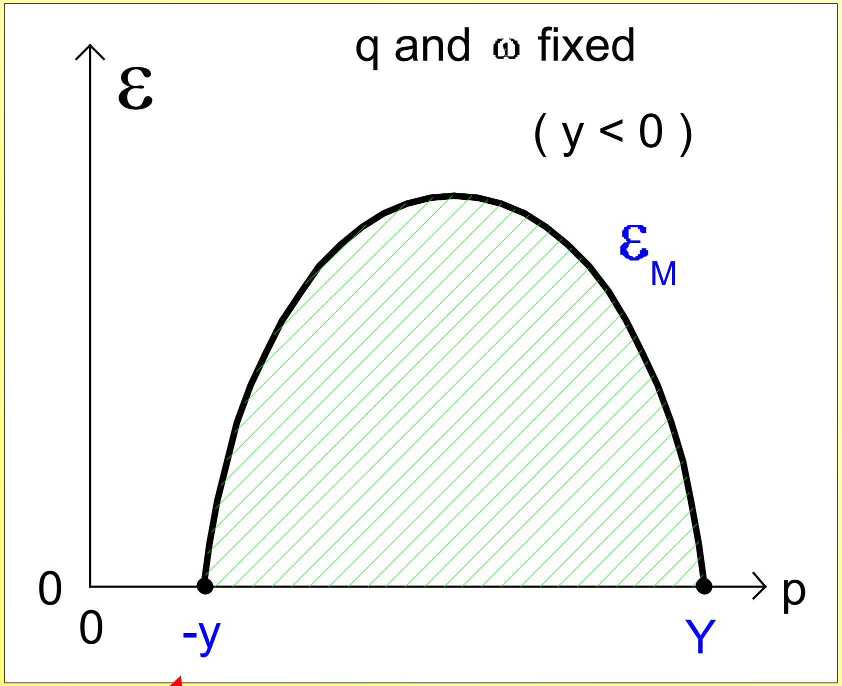
What is optimal?

From the discussions above one is led to a choice such as the one made in many analyses of scaling, namely, set **p to |y|** and ε **to 0**:

$$\Sigma_{eN}^{eff} = \frac{1}{A}\left[Z\overline{\sigma}_{ep}^{elastic} + N\overline{\sigma}_{en}^{elastic} \right]_{p=|y|,\,\varepsilon=0}$$



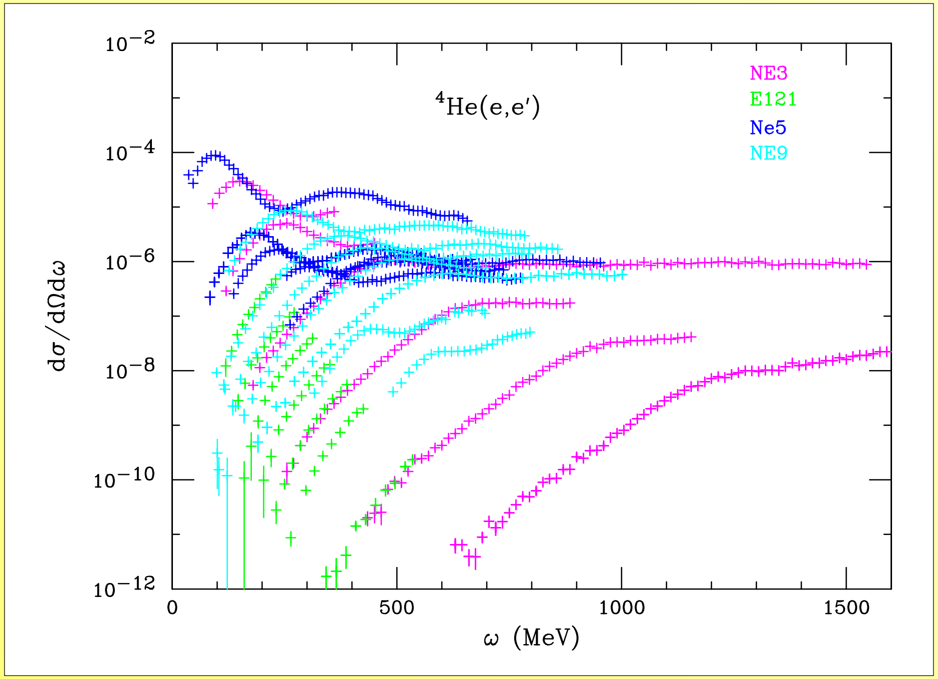
Evaluate the single-nucleon cross section at this point and remove from integral



Evaluate the single-nucleon cross section at this point and remove from integral

… then, dividing by the effective single-nucleon cross section leads to the definition of the **scaling function**:

$$F(q,y) \equiv \frac{d^2\sigma / d\Omega_e d\omega}{A\Sigma_{eN}^{eff}}$$



Example using ^{4}He data from SLAC:

when the inclusive cross section for various beam energies and electron scattering angles

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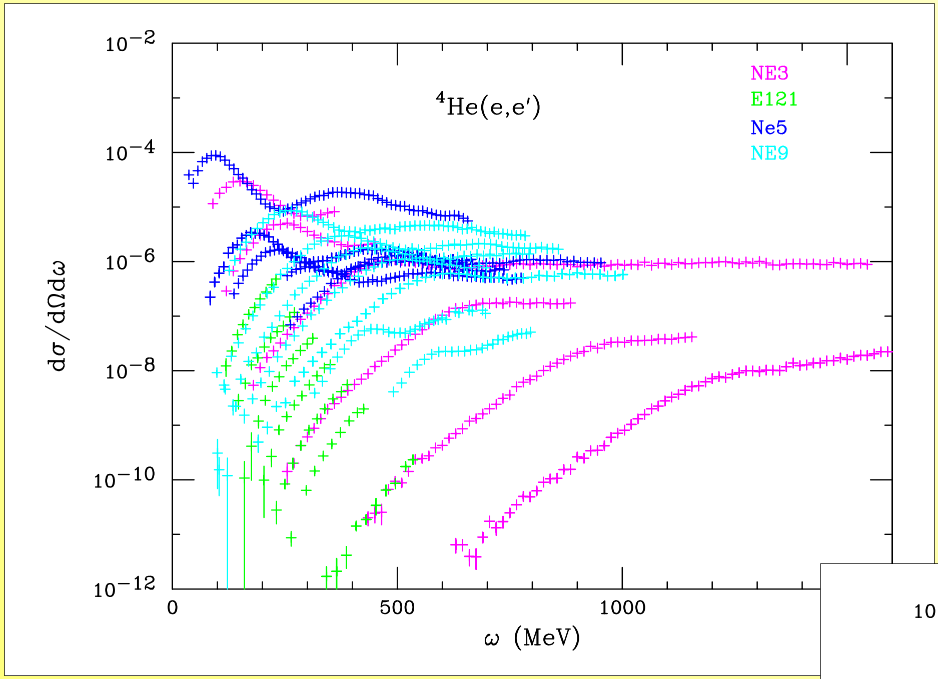
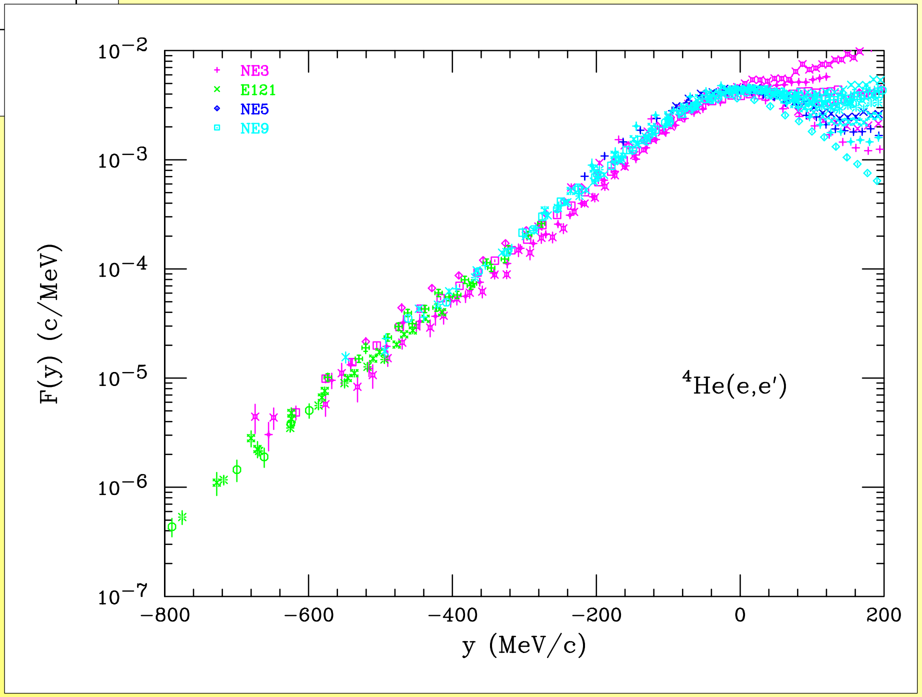


when the inclusive cross section for various beam energies and electron scattering angles

is used to obtain the function *F(q,y)*, and this is plotted as a function of *y* for various values of *q*, one finds



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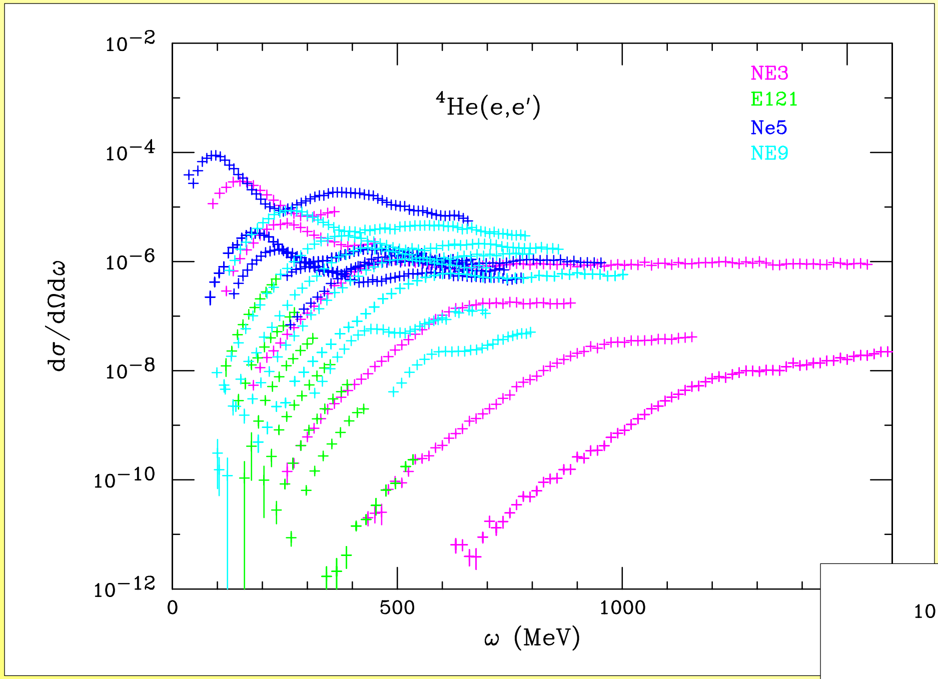

when the inclusive cross section for various beam energies and electron scattering angles

$\omega < \omega_{QE}$ (x > 1)

$\omega > \omega_{QE}$ (x < 1)

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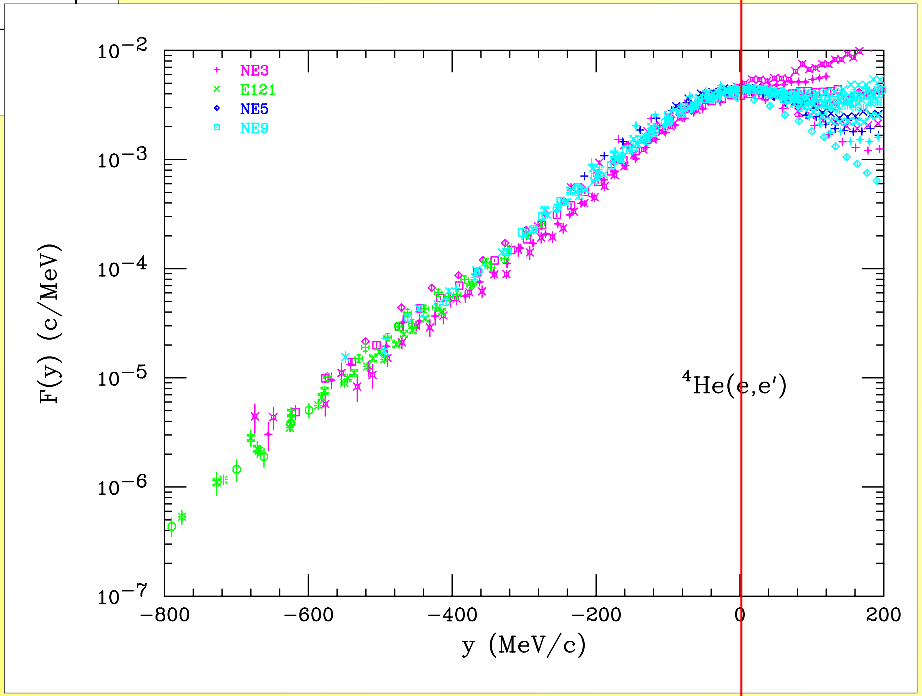

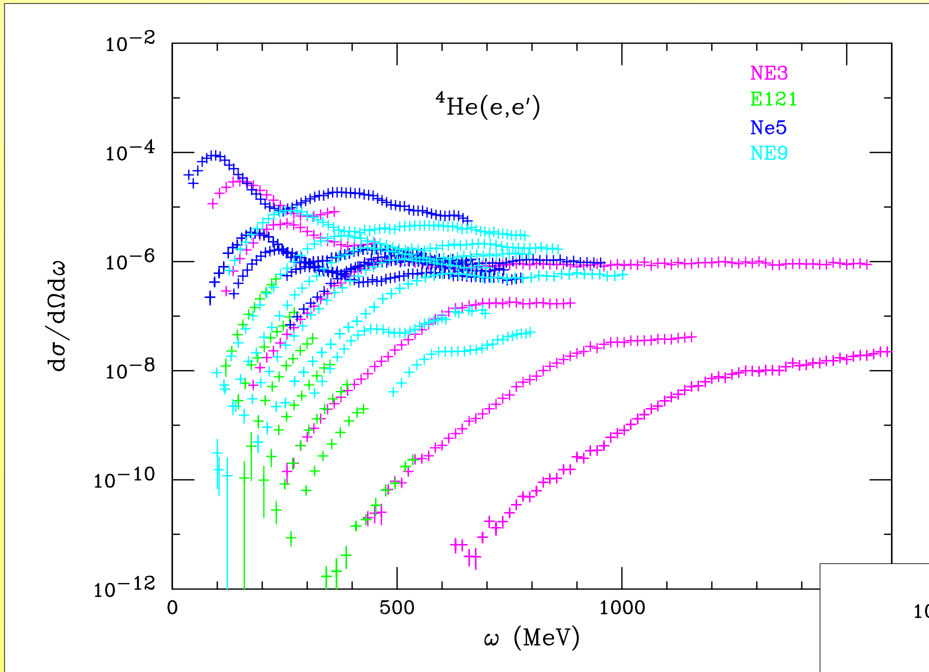

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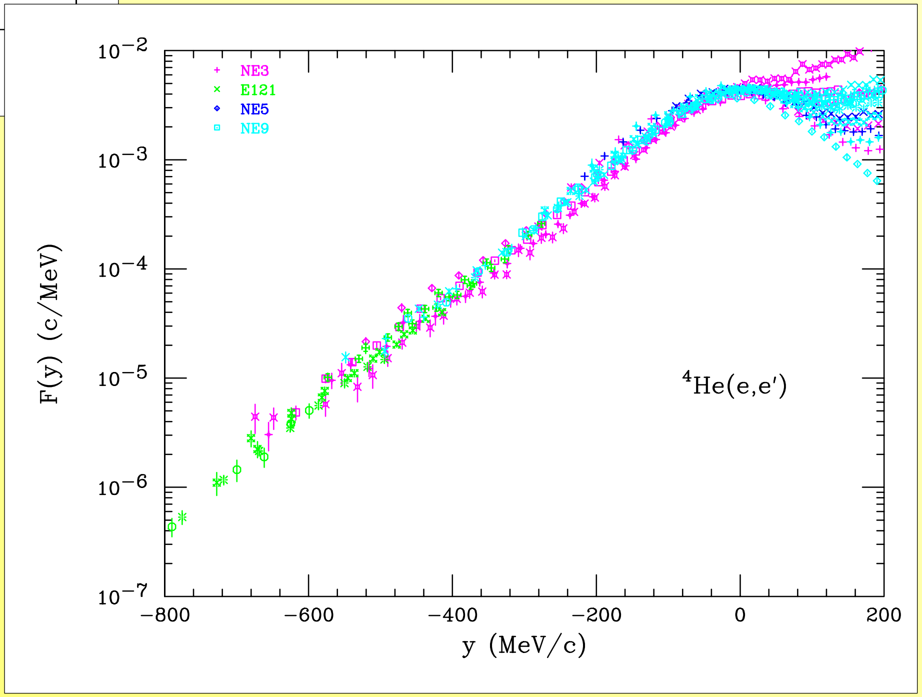


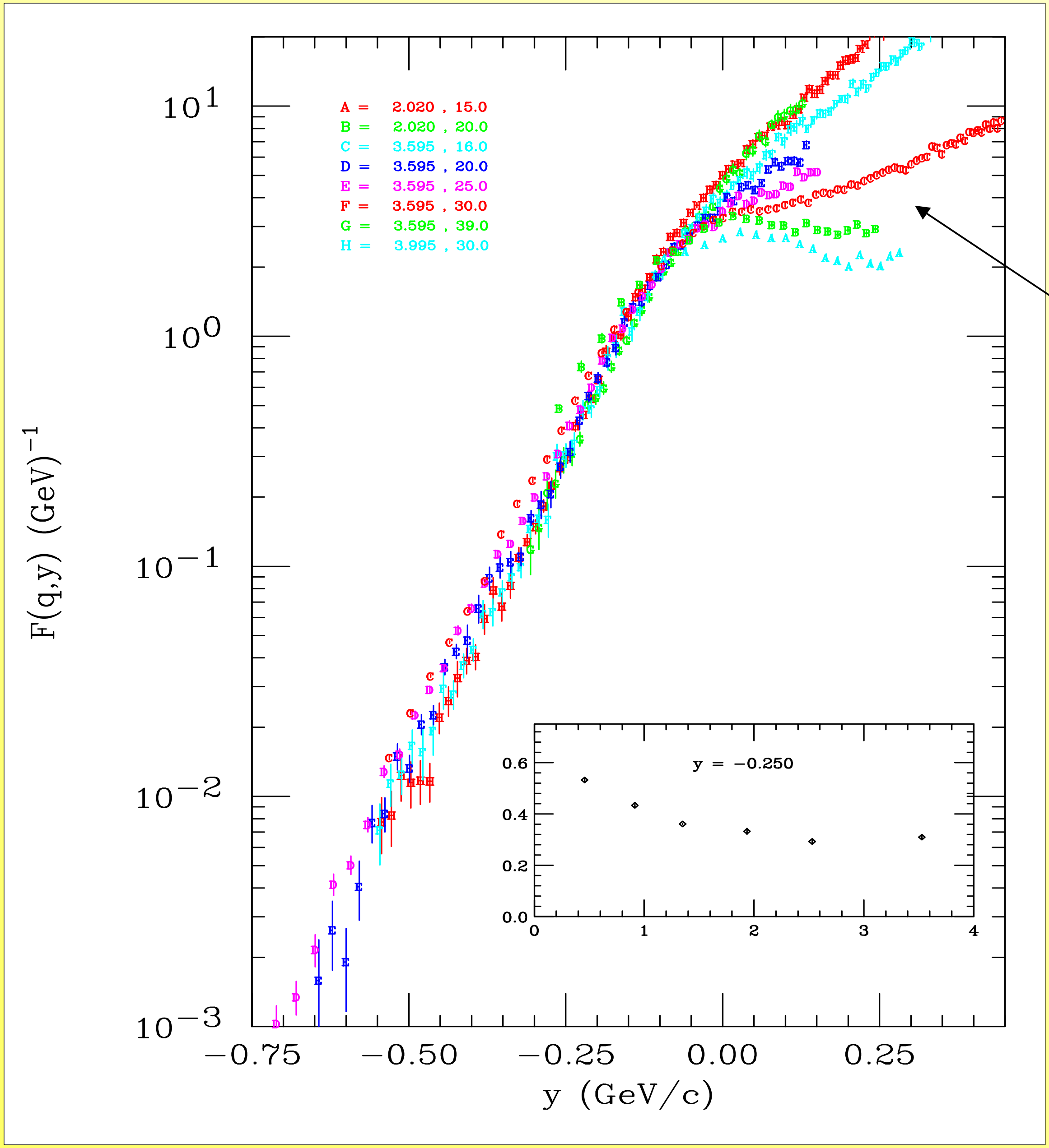
Independence of q

$\updownarrow$

SCALING OF THE 1st KIND

(y-scaling)

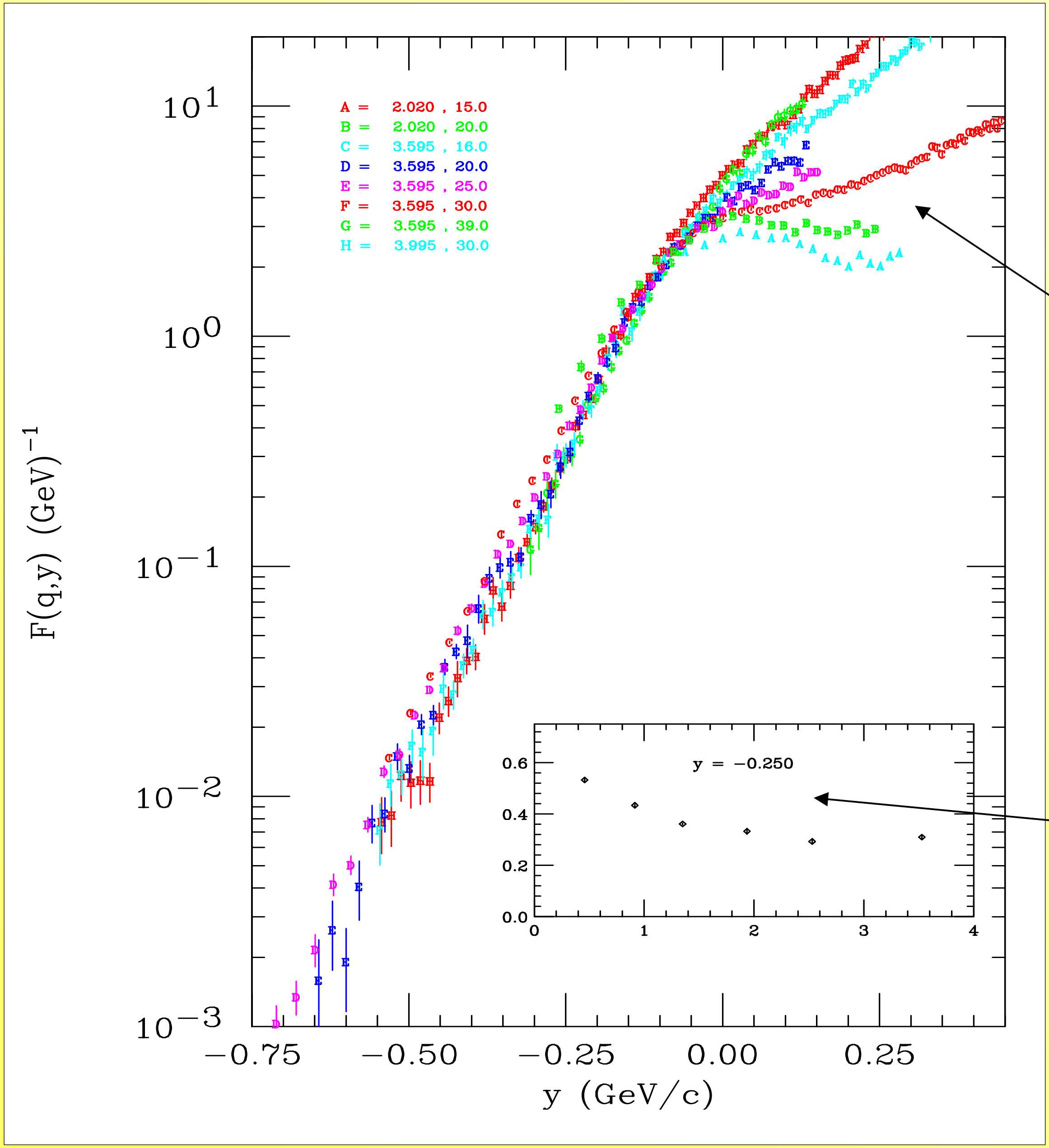
$$F(q,y) \xrightarrow[q\to\infty]{} F(y) \equiv F(\infty, y)$$



Example of ^{56}Fe

Note that at *y>0* the scaling is not good, due to the presence of resonances, meson production, *etc.* (see later, however)

Example of ^{56}Fe



Note that at *y>0* the scaling is not good, due to the presence of resonances, meson production, *etc.* (see later, however)

Scaling function at y = -250 MeV/c versus Q^2 in $(GeV/c)^2$

… approach to scaling

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Next we introduce a characteristic momentum scale for a given nuclear species

$$k_A = \sqrt{\langle k^2 \rangle_A}$$

and use this to define a dimensionless function

$$f(q,y) \equiv k_A \bullet F(q,y)$$

Correspondingly, one wishes to introduce a dimensionless scaling variable ***ψ*** and then to plot ***f(q,ψ)*** versus *ψ* for various values of momentum transfer *q*

The **Relativistic Fermi Gas (RFG)** model is used to motivate the choice of scaling variable.

In the RFG one has

$$\left[k_A\right]^{RFG} = k_F$$

… and a dimensionless scaling variable ψ' which yields exact 1st-kind scaling for the RFG.

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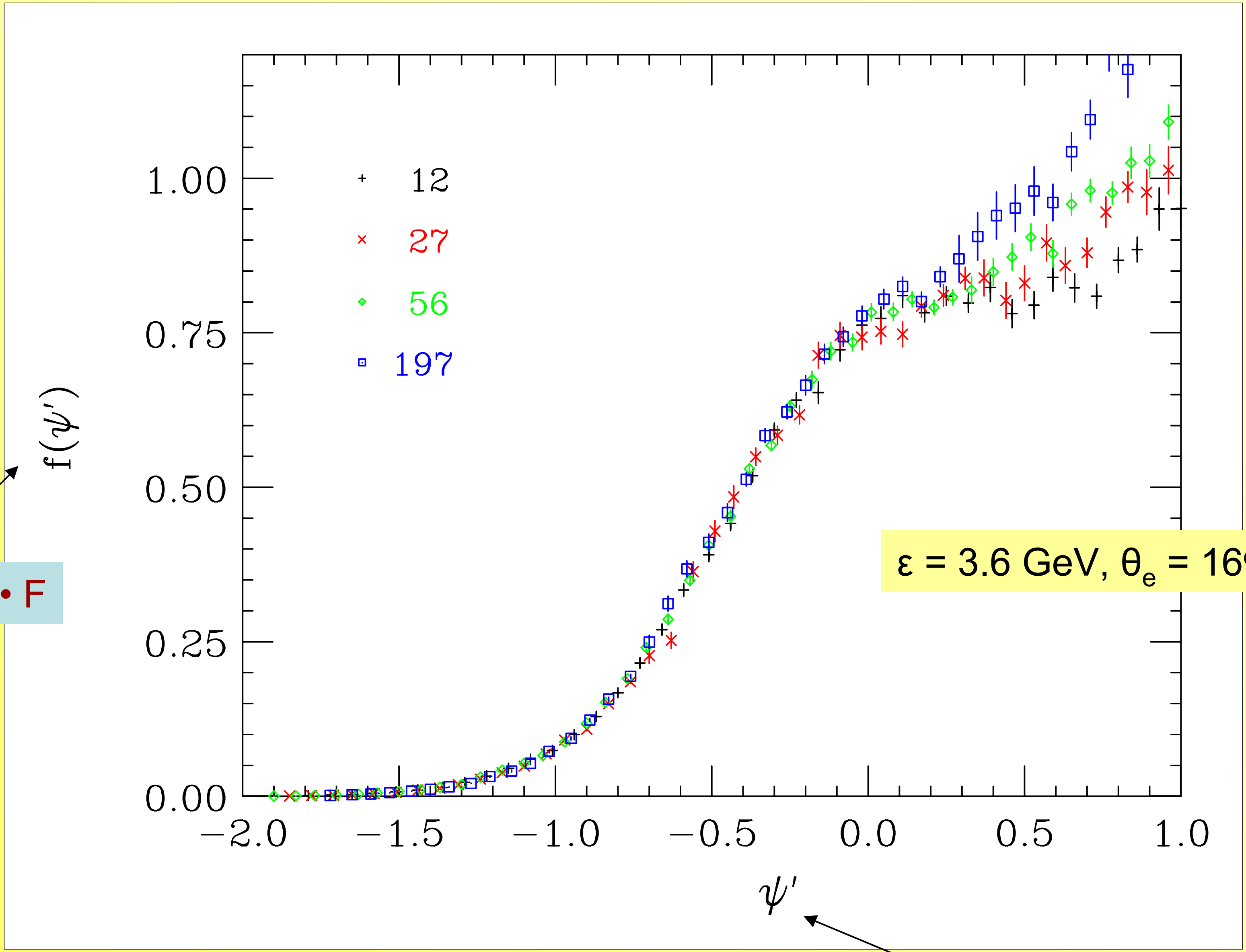
roughly $\psi' = y/k_A$

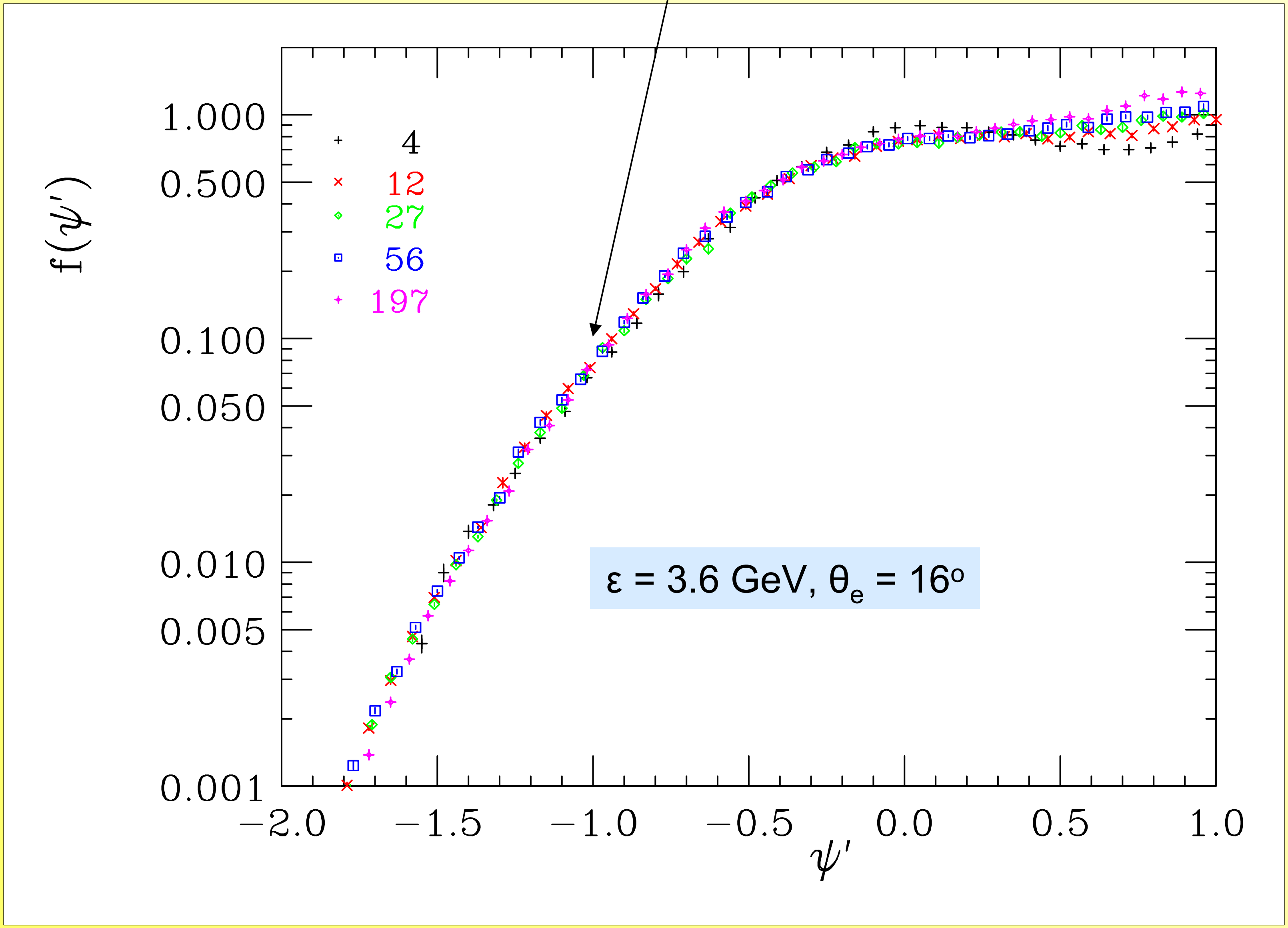


$k_A \bullet F$

$\sim y / k_A$

k_A = characteristic momentum scale for each nucleus

Scaling of the 2nd kind (A independence for ψ'<0)



In the scaling region ($\psi' < 0$) a universal behavior is seen, with

very little dependence on the nuclear species

$\updownarrow$

SCALING OF THE 2nd KIND

In the region above $\psi' = 0$ where resonances, meson production and the start of DIS enter the 2nd-kind scaling is not as good (see below)

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Although the amount of data separated into longitudinal (L) and transverse (T) responses is small, one can attempt a scaling analysis with what does exist. The inclusive cross section may be written

$$\frac{d^2\sigma}{d\Omega_e d\omega} = \sigma_M \left[v_L R_L(q,\omega) + v_T R_T(q,\omega) \right]$$

$$v_L = \left| Q^2 / q^2 \right|^2$$

$$v_T = \tfrac{1}{2}\left| Q^2 / q^2 \right| + \tan^2 \theta_e / 2$$

From which L and T scaling functions can be defined as above

$$F_L(q,y) \equiv \frac{R_L(q,\omega)}{A\left[\Sigma_{eN}^{eff}\right]_L / \sigma_M v_L}$$

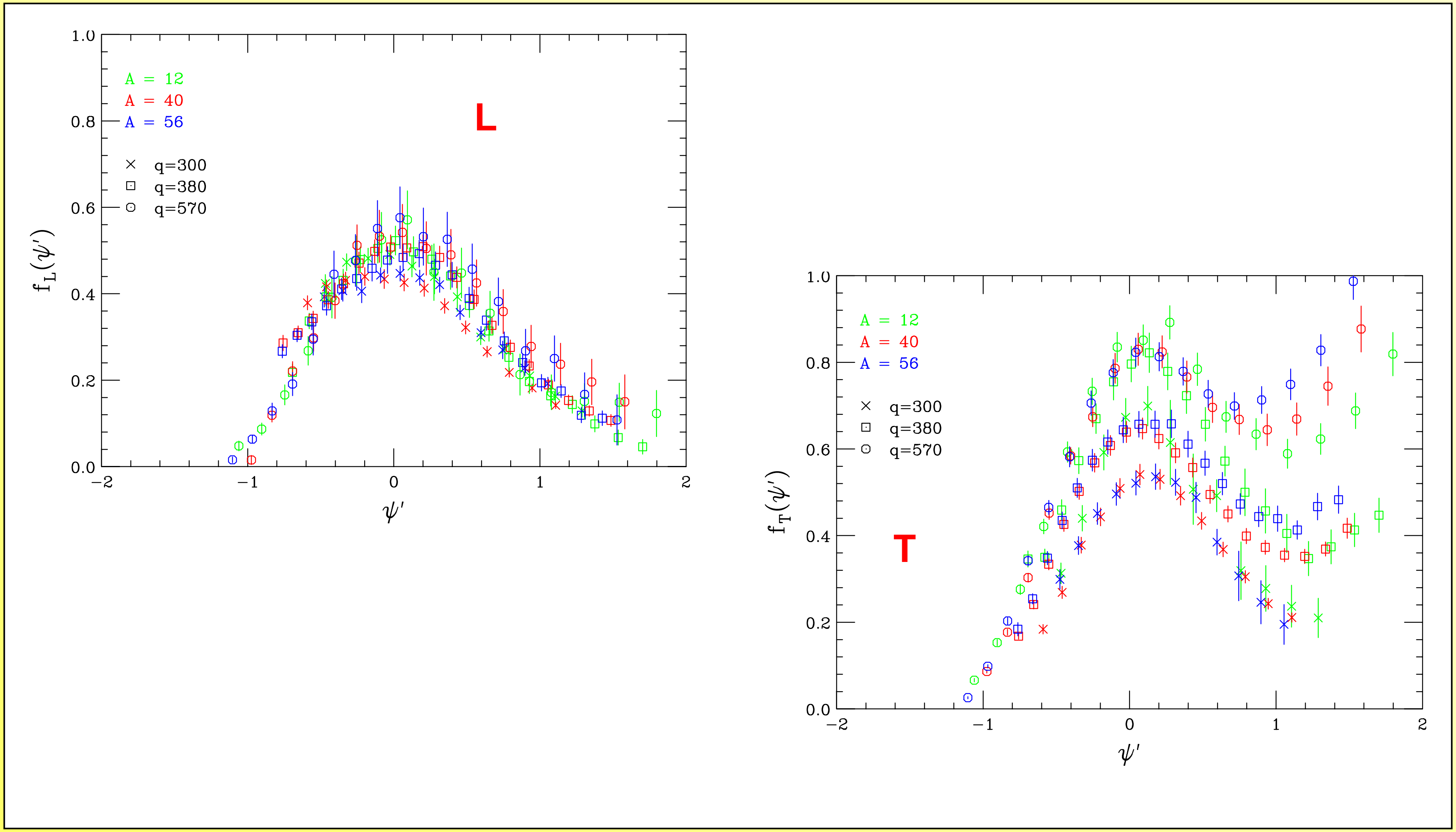
$$F_T(q,y) \equiv \frac{R_T(q,\omega)}{A\left[\Sigma_{eN}^{eff}\right]_T / \sigma_M v_T}$$

as can their dimensionless analogs

$$f_L(q,y) \equiv k_A \bullet F_L(q,y)$$

$$f_T(q,y) \equiv k_A \bullet F_T(q,y)$$

What results is the following:



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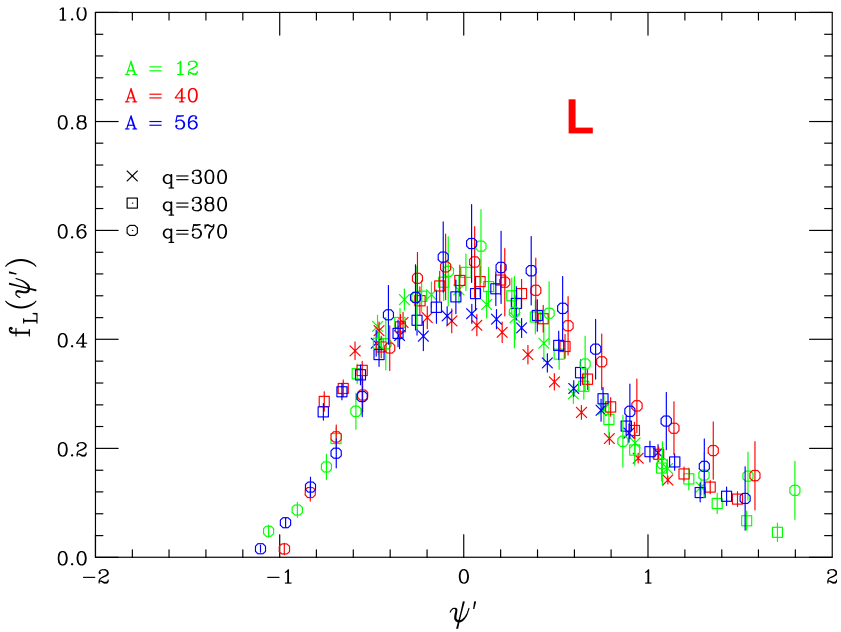
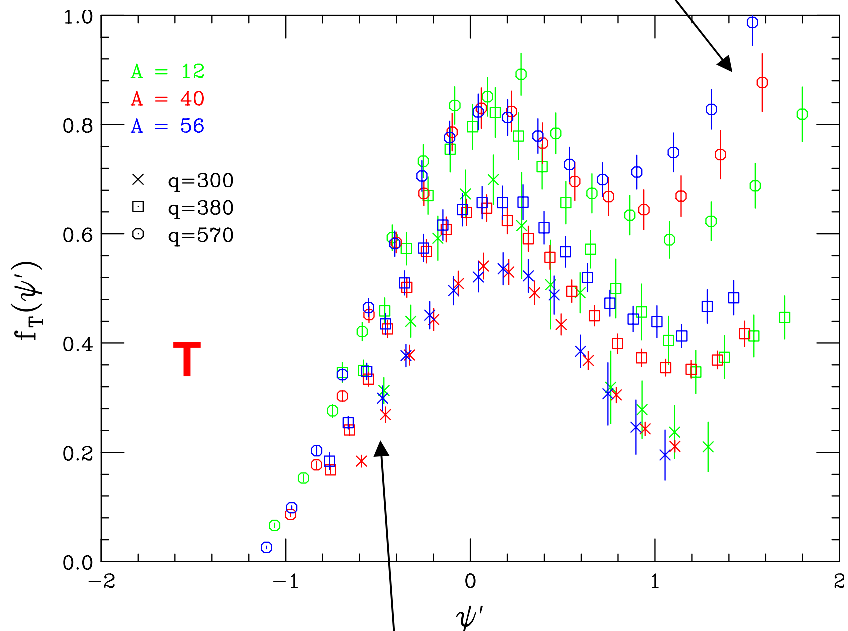
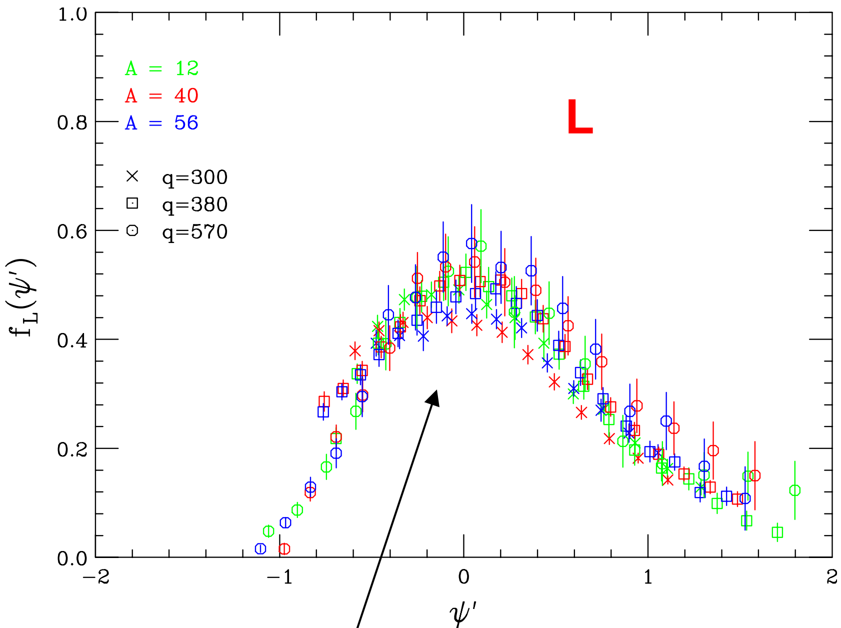


Inelastic contributions (mainly T)
+ MEC (dominantly T)

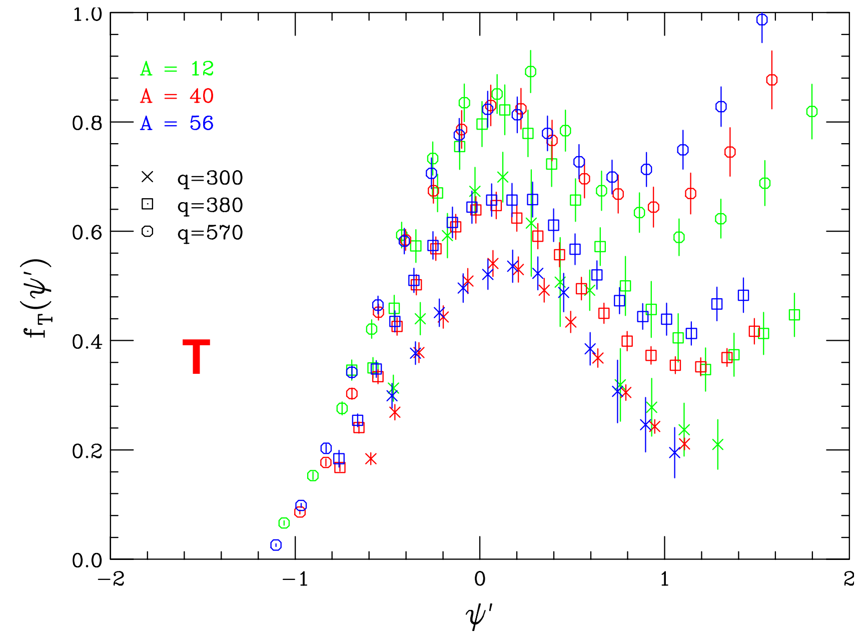


… however, still some residual below the QE peak

What results is the following:

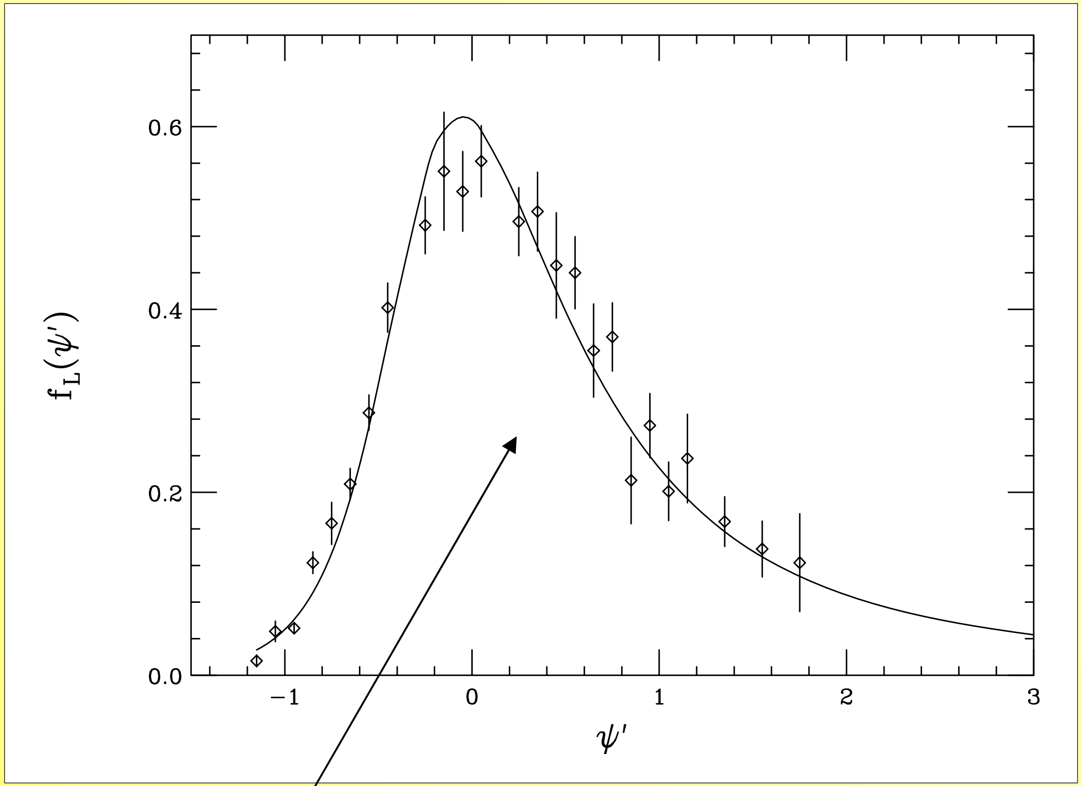


In contrast, the L results show a **universal behavior**



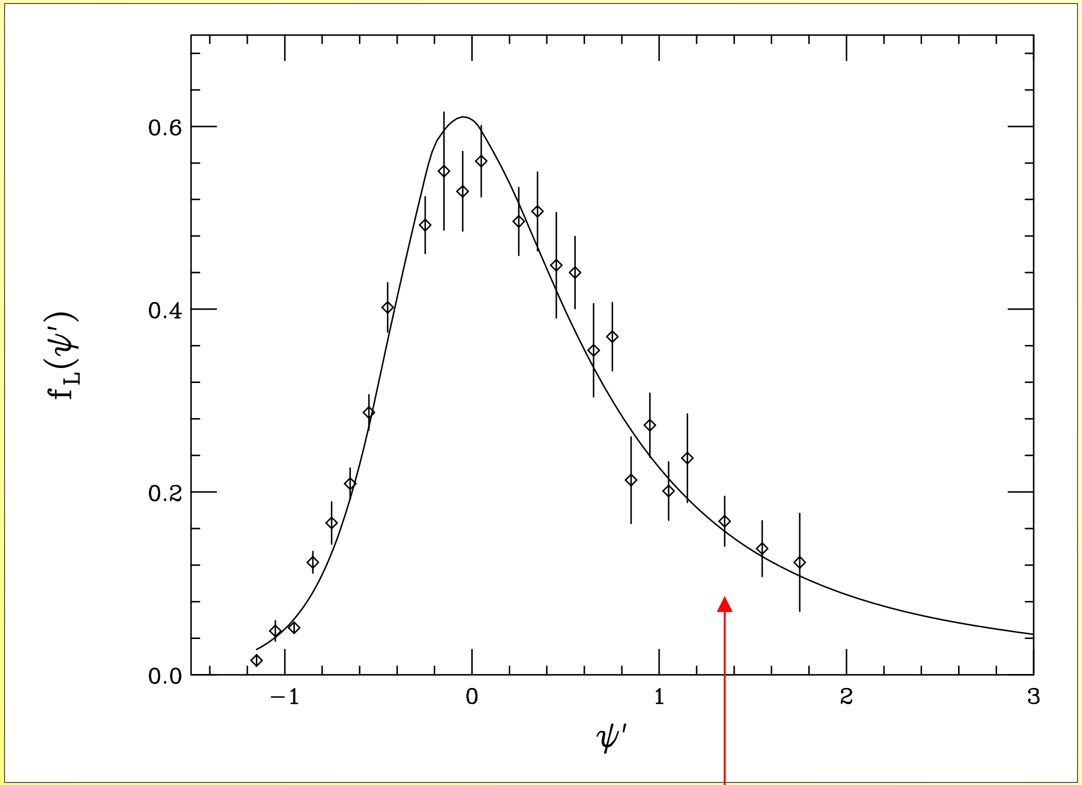
Longitudinal response only (little from MEC or pion production):

3 nuclei and
3 values of q



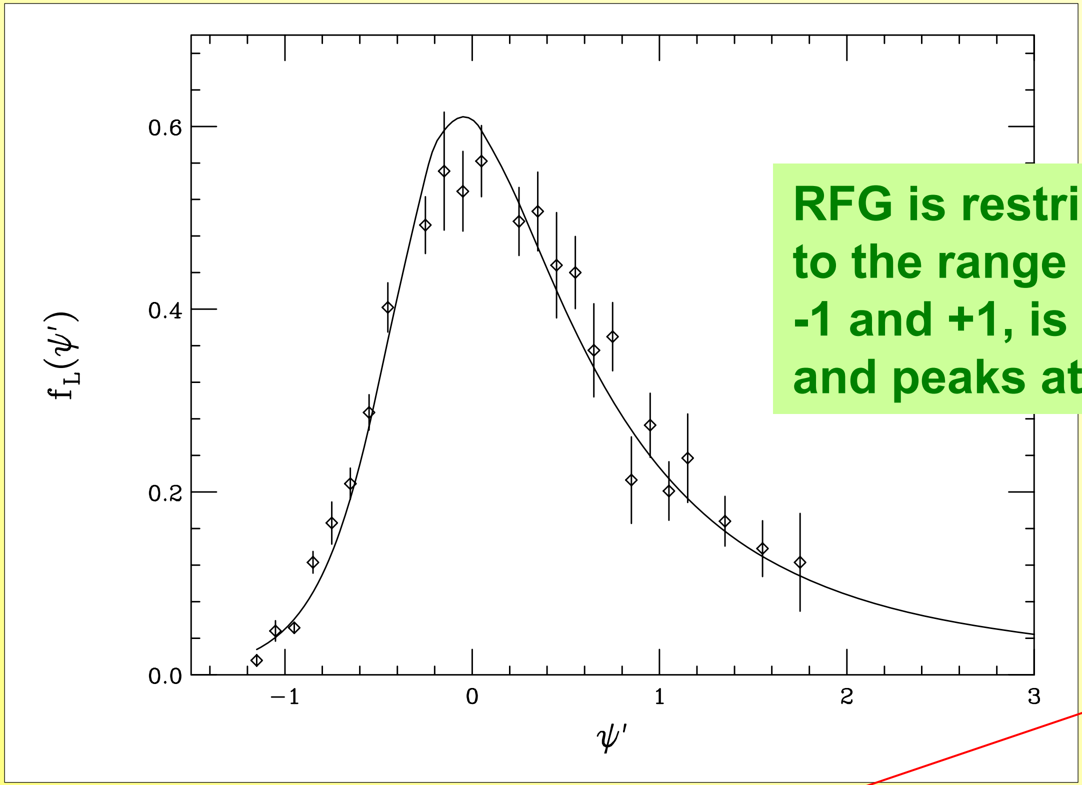
which is seen to be both independent of q (scaling of the 1st kind) and also independent of nuclear species (scaling of the 2nd kind)

$\leftrightarrow$ **SUPERSCALING**



Notes:

(1) Asymmetric shape; tail at high energy loss

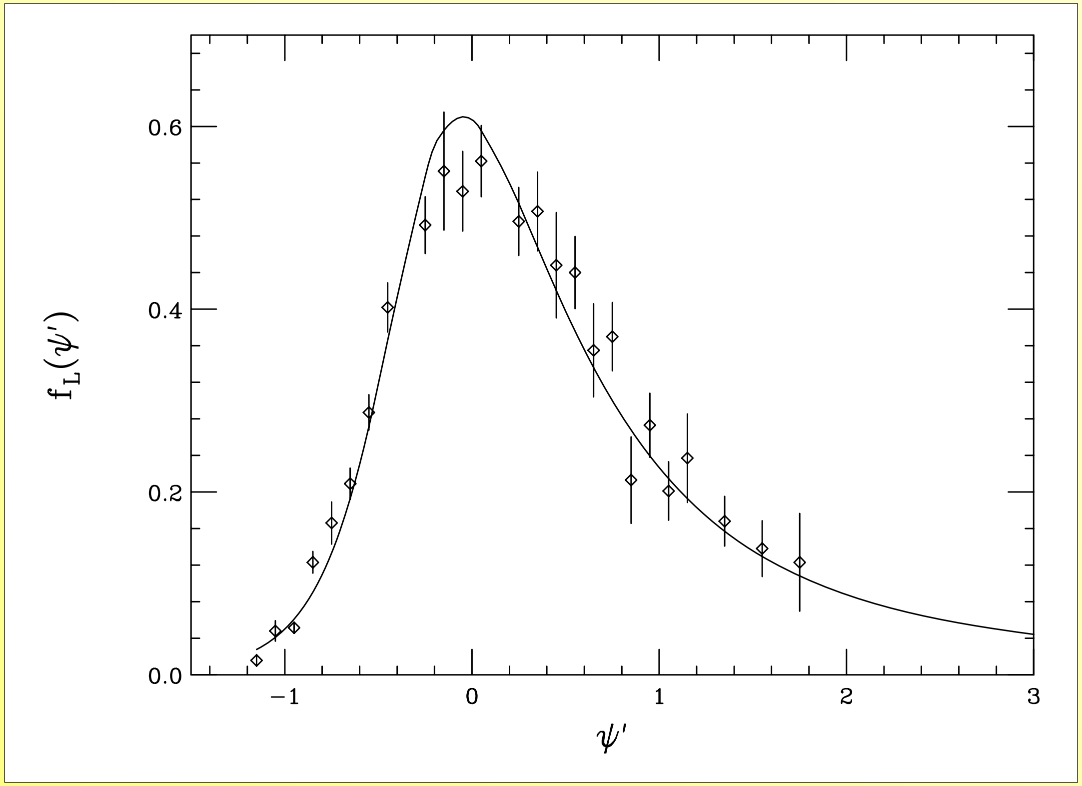


RFG is restricted to the range between -1 and +1, is symmetric and peaks at 0.75

Notes:

(1) Asymmetric shape; tail at high energy loss

(2) RFG very poor



Notes:

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(3) Best models yield this shape:

(a) **RMF approaches**

(b) Semi-rel approach

(c) BCS-inspired model

(d) Recent study with correlations

Note: in the RFG one has

$$[f_L]^{RFG} = [f_T]^{RFG} = [f]^{RFG}$$

which has been called **SCALING OF THE 0th KIND**

If it were not for

- contributions from resonances, meson production and DIS (which should not scale, since they involve different elementary cross sections, not elastic eN scattering, and since the scaling variables constructed above are appropriate only for QE scattering; see the discussions to follow), and for

- effects from meson-exchange currents (dominantly in T)

one might expect scaling of the 0th kind to be found.

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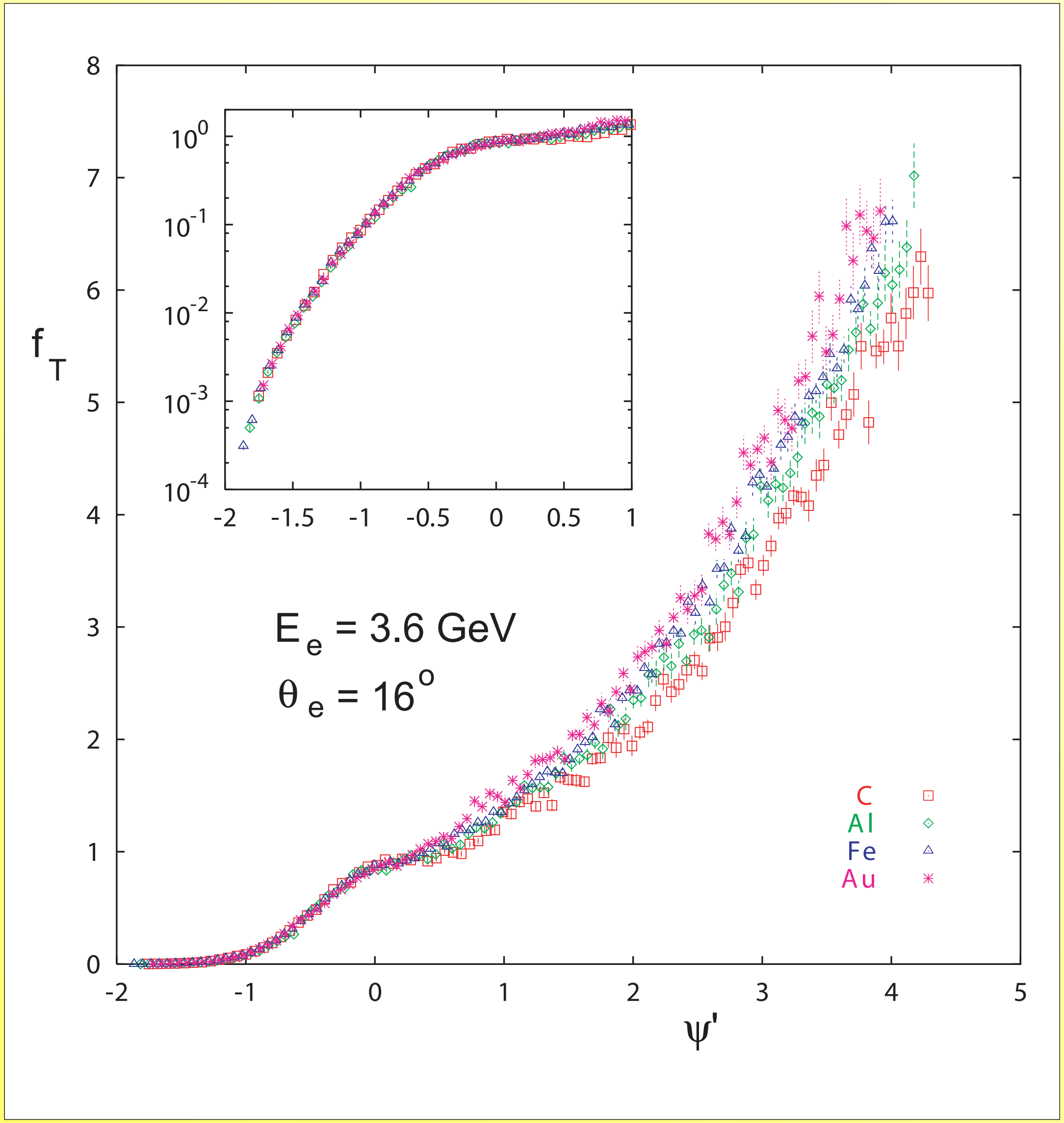
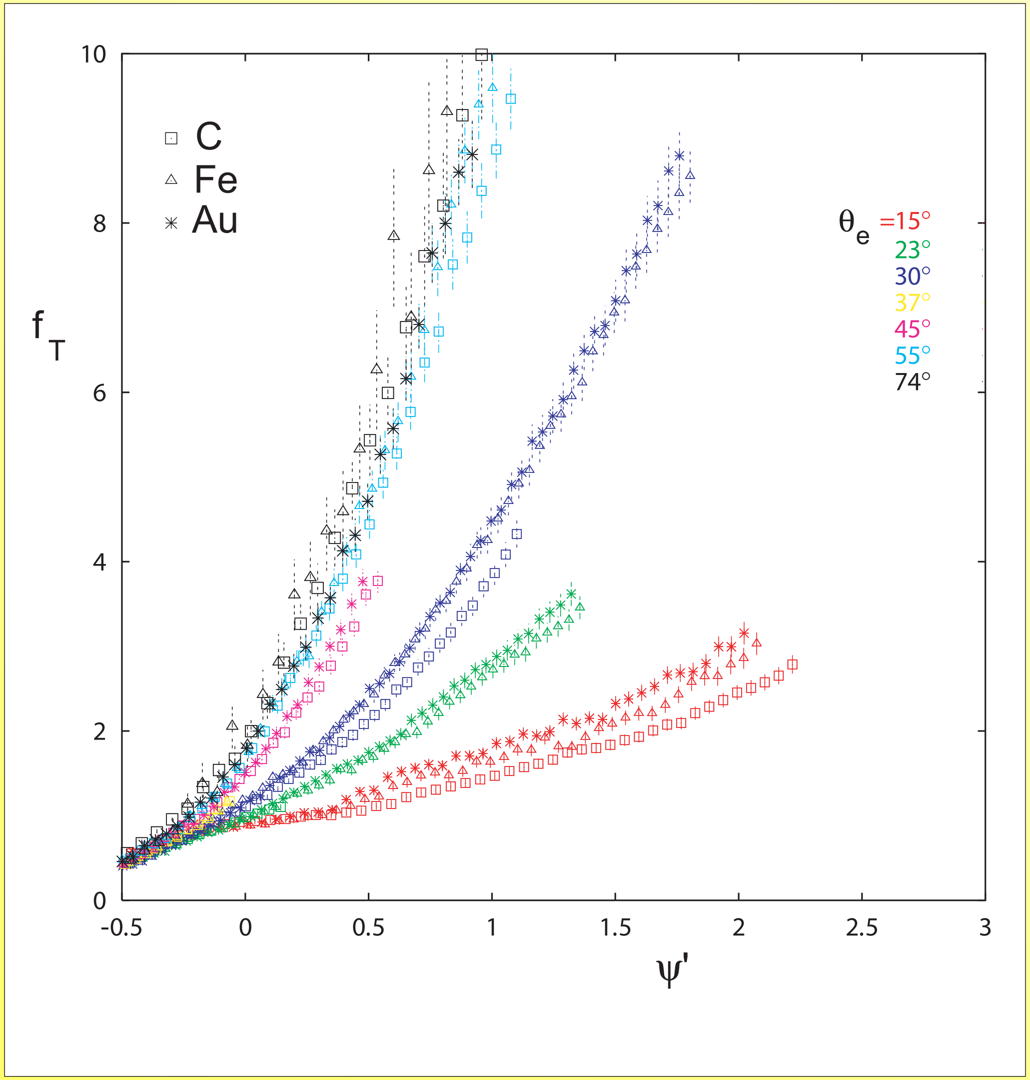


Large ψ' region



Breaking of 1st and 2nd kind scaling at high ψ'

In the region above the QE peak one certainly expects **inelastic** contributions to be important in the T response, although not so in the L response

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One also expects to have **2p-2h MEC** contributions which add to the response discussed above; again, these are mainly T, not L. Typically they contribute 10-15% of the total and are one of the reasons for the scaling violations in the T response seen above.

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… the net result of adding together the **universal L scaling function**, the **inelastic contributions** obtained using this as well, and the **2p-2h MEC contributions** is in reasonable agreement with experiment (see below).

SuperScaling Approach (SuSA)

(1) Assume a **universal** scaling function, either phenomenological from the longitudinal results shown above, or from models
(2) Use this together with elastic eN as above or inelastic eN → e'X single-nucleon cross sections to obtain the QE and inel contributions
(3) Add 2-particle emission MEC contributions
(4) Use this universal approach to compare with inclusive ee' data
(5) Replace the single-nucleon cross sections in (2) with CC or NC neutrino reaction cross sections to obtain the SuSA predictions for the neutrino-nucleus cross sections

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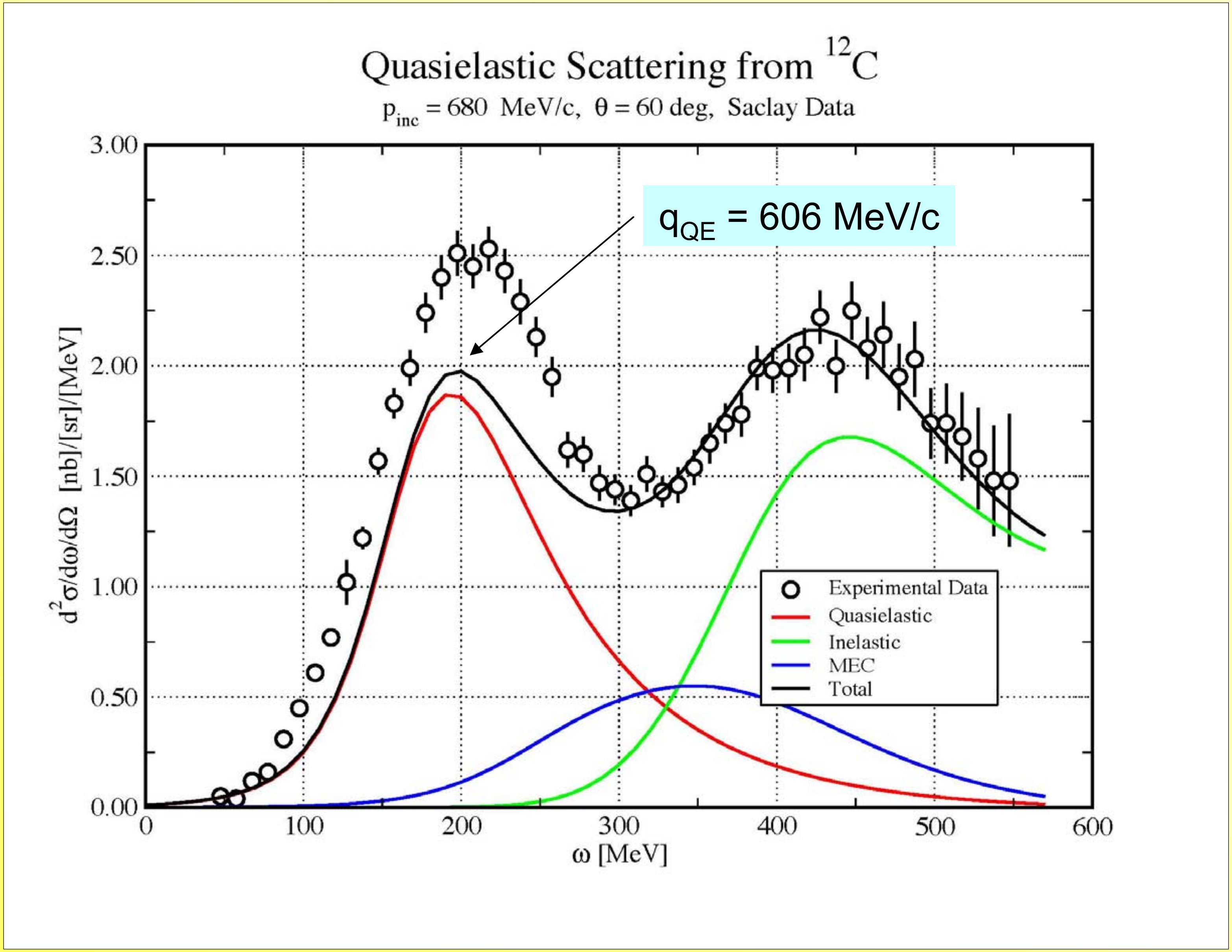
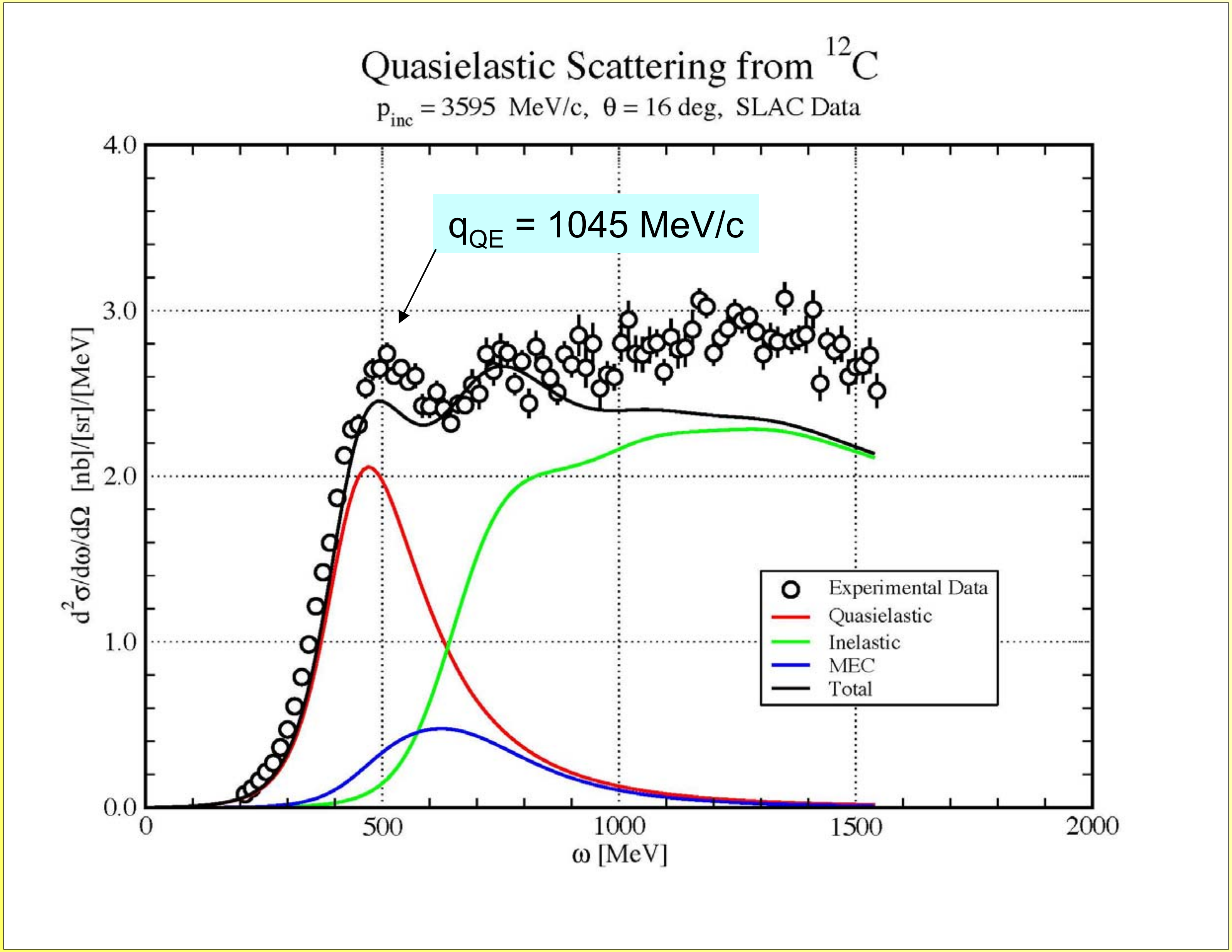
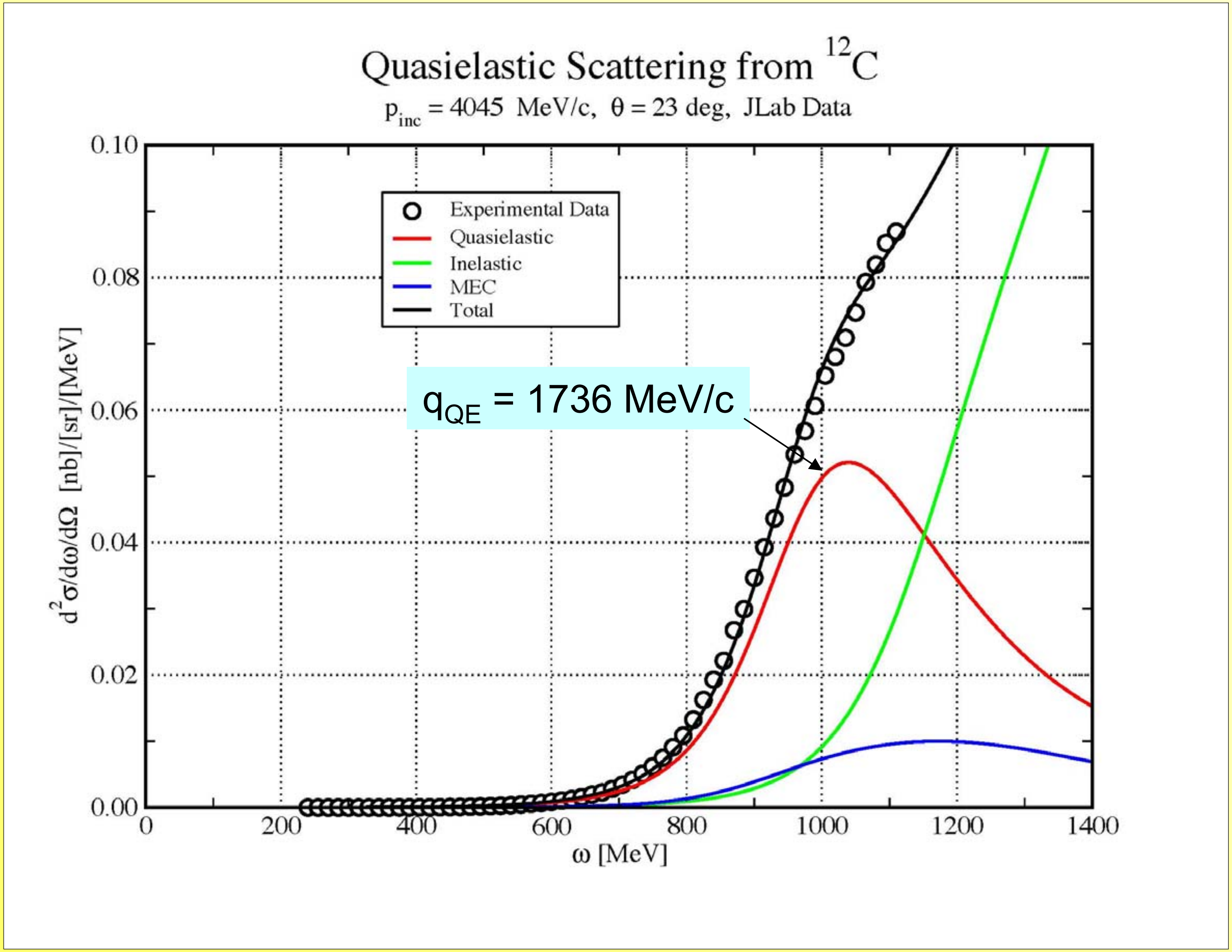
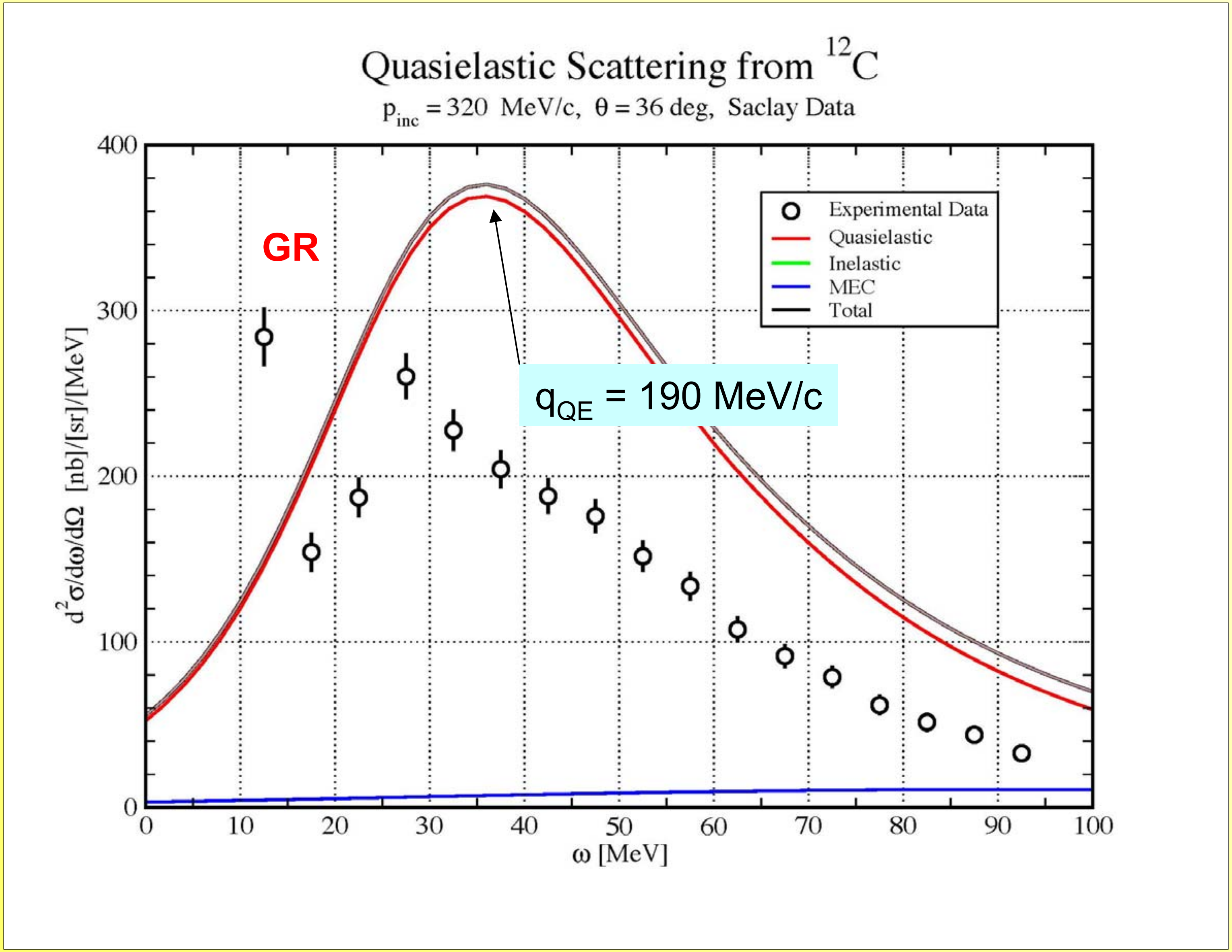
… of course, if the test in (4) fails, one should not expect to have very good predictions for neutrino reactions, as is the case for simplistic models such as the RFG

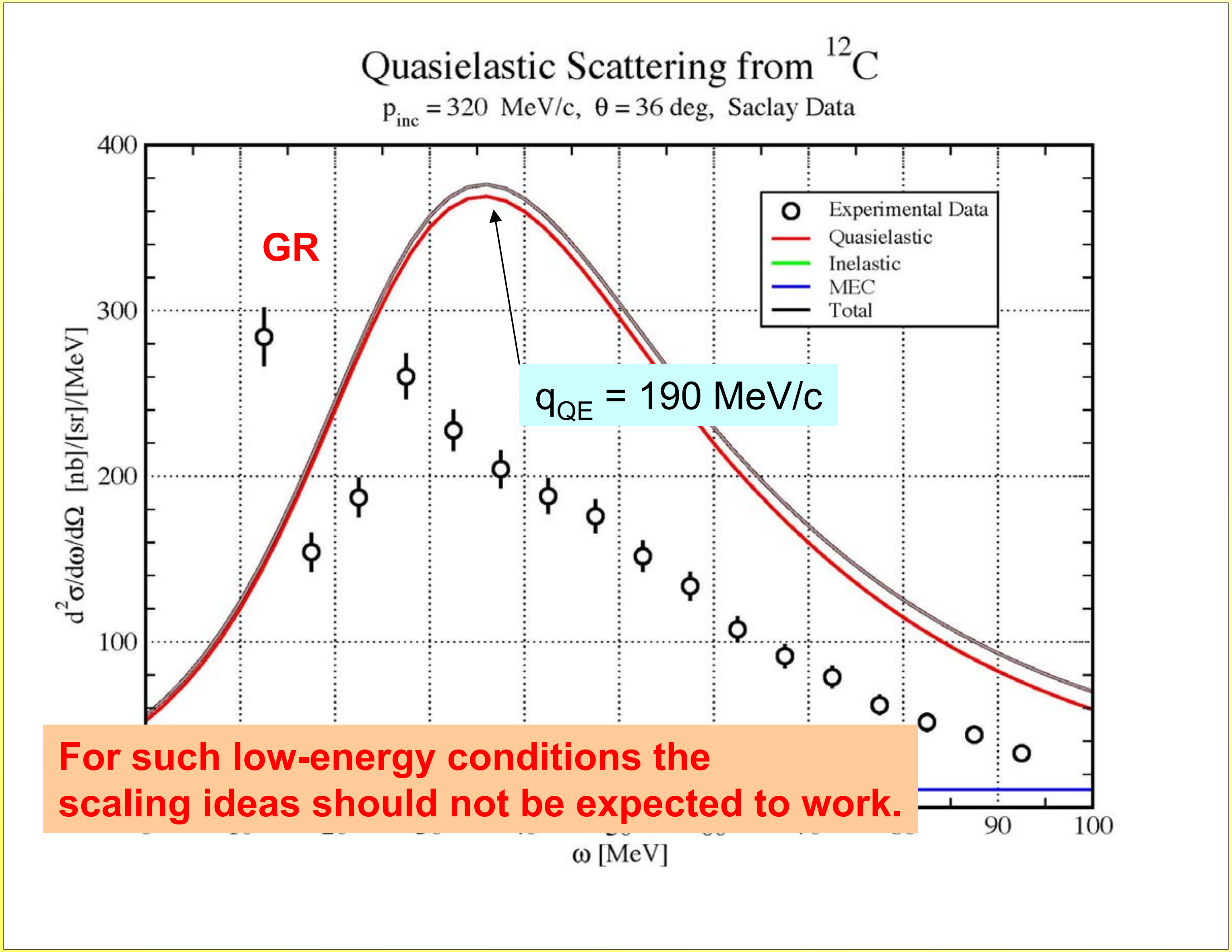

Quasielastic Scattering from ^{12}C
p_{inc} = 680 MeV/c, θ = 60 deg, Saclay Data
q_{QE} = 606 MeV/c
$d^2\sigma/d\omega/d\Omega$ [nb]/[sr]/[MeV]
ω [MeV]
Experimental Data
Quasielastic
Inelastic
MEC
Total
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0
100
200
300
400
500
600


Quasielastic Scattering from 12C
pinc = 3595 MeV/c, θ = 16 deg, SLAC Data
qQE = 1045 MeV/c
d2σ/dω/dΩ [nb]/[sr]/[MeV]
ω [MeV]
0.0
1.0
2.0
3.0
4.0
0
500
1000
1500
2000
Experimental Data
Quasielastic
Inelastic
MEC
Total

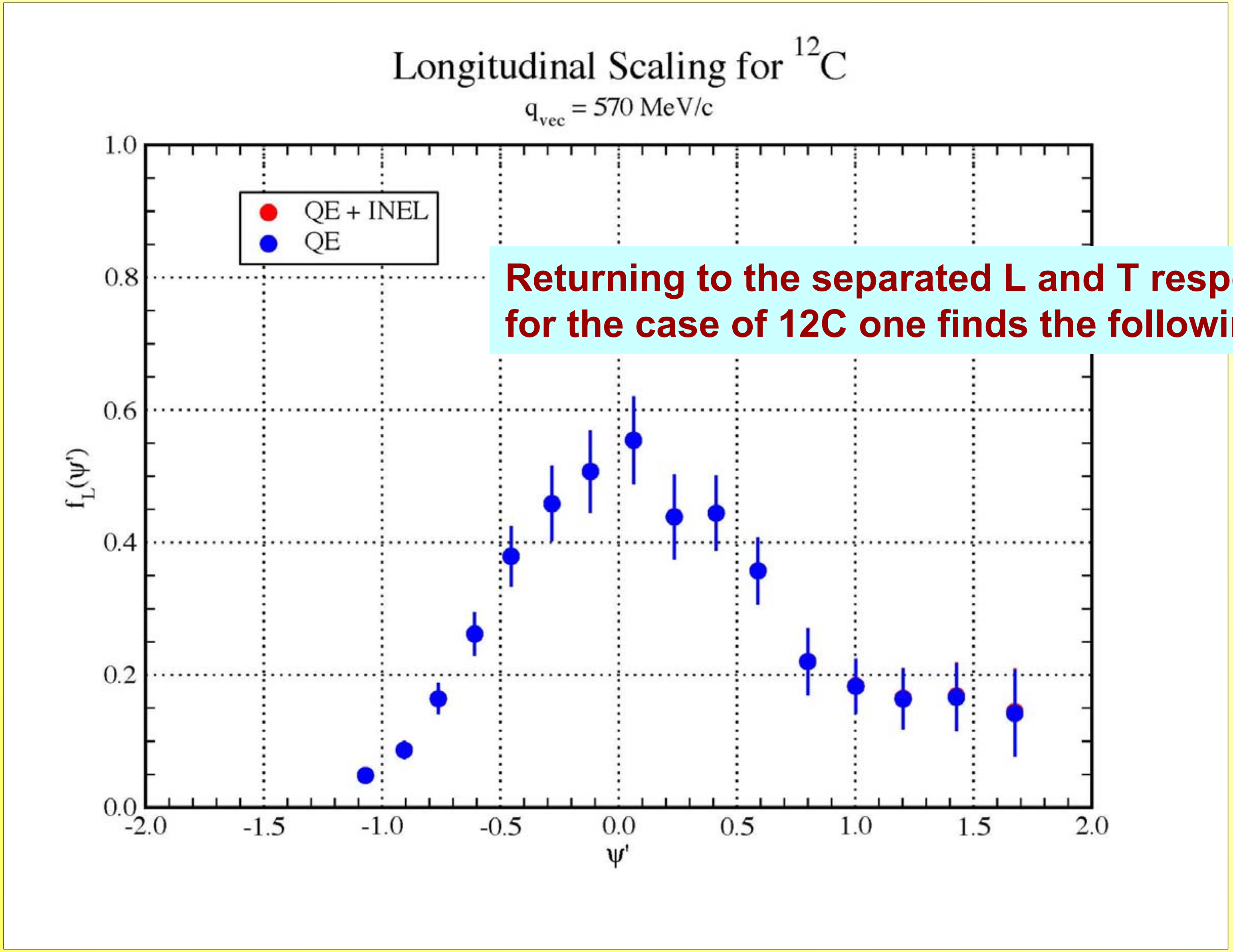


Quasielastic Scattering from ^{12}C
p_{inc} = 4045 MeV/c, θ = 23 deg, JLab Data
Experimental Data
Quasielastic
Inelastic
MEC
Total
q_{QE} = 1736 MeV/c
$d^2\sigma/d\omega/d\Omega$ [nb]/[sr]/[MeV]
ω [MeV]
0.00
0.02
0.04
0.06
0.08
0.10
0
200
400
600
800
1000
1200
1400

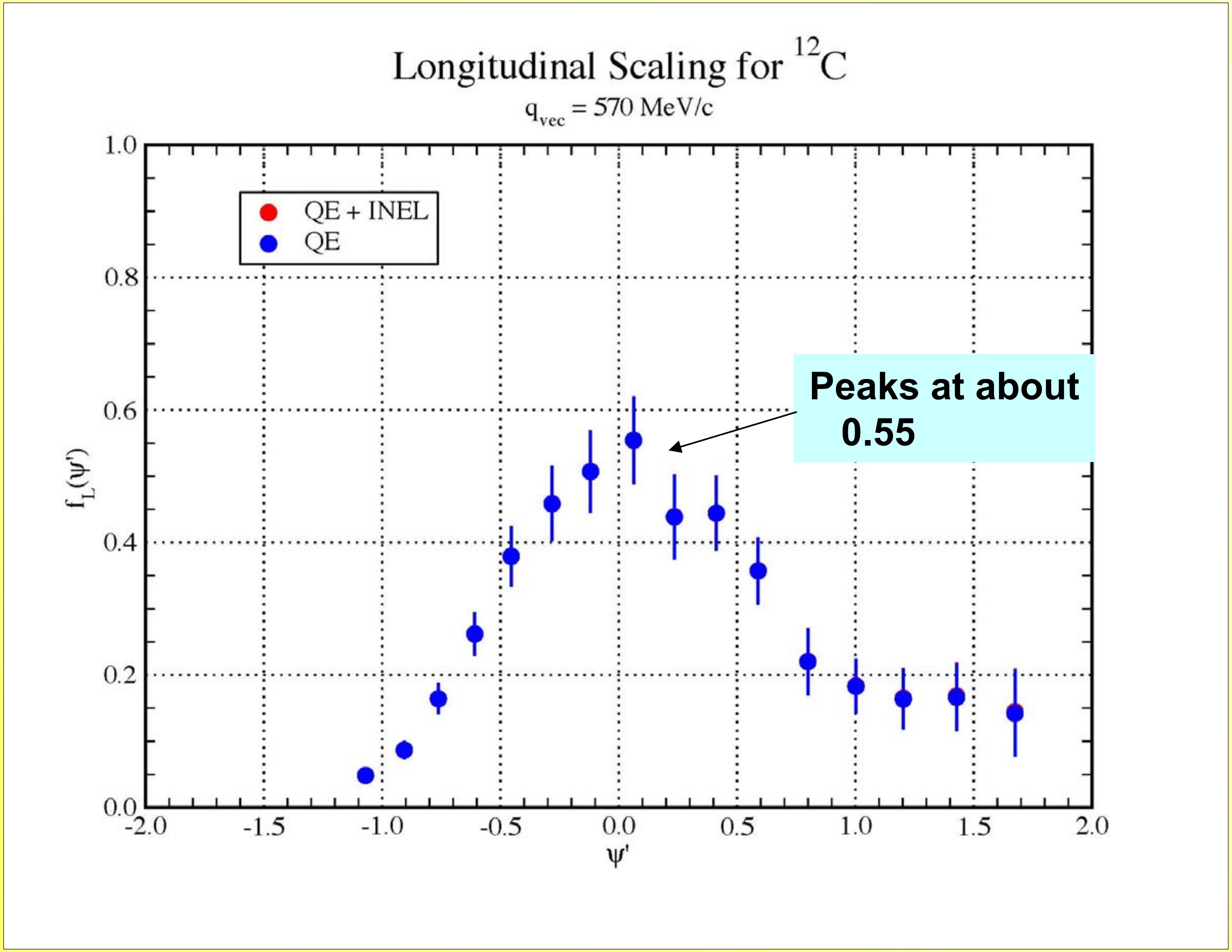

Quasielastic Scattering from ^{12}C
p_{inc} = 320 MeV/c, θ = 36 deg, Saclay Data
GR
q_{QE} = 190 MeV/c
Experimental Data
Quasielastic
Inelastic
MEC
Total
d²σ/dω/dΩ [nb]/[sr]/[MeV]
ω [MeV]
0
100
200
300
400
0
10
20
30
40
50
60
70
80
90
100



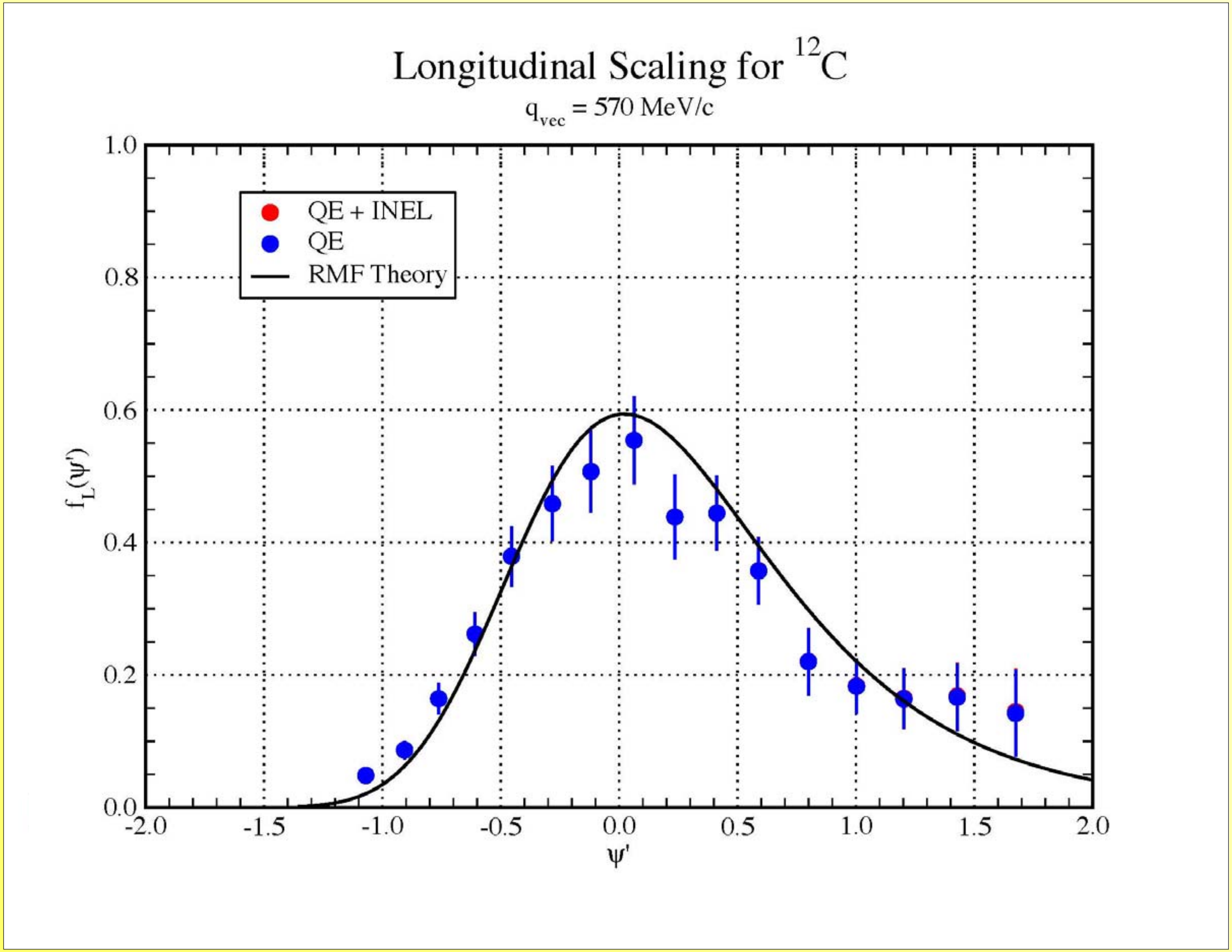
For such low-energy conditions the scaling ideas should not be expected to work.

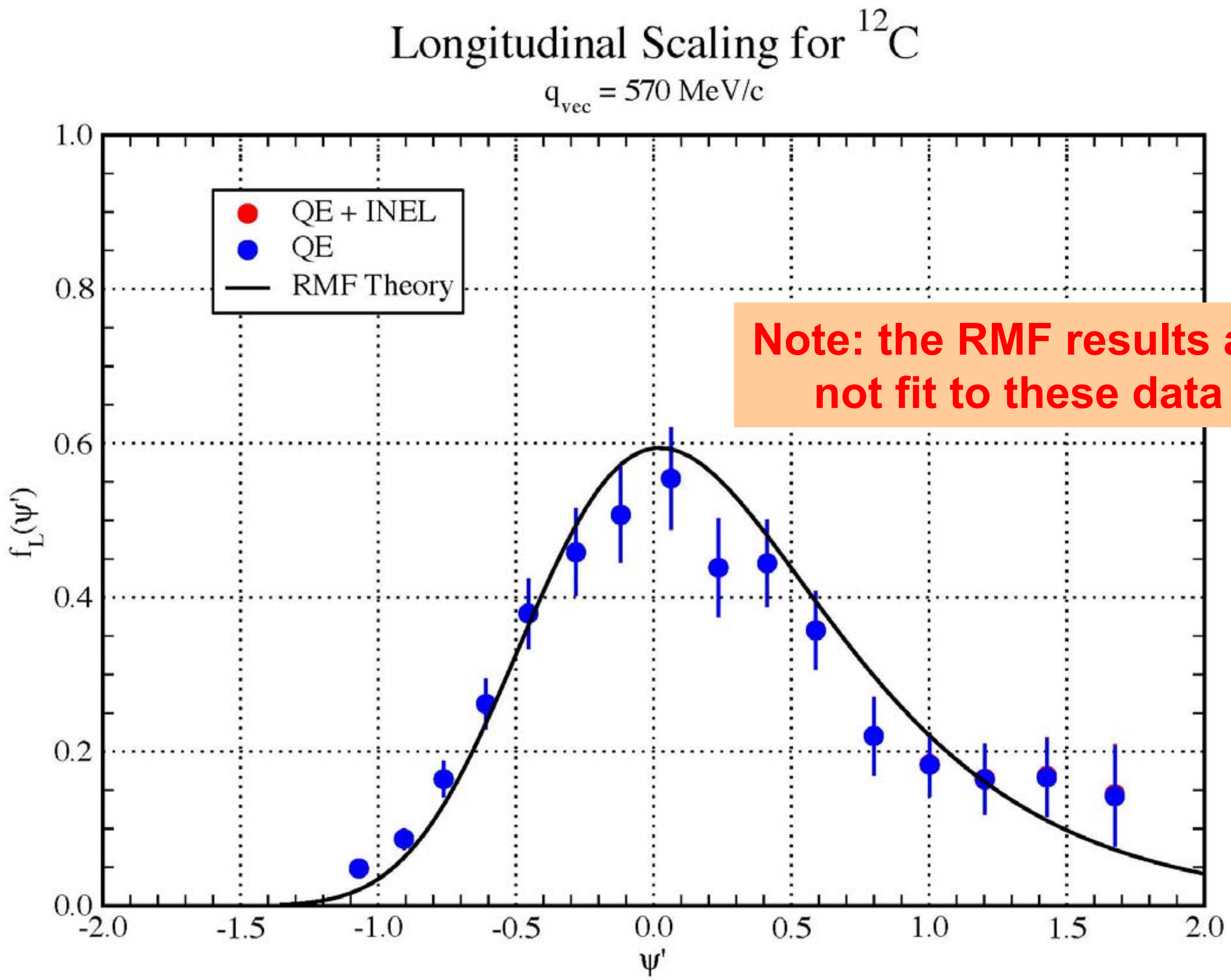


Returning to the separated L and T responses, for the case of 12C one finds the following:

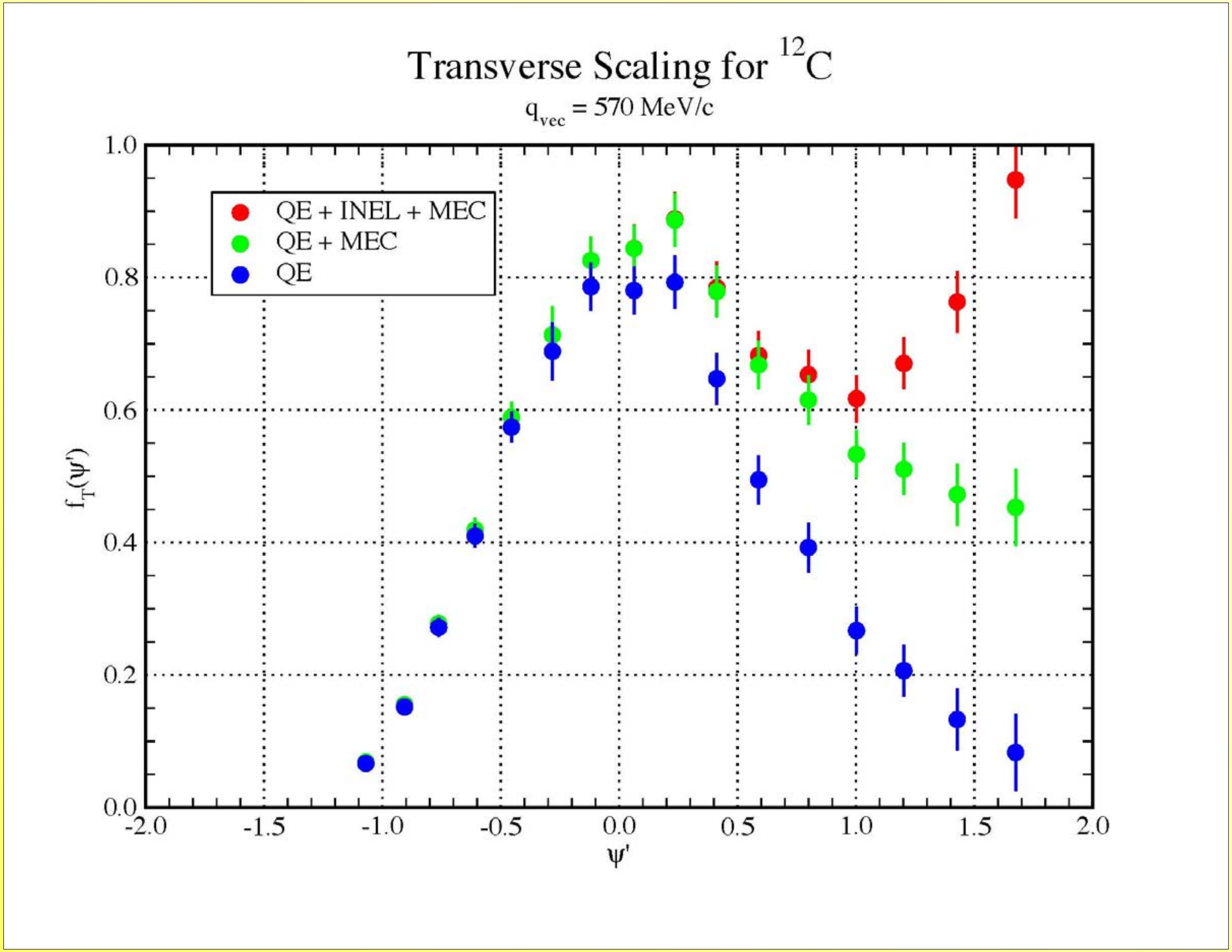


Fermilab 2011

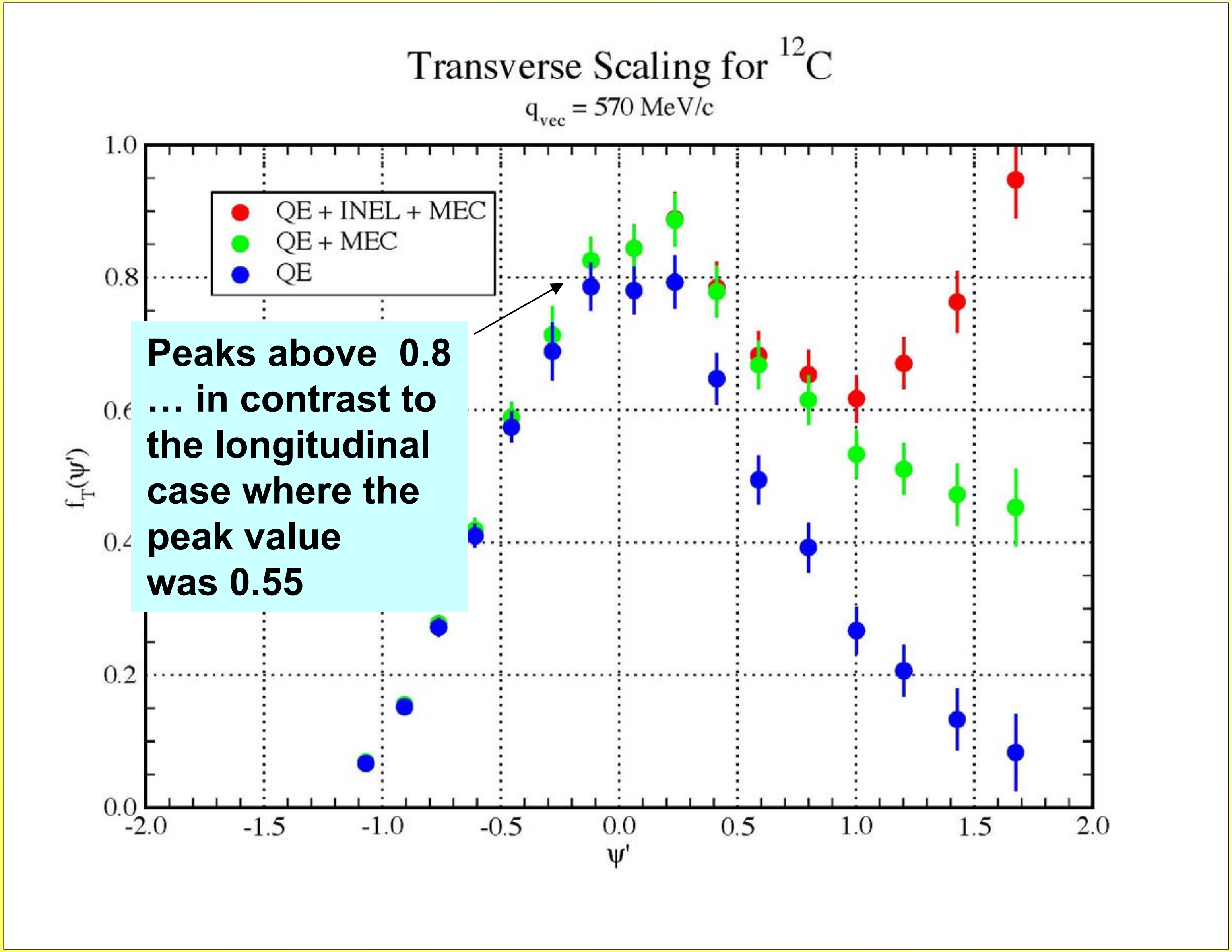

Longitudinal Scaling for ^{12}C
q_{vec} = 570 MeV/c
QE + INEL
QE
RMF Theory
$f_L(\psi')$
ψ'
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0



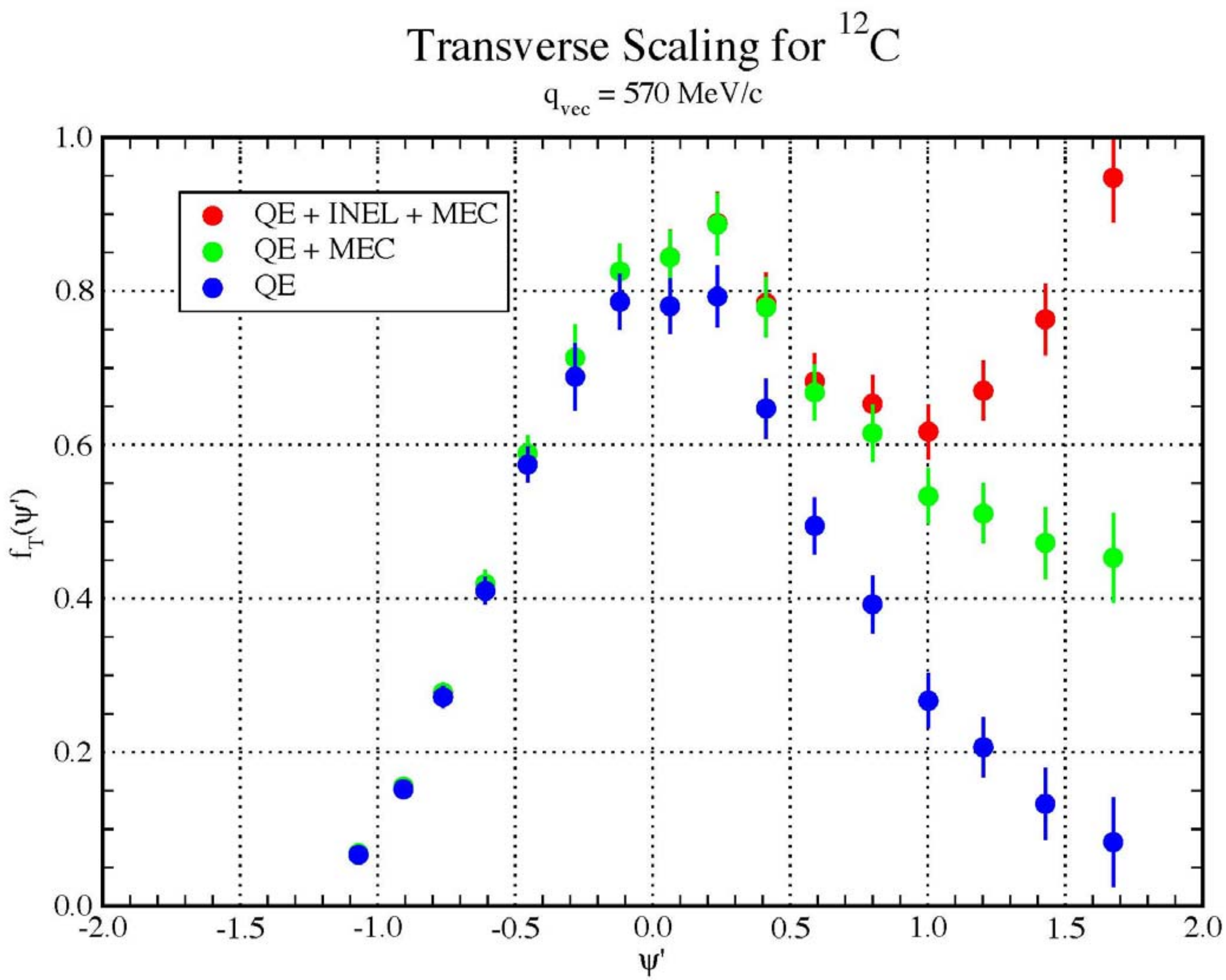
Note: the RMF results are not fit to these data



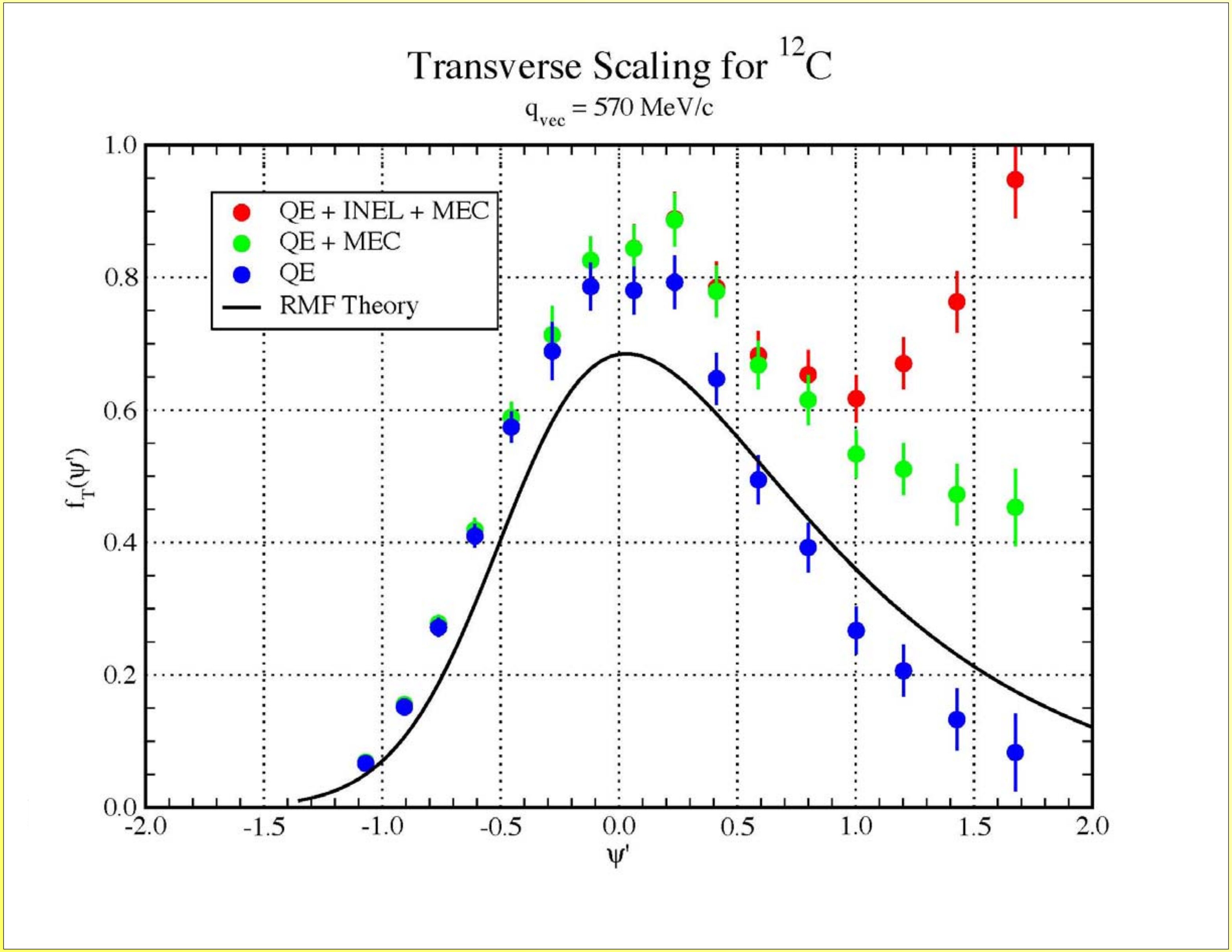
Transverse Scaling for ^{12}C
q_{vec} = 570 MeV/c
QE + INEL + MEC
QE + MEC
QE
$f_T(\psi')$
ψ'
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0

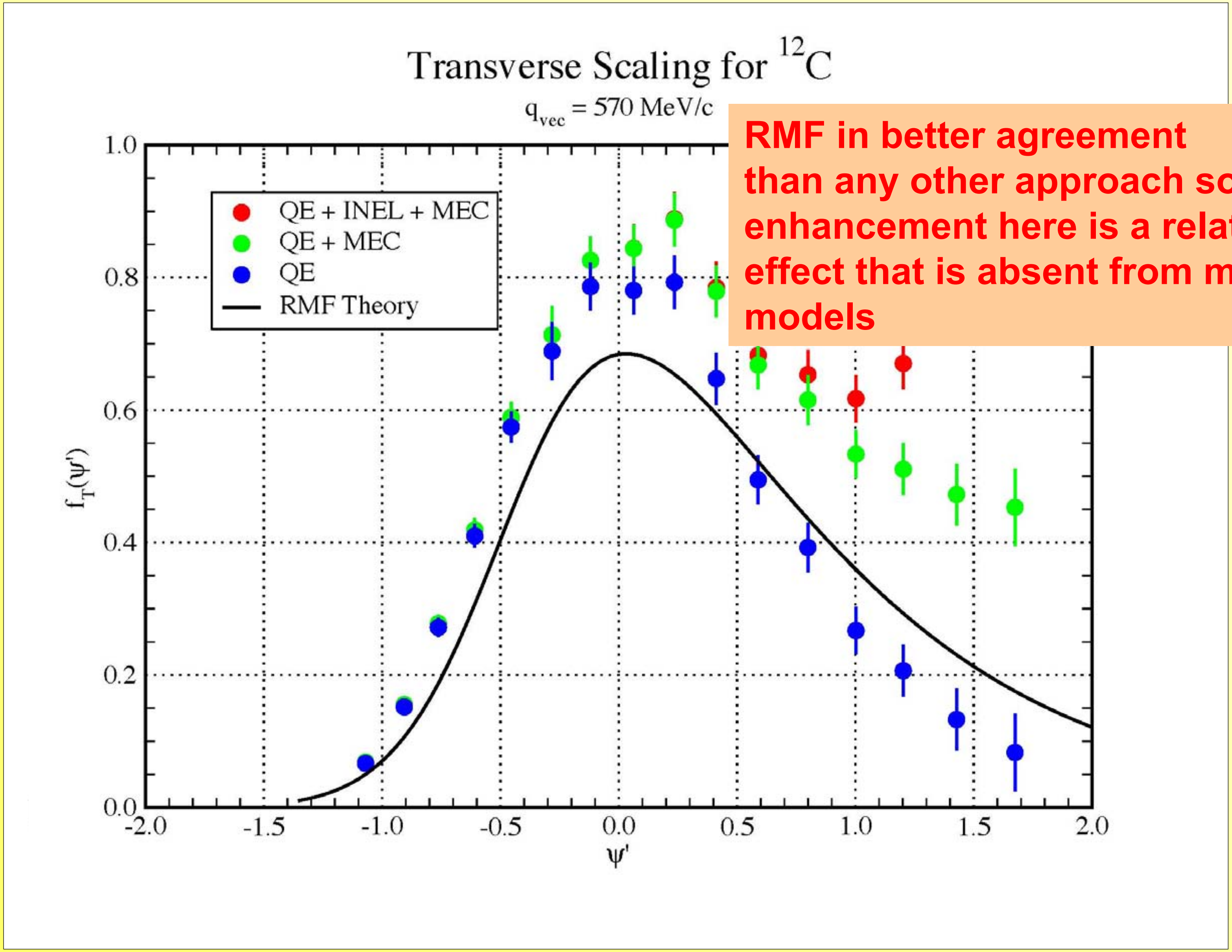


Peaks above 0.8 … in contrast to the longitudinal case where the peak value was 0.55



First clear evidence for violations of scaling of the 0th kind


Transverse Scaling for 12C
q_vec = 570 MeV/c
QE + INEL + MEC
QE + MEC
QE
RMF Theory
f_T(ψ')
ψ'
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0



RMF in better agreement than any other approach so far; enhancement here is a relativistic effect that is absent from most models

Outline:

- Introduction
- Scaling of the 1st kind (y-scaling)
- Scaling of the 2nd kind
- Scaling of the 0th kind
  and Superscaling
- Non-QE scaling
  - Inelastic scattering
  - 2p-2h MEC effects
- Predicting ν cross sections using scaling
  and scaling of the 3rd kind

Just as for the electron scattering reactions in the QE and Δ regions, we **use the scaling functions determined above**, but now multiply by the corresponding **charge-changing neutrino reaction cross sections** for the Z protons and N neutrons in the nucleus.

For the QE region we have the elementary reactions

$$\nu_\mu + n \to p + \mu^-$$

$$\bar{\nu}_\mu + p \to n + \mu^+$$

While in the Δ region we have

$$\nu_\mu + p \to \Delta^{++} + \mu^-$$

$$\nu_\mu + n \to \Delta^{+} + \mu^-$$

$$\bar{\nu}_\mu + p \to \Delta^{0} + \mu^+$$

$$\bar{\nu}_\mu + n \to \Delta^{-} + \mu^+$$

… and so on.

Note that these reactions are **isovector** only, whereas electron scattering contains both isoscalar and isovector contributions (the transverse EM response is, in fact, predominantly isovector at high energy).

Thus, in going from electron scattering where the universal scaling function came from the L response (essentially 50% isoscalar and 50% isovector) to CC neutrino reactions we have had to invoke

Scaling of the 3rd Kind

where the isospin nature of the scaling functions is assumed to be universal.

The nuclear response function may be decomposed into a generalization of the familiar Rosenbluth expression from studies of electron scattering (see above):

$$R_\chi = \left[\widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_{T} R_{T}\right] + \chi\left[\widehat{V}_{T'} R_{T'}\right]$$

$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

changes sign in going from neutrinos to anti-neutrinos

The cross section is dominantly **transverse** (T, T')

$$R_\chi = \left[\widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_T R_T\right] + \chi\left[\widehat{V}_{T'} R_{T'}\right]$$

$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

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$$R_\chi = \left[\hat{V}_{CC} R_{CC} + 2\hat{V}_{CL} R_{CL} + \hat{V}_{LL} R_{LL} + \hat{V}_{T} R_{T}\right] + \chi\left[\hat{V}_{T'} R_{T'}\right]$$

$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

… and has VV, AA and VA contributions

The cross section is dominantly **transverse** (T, T')

$$R_\chi = \left[\widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_T R_T\right] + \chi\left[\widehat{V}_{T'} R_{T'}\right]$$

$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

… and has **VV**, AA and VA contributions

The **VV** response has the same (isovector) contributions as occur for electron scattering, including the **2p-2h MEC contributions**

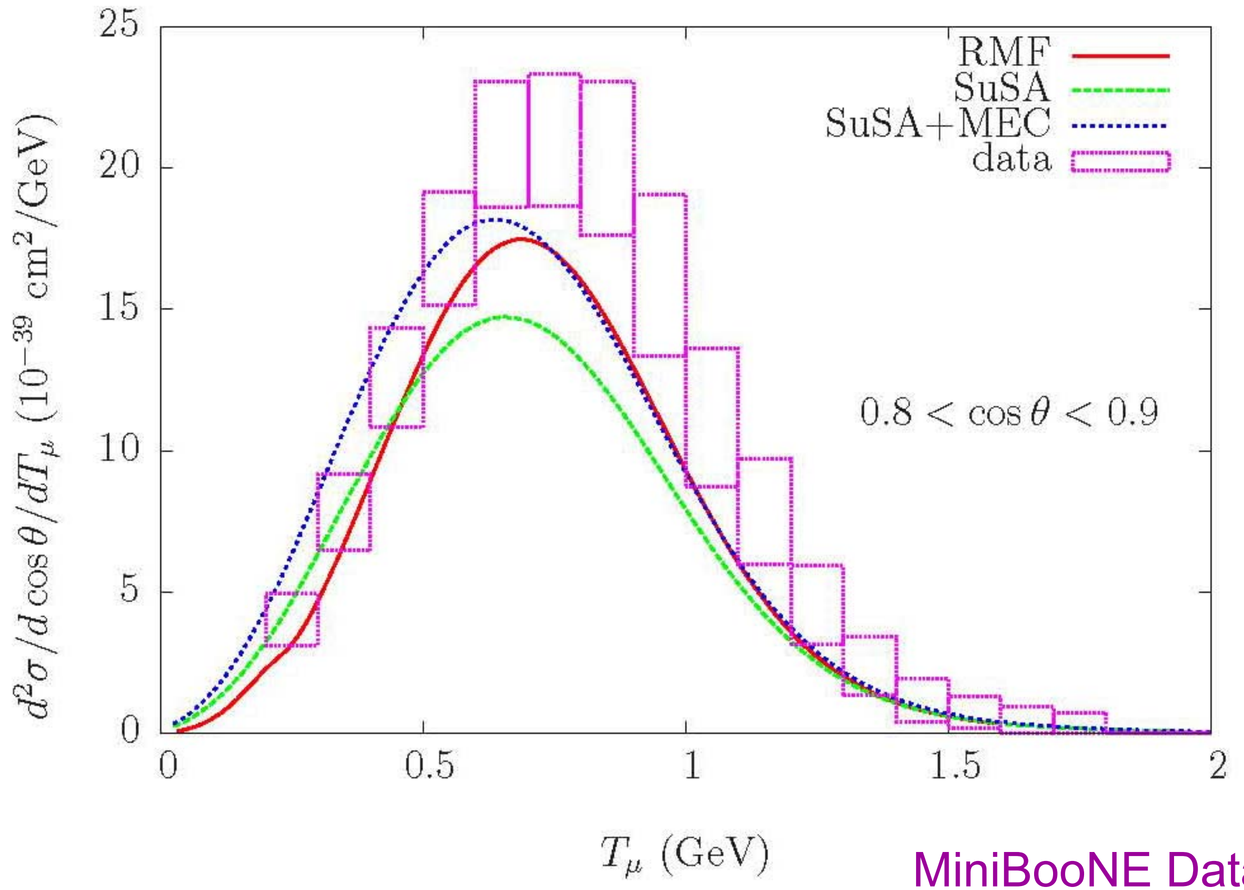
The cross section is dominantly **transverse** (T, T')

$$R_{\chi} = \left[\widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_{T} R_{T} \right] + \chi \left[\widehat{V}_{T'} R_{T'} \right]$$

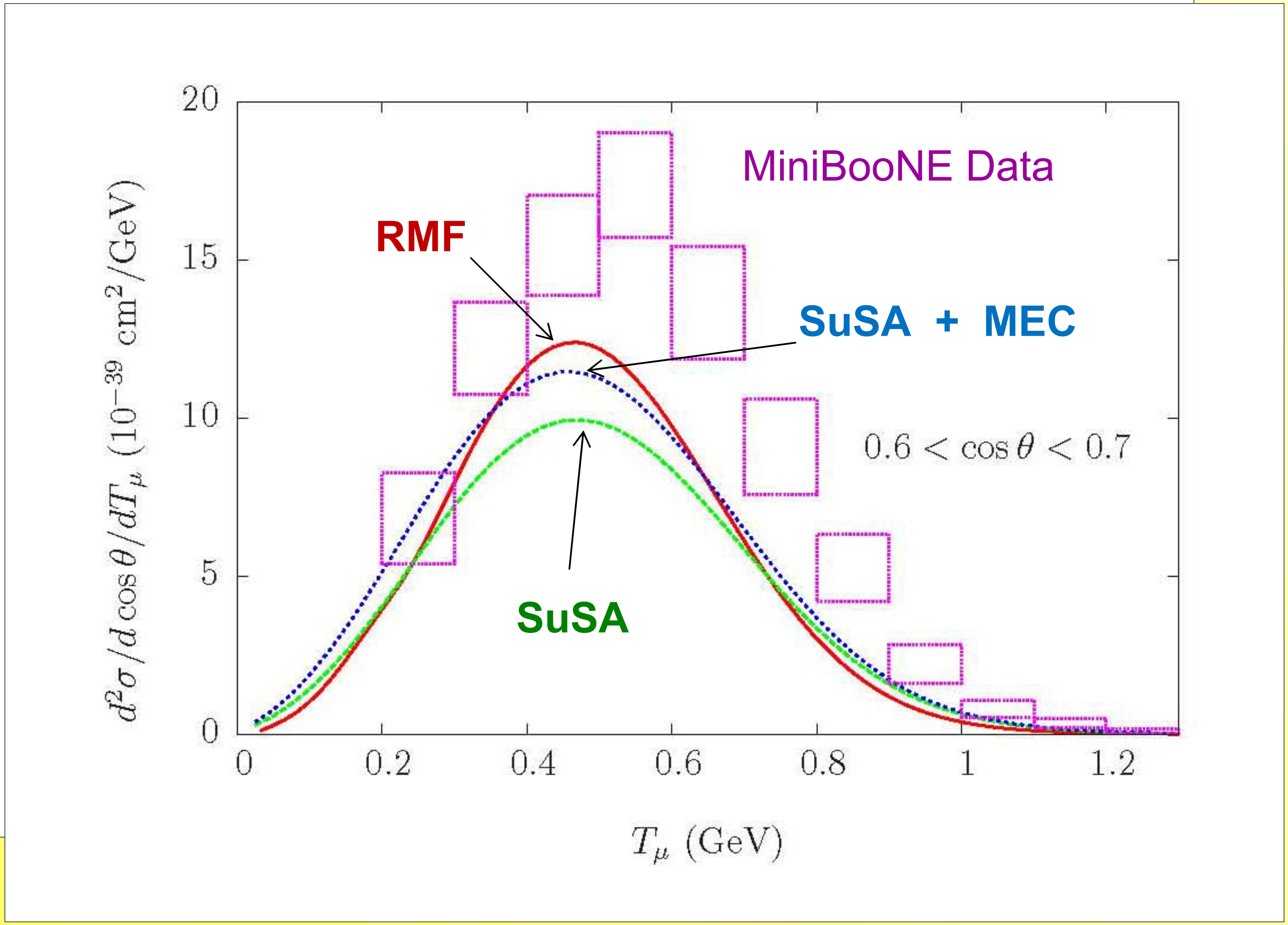
$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

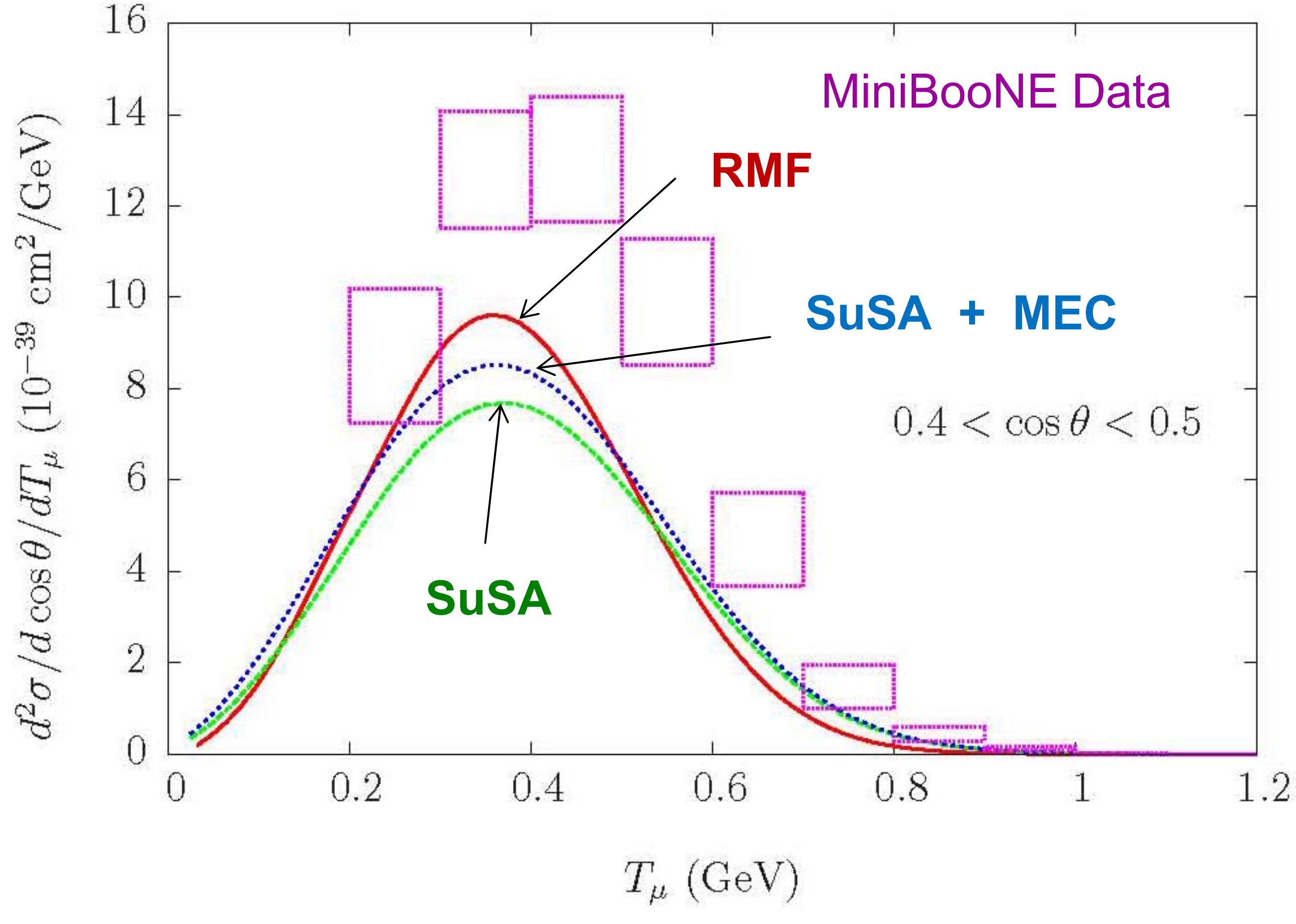
… and has VV, **AA** and **VA** contributions

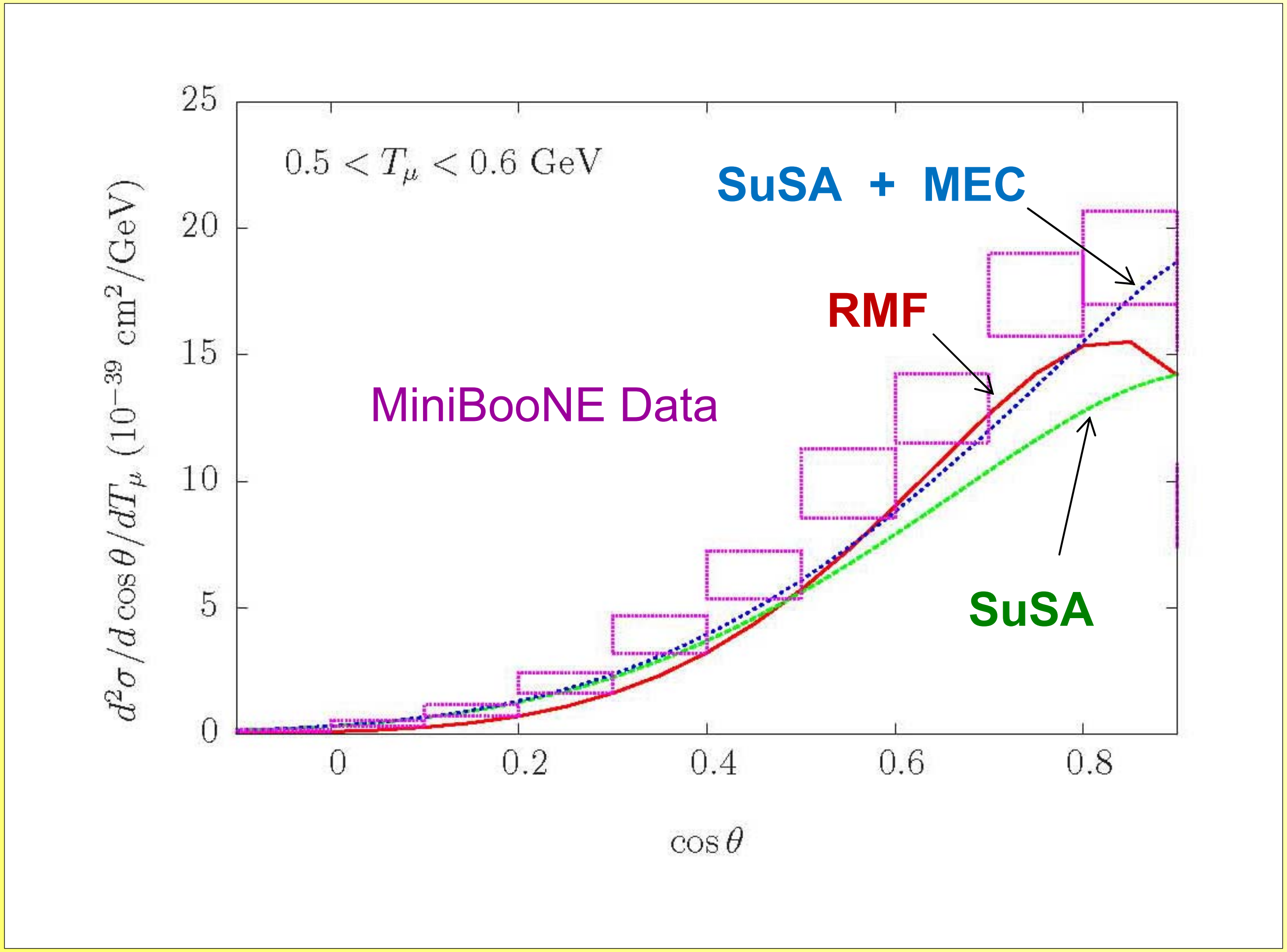
The VV response has the same (isovector) contributions as occur for electron scattering, including the 2p-2h MEC contributions; however, the **transverse axial-vector matrix elements have no MEC pieces** in leading order and thus the **AA** and **VA** contributions do not contain the scaling violations from MEC



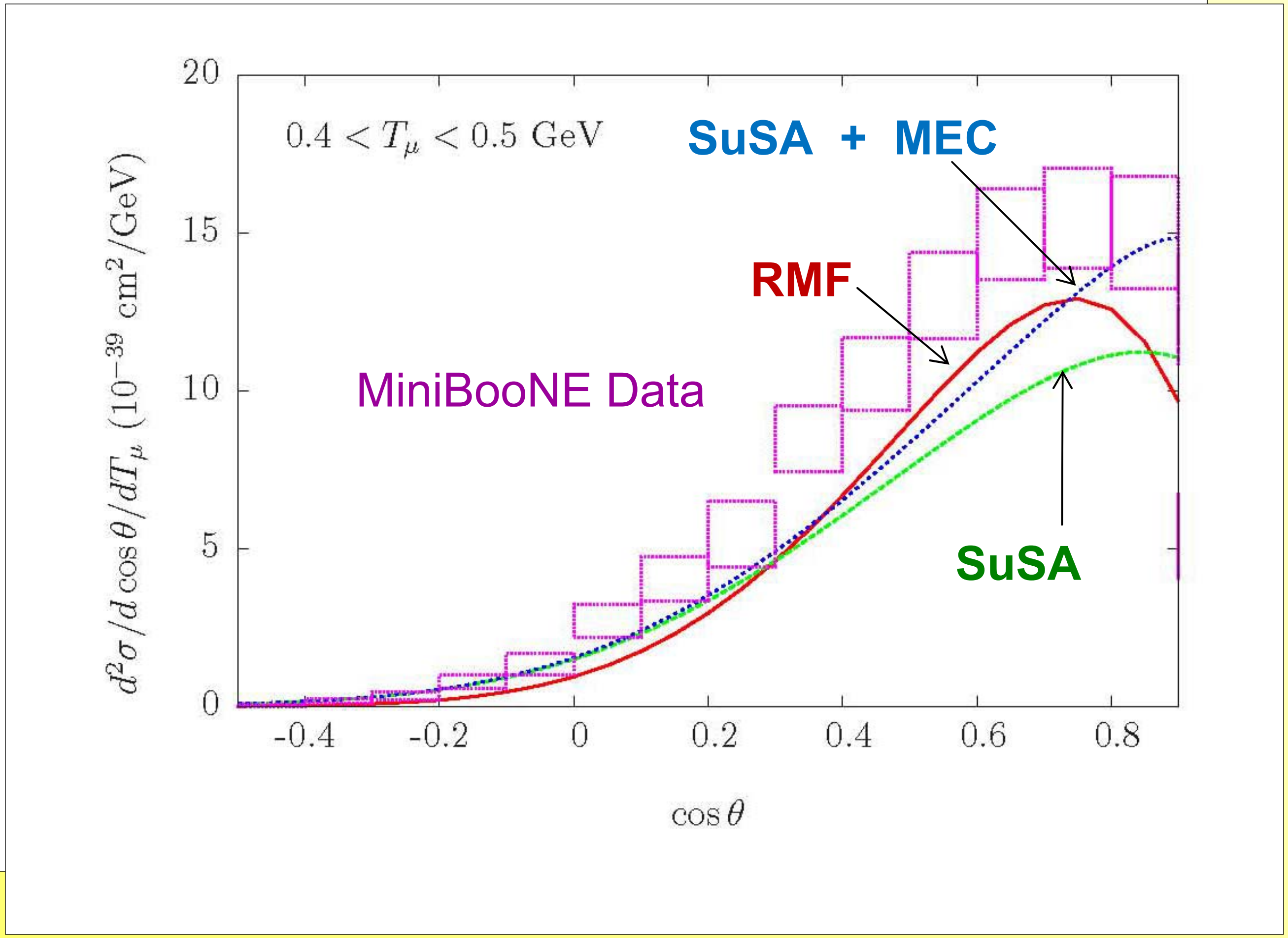
MiniBooNE Data

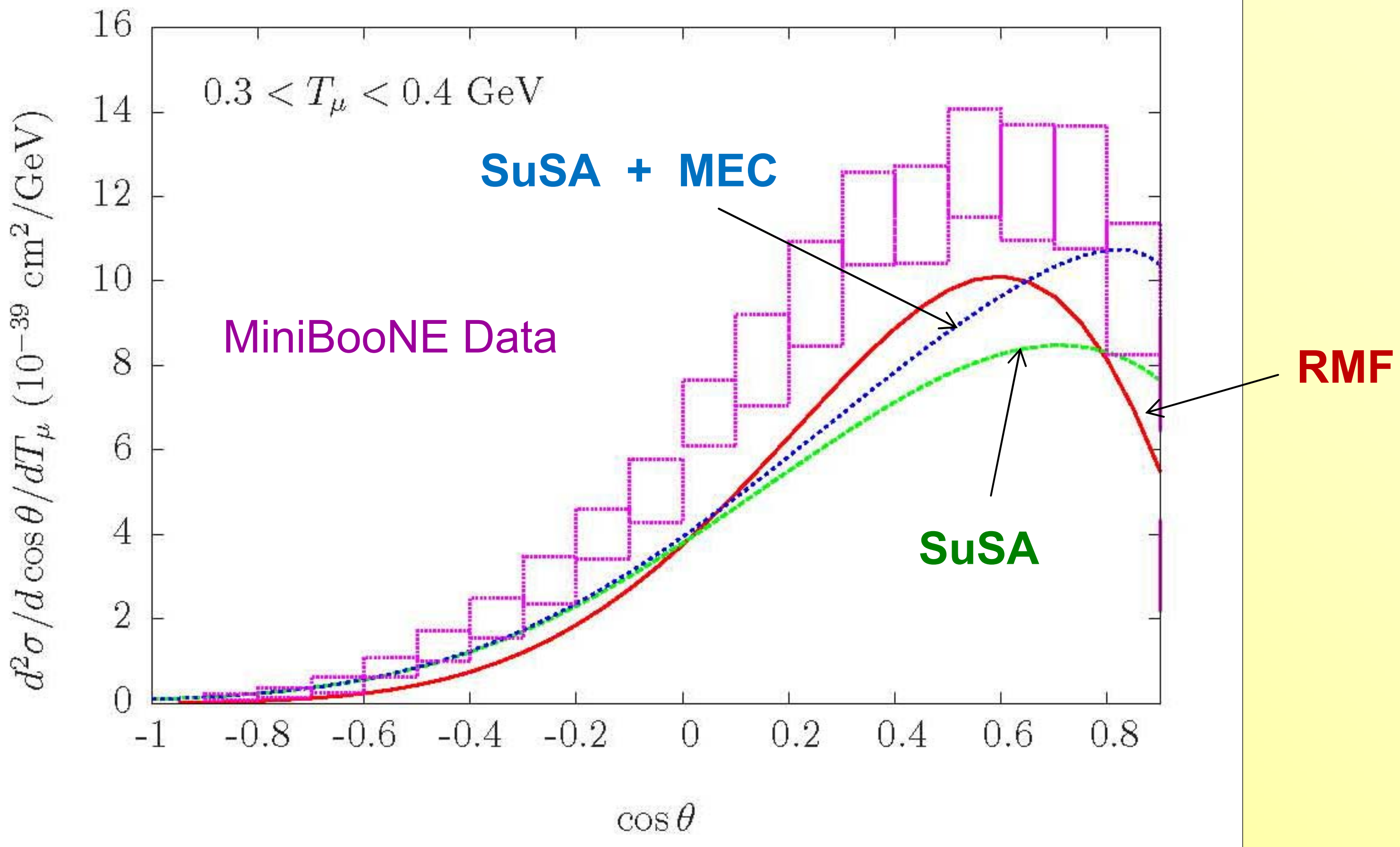

MiniBooNE Data
RMF
SuSA + MEC
SuSA
$0.6 < \cos\theta < 0.7$
$d^2\sigma/d\cos\theta/dT_\mu$ (10^{-39} cm^2/GeV)
T_μ (GeV)
0
5
10
15
20
0
0.2
0.4
0.6
0.8
1
1.2


MiniBooNE Data
RMF
SuSA + MEC
SuSA
$0.4 < \cos\theta < 0.5$
$d^2\sigma/d\cos\theta/dT_\mu\ (10^{-39}\ \mathrm{cm}^2/\mathrm{GeV})$
T_μ (GeV)
0
2
4
6
8
10
12
14
16
0
0.2
0.4
0.6
0.8
1
1.2

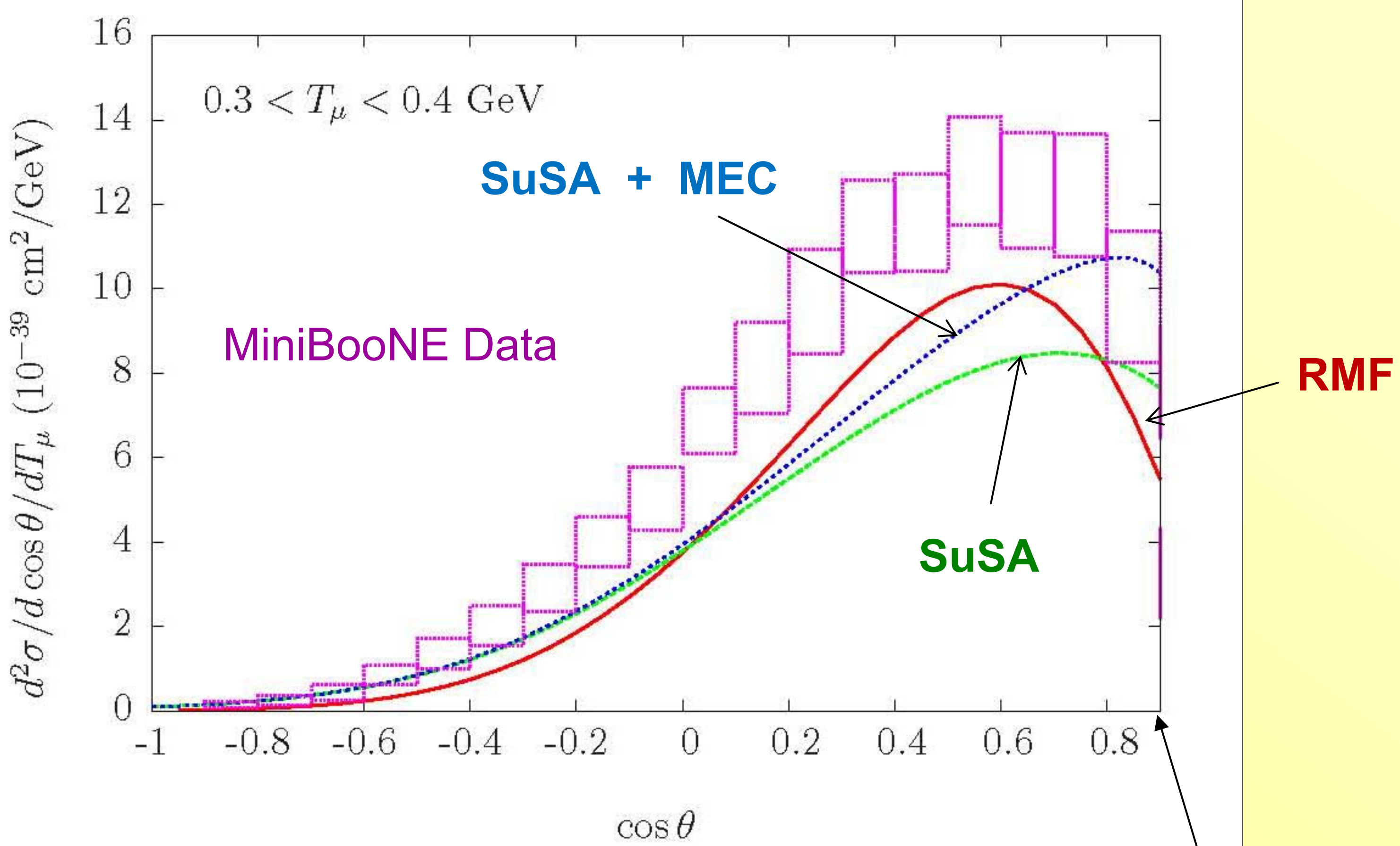


$0.5 < T_\mu < 0.6$ GeV
SuSA + MEC
RMF
MiniBooNE Data
SuSA
$d^2\sigma/d\cos\theta/dT_\mu$ (10^{-39} cm^2/GeV)
$\cos\theta$
0
5
10
15
20
25
0
0.2
0.4
0.6
0.8


$0.4 < T_\mu < 0.5$ GeV
SuSA + MEC
RMF
MiniBooNE Data
SuSA
$d^2\sigma/d\cos\theta/dT_\mu$ $(10^{-39}$ cm^2/GeV)
$\cos\theta$
20
15
10
5
0
-0.4
-0.2
0
0.2
0.4
0.6
0.8



$0.3 < T_\mu < 0.4$ GeV
SuSA + MEC
MiniBooNE Data
RMF
SuSA
$d^2\sigma/d\cos\theta/dT_\mu$ (10^{-39} cm^2/GeV)
$\cos\theta$
16
14
12
10
8
6
4
2
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8



Note: results here cut off at 0.9, since between 0.9 and 1 roughly ½ of the cross section arises from excitations below 50 MeV

SuperScaling Approach (SuSA) Collaboration:

- Maria Barbaro and Alfredo Molinari, University of Torino, Torino, Italy
- Juan Caballero and Chiara Maieron, University of Sevilla, Sevilla, Spain
- Quique Amaro, University of Granada, Granada, Spain
- Elvira Moya de Guerra and Jose Udias, Complutense University, Madrid, Spain
- Ingo Sick, University of Basel, Basel, Switzerland
- with Claude Williamson and TWD @ M.I.T.


. . . thank you