An Overview of Neutrino-Hadron Interactions at Low and Intermediate Energies

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Precise Neutrino-Hadron Interactions

The need for precise estimates and measurements of the cross sections.

Among the topics to be covered today will be pion production at small Q^2.

Remarks on CP violation.

Coherent pion Production.

The transition from resonaces to DIS.

1.Introduction

- The long base-line experiments require identical comparisons in the near and the far away detectors to decipher differences caused by the oscillations. As an example I show the distortion on the spectrum in the muon energy caused by the oscillation from KEK to the SK detector.

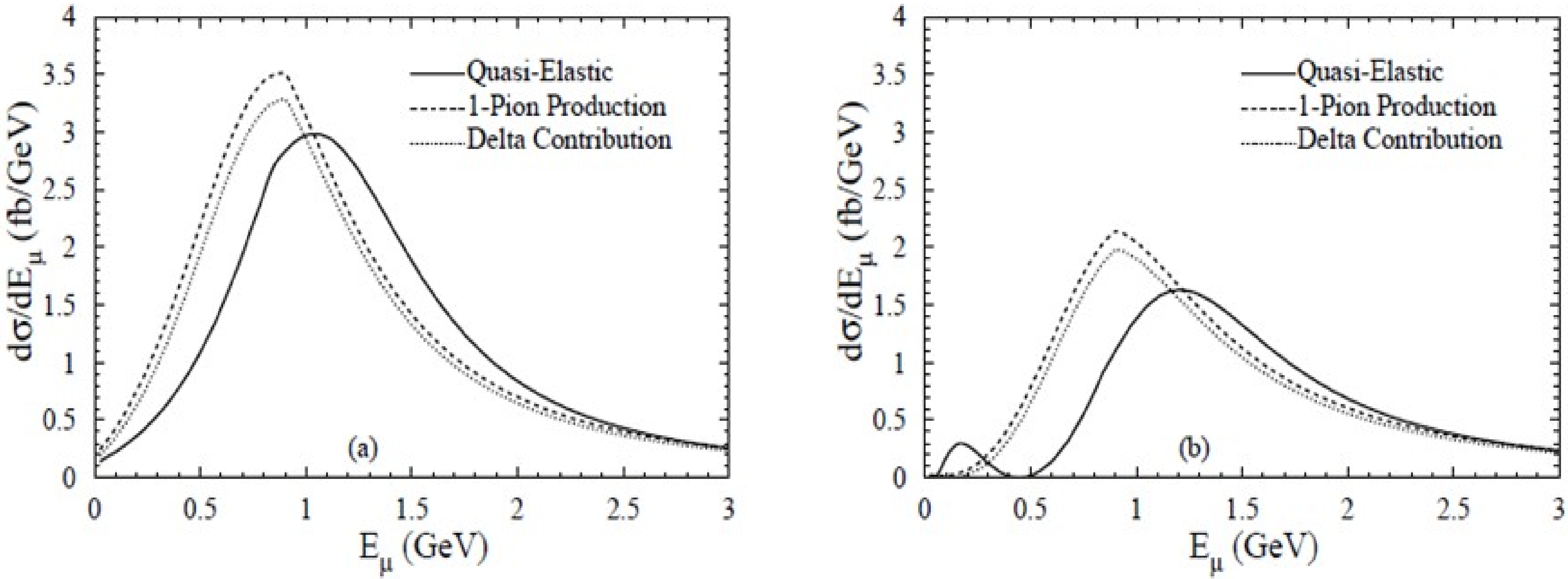


Figure 2: Approximate prediction of the muon energy spectra for the (a) Nearby and (b) SK

Two quantities to measure

$$P(\overline{\nu}_\mu \to \overline{\nu}_\mu) = 1 - \sin^2(2\overline{\theta})\sin^2\left(\frac{1.267\Delta\overline{m}^2 L}{E}\right)$$

$$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\overline{\nu}_\mu \to \overline{\nu}_e)} \simeq \frac{\Delta m_{12}^2 L}{4E_\nu} \cdot \frac{\sin 2\theta_{12}}{\sin\theta_{13}} \cdot \delta_{CP}.$$

$$U_{\mathrm{vac}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}c_{23} \end{pmatrix}$$

2. Propreties of neutrino reactions for pion production

- Vector form factors determined by CVC and electroproduction data published by (Lalakulich+Paschos+Piranishvili)

- Matrix elements of the axial current have 4
  form factors, out of them two contribute to
  the divergence because the other two orthogonal to q,mu
  (Special Property) : The neutrino reactions at small Q^2 select these two form factors.

Explanation on next page.

$$\left\langle \Delta^{++} \,\middle|\, \mathcal{A}_\mu^+ \,\middle|\, p \right\rangle = \sqrt{3}\, \overline{\psi}_\lambda(p') i g_\mu^\lambda C_5^A(Q^2) u(p) + \sqrt{3} \frac{i f_\pi q_\mu}{q^2 - m_\pi^2} \left\langle \Delta \,\middle|\, j_\pi \,\middle|\, p \right\rangle .$$

$$\begin{aligned} \epsilon^\mu(\lambda = 0) \left\langle \Delta^{++} \,\middle|\, \mathcal{A}_\mu \,\middle|\, p \right\rangle &= \epsilon^\mu(\lambda = 0) i\sqrt{3}\overline{\psi}_\mu(p') C_5^A(Q^2) u(p) \\ &\approx i\sqrt{3} \frac{q^\mu}{\sqrt{Q^2}} \overline{\psi}_\mu(p') C_5^A(Q^2) u(p) + \mathcal{O}\left(\frac{Q^2}{\nu^2}\right) \\ &\approx -\frac{f_\pi \sqrt{2}}{\sqrt{Q^2}} A(\pi^+ p \to \Delta^{++}) \end{aligned}$$

Pion production at small Q^2

- Cross Section with zero helicity and longitudinal polarization. The formula is exact without transverse polarizations.

$$\frac{\mathrm{d}\sigma^A}{\mathrm{d}Q^2\mathrm{d}\nu} = \frac{G_F^2|V_{ud}|^2}{2\pi}\frac{1}{4\pi}\frac{\nu}{E_\nu^2}\frac{f_\pi^2}{Q^2}\left\{\tilde{L}_{00} + 2\tilde{L}_{l0}\frac{m_\pi^2}{Q^2+m_\pi^2} + \tilde{L}_{ll}\left(\frac{m_\pi^2}{Q^2+m_\pi^2}\right)^2\right\}\sigma(\pi^+ p \to X^{++}). \tag{12}$$

including the muon mass the factor infront of the cross section is

$$\tilde{L}_{00} = 4\left[\frac{[Q^2(2E_\nu - \nu) - \nu m_\mu^2]^2}{Q^2(Q^2+\nu^2)} - Q^2 - m_\mu^2\right]$$

$$W_2^A(Q^2,\nu) = \frac{2f_\pi^2}{\pi}\frac{\nu}{Q^2+\nu^2}\sigma(\pi^+ p \to X^{++})$$

Compare factors infront for Paschos-Schalla vs Berger-Sehgal

E=1 GeV	Q^2 =0.010GeV^2				Q^2 =0.10 GeV^2	
v GeV	Q^2 /min	Pasc+sch	BS		Pasc+Sch	BS
0.20	0.0028	.329	.215		.150	.189
0.25	0.0037	.240	.161		.140	.163
0.30	0.0048	.171	.122		.130	.136
0.35	0.0061	.119	.095		.110	.117
0.40	0.0074	.079	.073		.095	.097
0.45	0.0091	.047	.056		.085	.078
0.50	0.0112	.022			.070	.064
0.55	0.136				.058	.051

Alternative general formula

$$\frac{d\sigma}{dQ^2 dW} = \frac{G^2}{4\pi}\cos^2\theta_C \frac{W}{m_N E^2}\left\{\mathcal{W}_1(Q^2+m_\mu^2)\right.$$
$$+\frac{\mathcal{W}_2}{m_N^2}\left[2(k\cdot p)(k'\cdot p)-\frac{1}{2}m_N^2(Q^2+m_\mu^2)\right]$$
$$-\frac{\mathcal{W}_3}{m_N^2}\left[Q^2 k\cdot p-\frac{1}{2}q\cdot p(Q^2+m_\mu^2)\right]$$
$$\left.+\frac{\mathcal{W}_4}{m_N^2}m_\mu^2\frac{(Q^2+m_\mu^2)}{2}-2\frac{\mathcal{W}_5}{m_N^2}m_\mu^2(k\cdot p)\right\}$$

2a. Adler sum rule

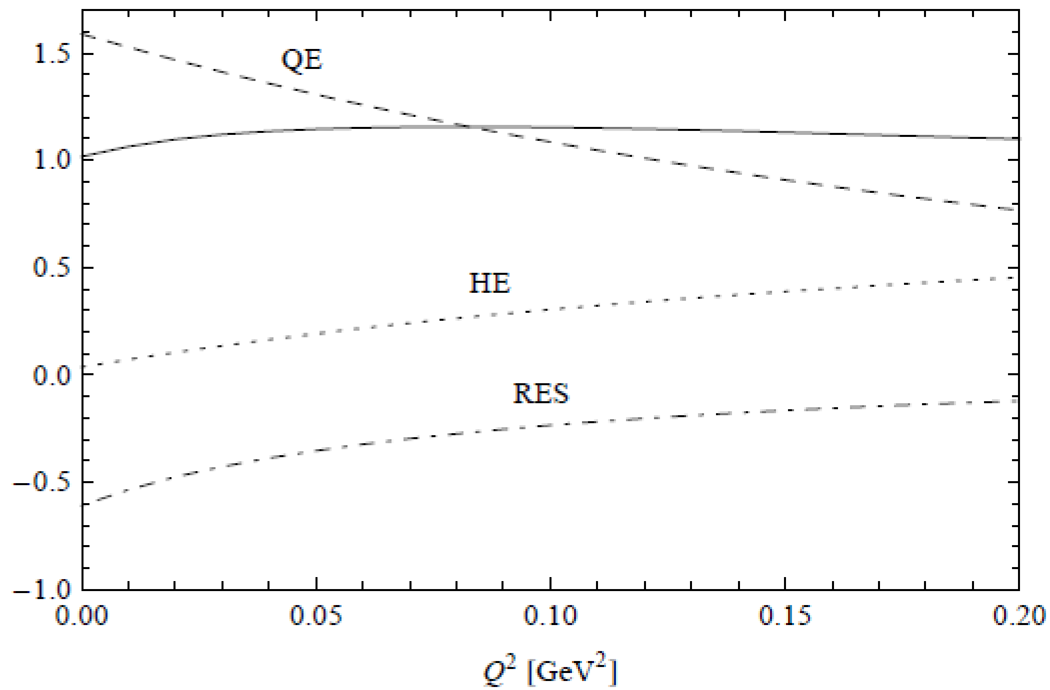
$$W_2^A(Q^2,\nu) = \frac{2f_\pi^2}{\pi}\frac{\nu}{Q^2+\nu^2}\sigma(\pi^+ p \to X^{++})$$

$$\left[g_A(Q^2)\right]^2 + \frac{2f_\pi^2}{\pi}\int_{\nu_{th}}^{\infty} \mathrm{d}\nu \frac{\nu}{Q^2+\nu^2}\left[\sigma^{\pi^+ n}(\nu) - \sigma^{\pi^+ p}(\nu)\right] = 1.$$

Values for Ev=1.0 GeV and Q^2= 0.010GeV^2 and typical pion-nucleon cross sections used

ν [GeV]	W [GeV]	$\sigma(\pi^+ p)$ [mb]	$\sigma(\pi^- p)$ [mb]	$\frac{\nu f_\pi^2 \tilde{L}_{00}}{E_\nu Q^2}$ [GeV]
0.20	1.118	16	12	0.329
0.25	1.159	77	30	0.240
0.30	1.199	189	67	0.171
0.35	1.238	175	63	0.119
0.40	1.275	95	37	0.079
0.45	1.311	60	28	0.047
0.50	1.347	42	26	0.022
0.55	1.381	31	28	0.002

Comparison with Adler sum rule



The difference below indicates the magnitude of the high energy contribution

- 1- $[g_A(Q^2)]^2 + \frac{2f_\pi^2}{\pi}\int_{\nu_{th}}^{\infty} \mathrm{d}\nu \frac{\nu}{Q^2+\nu^2}\left[\sigma^{\pi^+ n}(\nu) - \sigma^{\pi^+ p}(\nu)\right]$

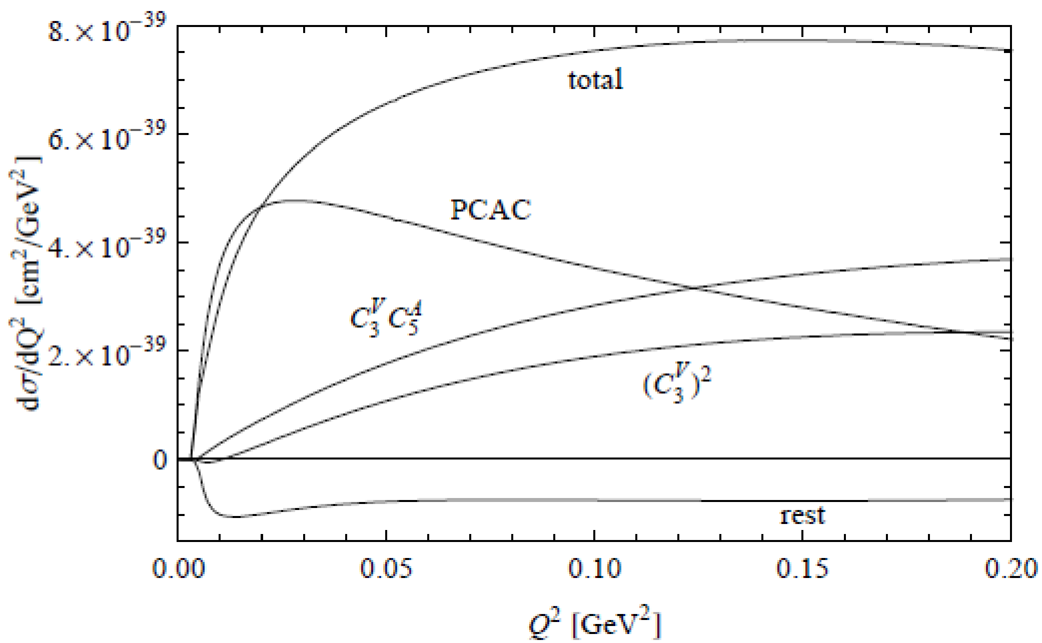
The difference gives how big is the high energy contribution (from A. Bodek)

0	0
0.05	0.075067224
0.10	0.152807688
0.15	0.236130089
0.20	0.304015692
0.25	0.378436883
0.30	0.433828477
0.35	0.487216568
0.40	0.540360896
0.45	0.575967877

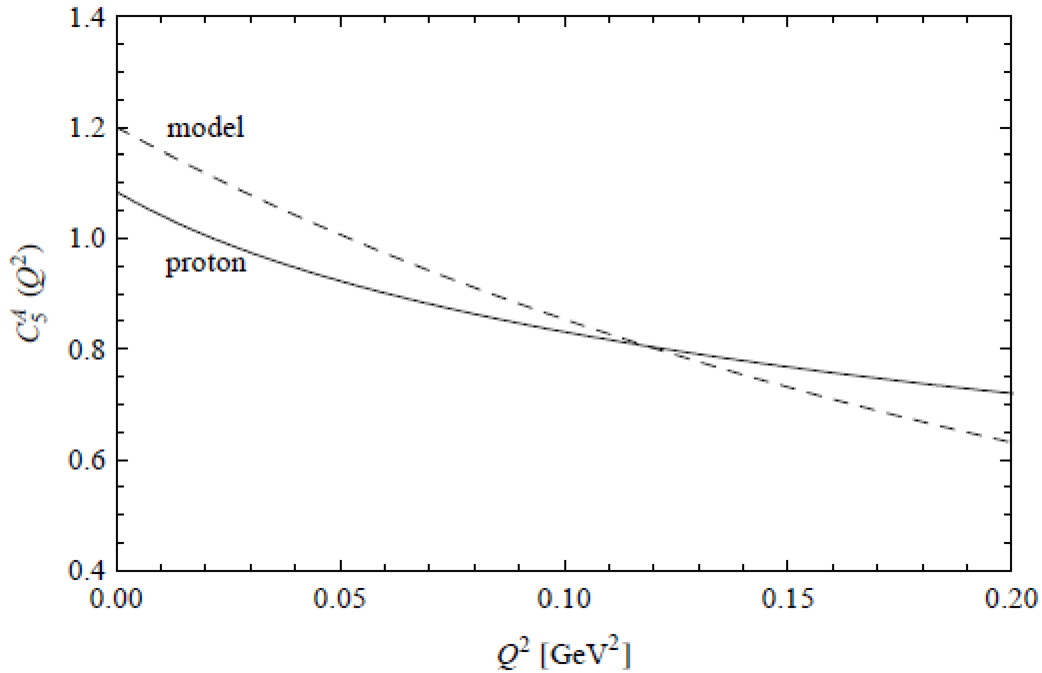
Extended estimates from Bodek

- Q2 axial Integral of d-u above W>1.4
- 0 0
- 0.05 0.075067224
- 0.10 0.152807688
- 0.15 0.236130089
- 0.20 0.304015692
- 0.25 0.378436883
- 0.30 0.433828477
- 0.35 0.487216568
- 0.40 0.540360896
- 0.45 0.575967877
- 0.50 0.614372485
- 0.55 0.645610486
- 0.60 0.67523349
- 0.65 0.70076928

Vector, Axial, Interference



Determine C5(A, Q^2)



2b. Comparison with experiments

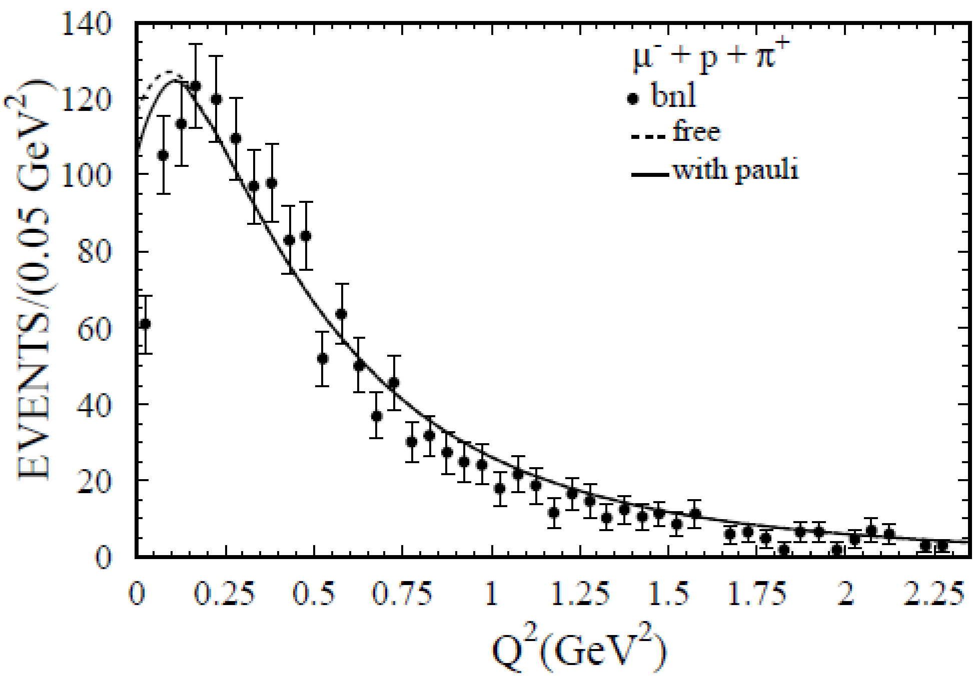


Figure 6: Q^2-spectrum of the process $\nu p \to \mu^- p \pi^+$.

Brookhaven exp.

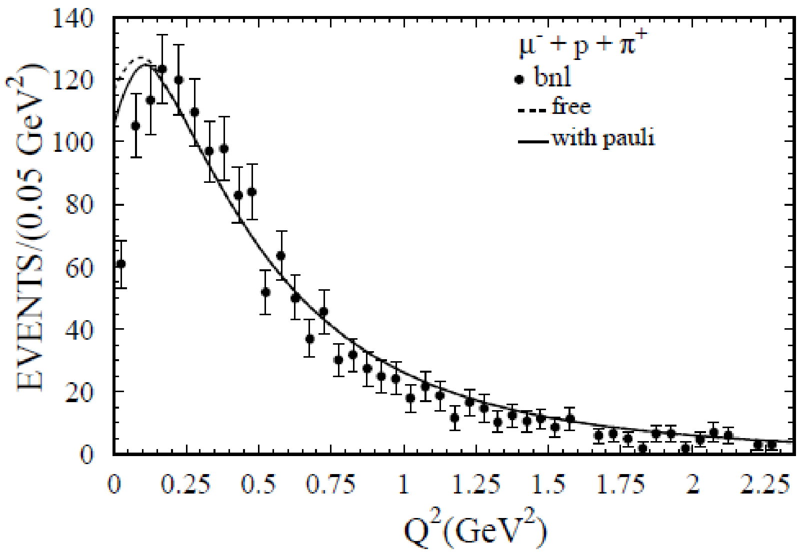
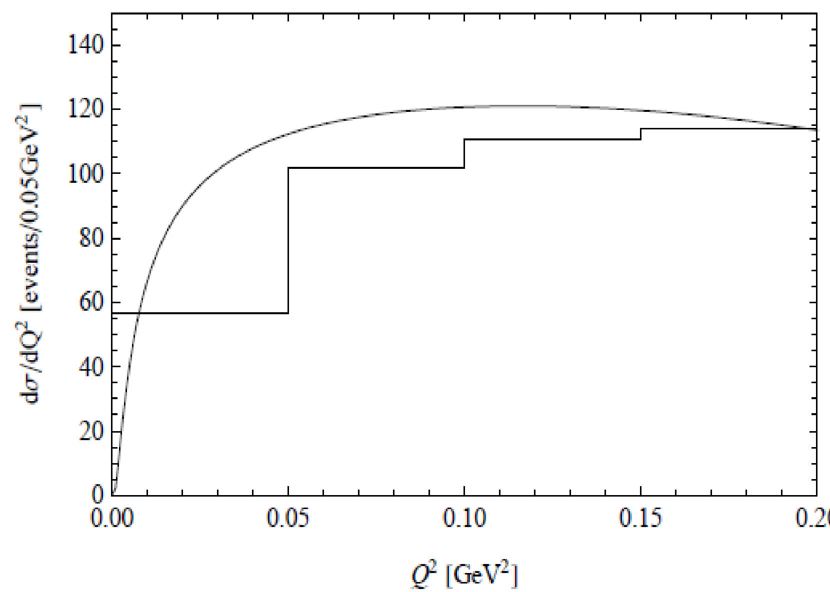
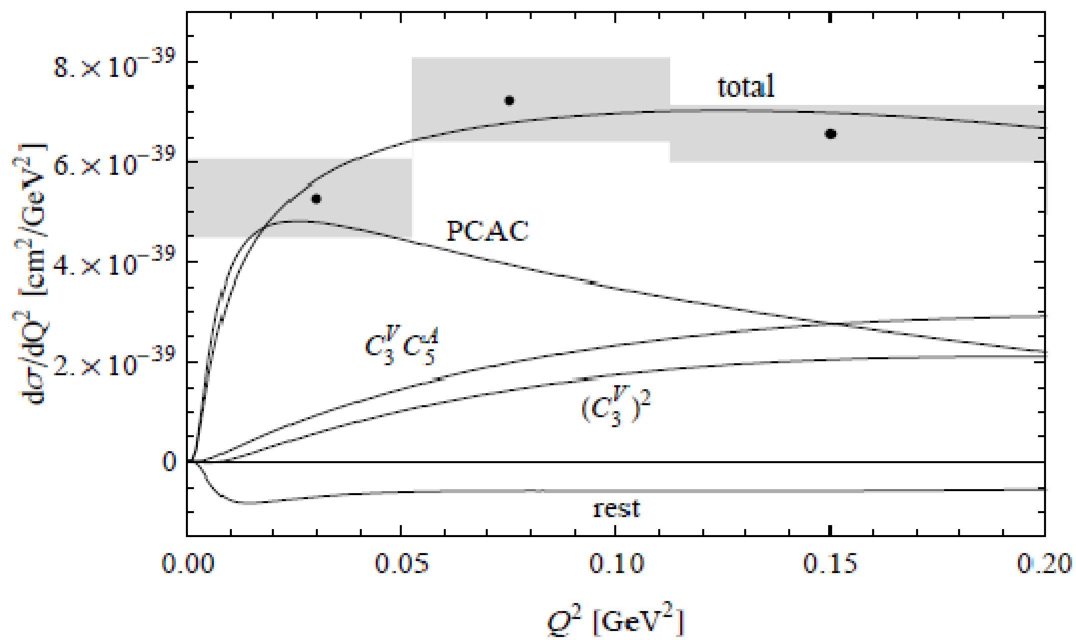


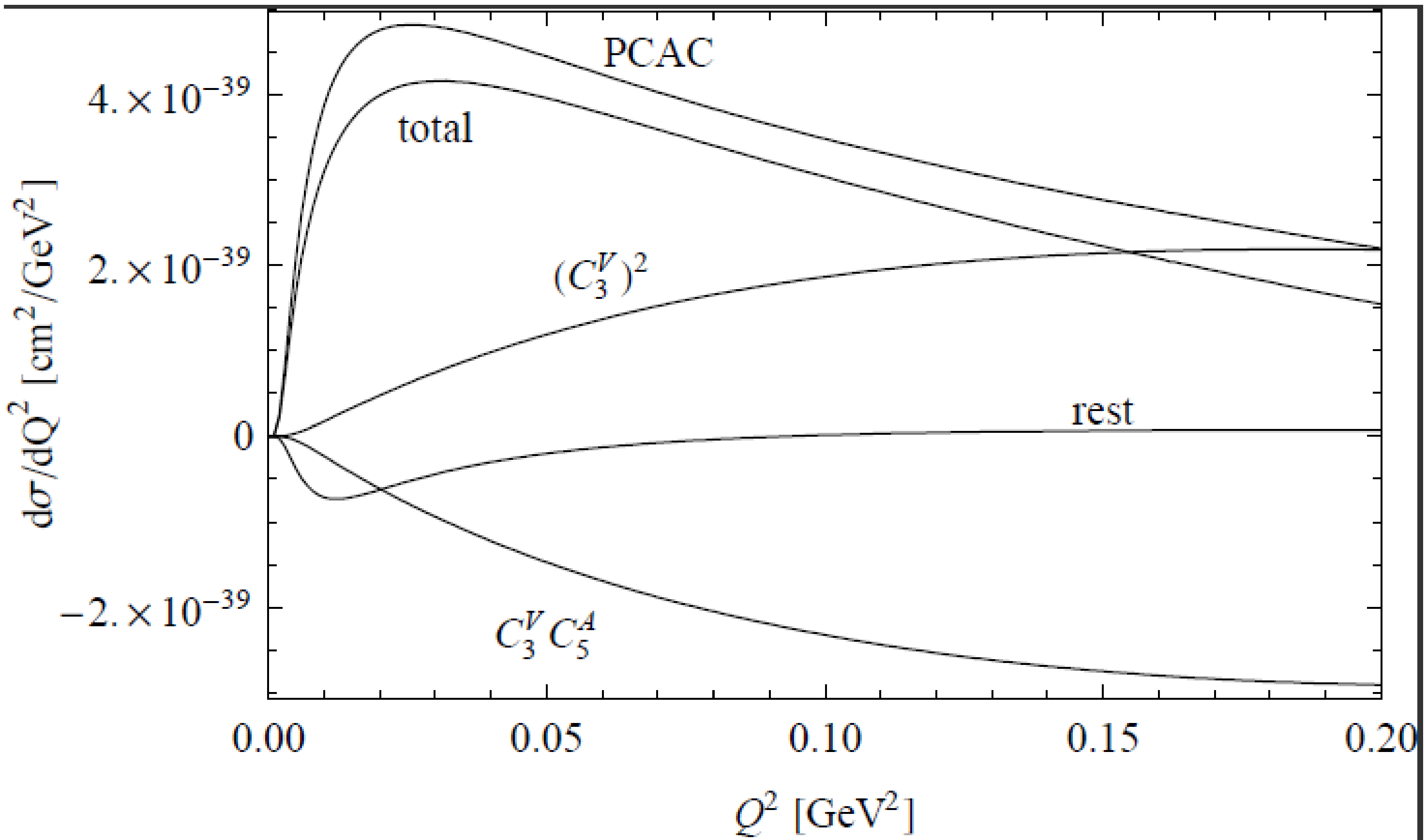
Figure 6: Q^2-spectrum of the process $\nu p \to \mu^- p \pi^+$.



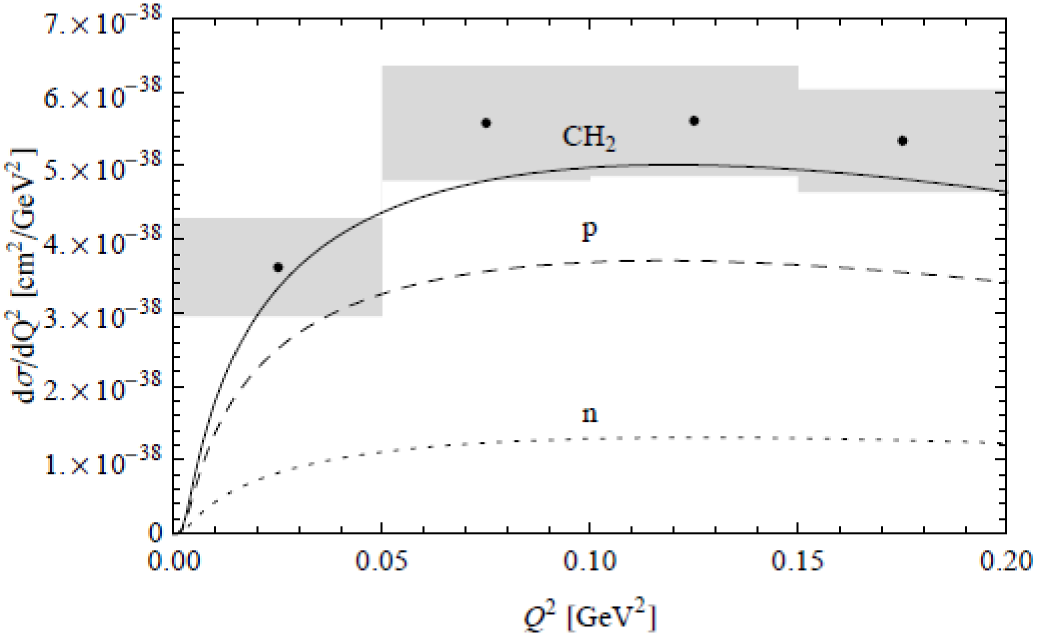
Argonne data



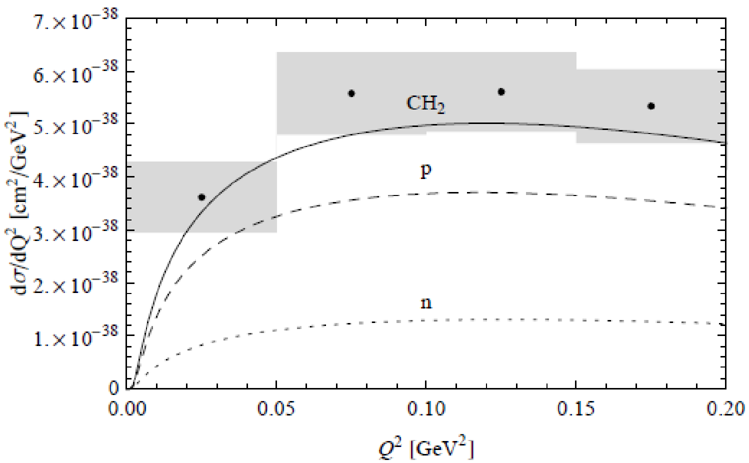
Argonne antineutrino

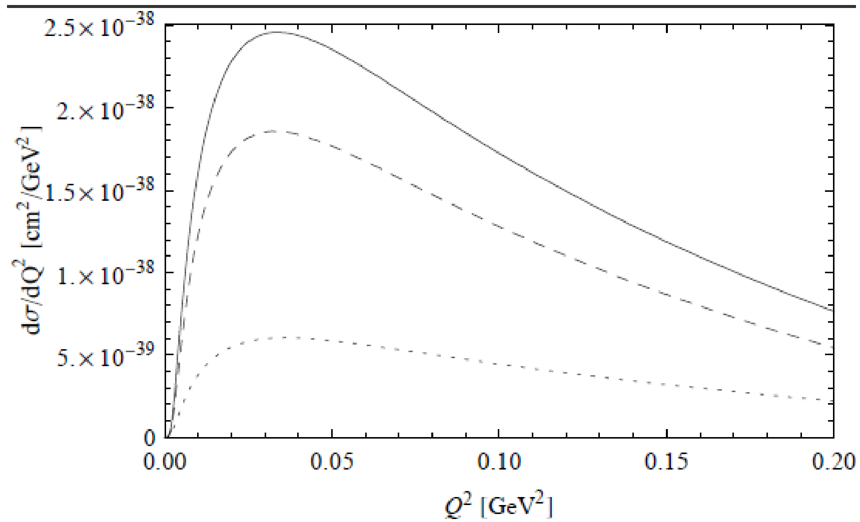


MiniBoone data

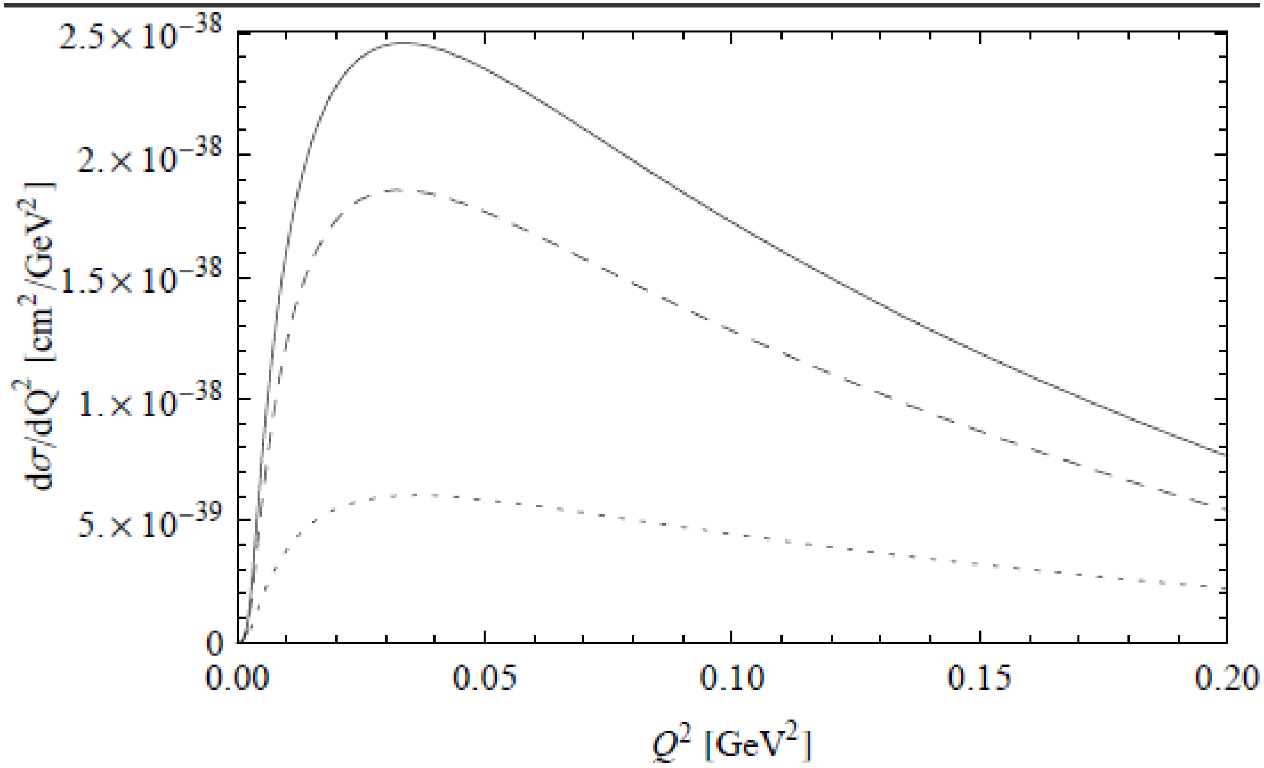


neutrino vs antineutrino





Antineutrinos at MiniBoone



3. Remark on CP Violation

The transition for ν_μ to ν_e over the distance of MINOS

$$| < \nu_e|\nu_\mu > |^2 \approx 2\ s_{13}^2\, sin^2 \frac{\Delta_{23}L}{4E} - \frac{1}{2}\, sin2\theta_{12}\ s_{13}\, sin\delta\, sin\frac{\Delta_{12}L}{2E}$$

For MINOS distance: $sin^2 \frac{\Delta_{23}L}{4E} \approx 1$ and $sin\frac{\Delta_{12}L}{2E} \approx 1.3 \times 10^{-1}$

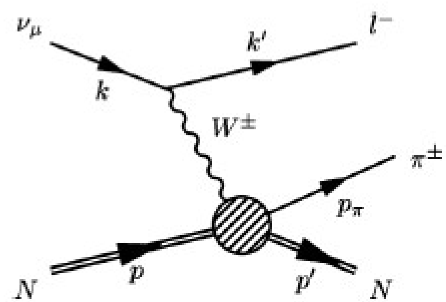
For maximal θ_{13} it gives

$$| < \nu_e|\nu_\mu > |^2 = 0.10 - 1.1 \times 10^{-2} sin\delta$$

- The antineutrino cross section is smaller and has a different shape which may provide a signature for the signal.
- Comparisons with neutral current cross sections are also helpful.

4. Coherent Pion Production

- The process occurs for small Q^2 <0.15 Gev^2



Feynman diagram of the charged current reaction.

$$\left(\frac{Q^2 + m_\pi^2}{2\nu}\right)^2 \leq |t| \leq \infty$$

$$\frac{d\sigma_\pi}{dt} = a \exp[-b|t|]$$

Same formula with nuclear cross section

- The cross section now refers to pion- nucleus elastic scattering

(π N→π N)

$$\frac{\mathrm{d}\sigma_{NC}}{\mathrm{d}Q^2\mathrm{d}\nu\mathrm{d}t} = \frac{G_F^2}{4(2\pi)^2}\frac{\nu}{E_\nu^2}\frac{f_\pi^2}{Q^2}\tilde{L}_{00}\frac{\mathrm{d}\sigma_\pi}{\mathrm{d}t}.$$

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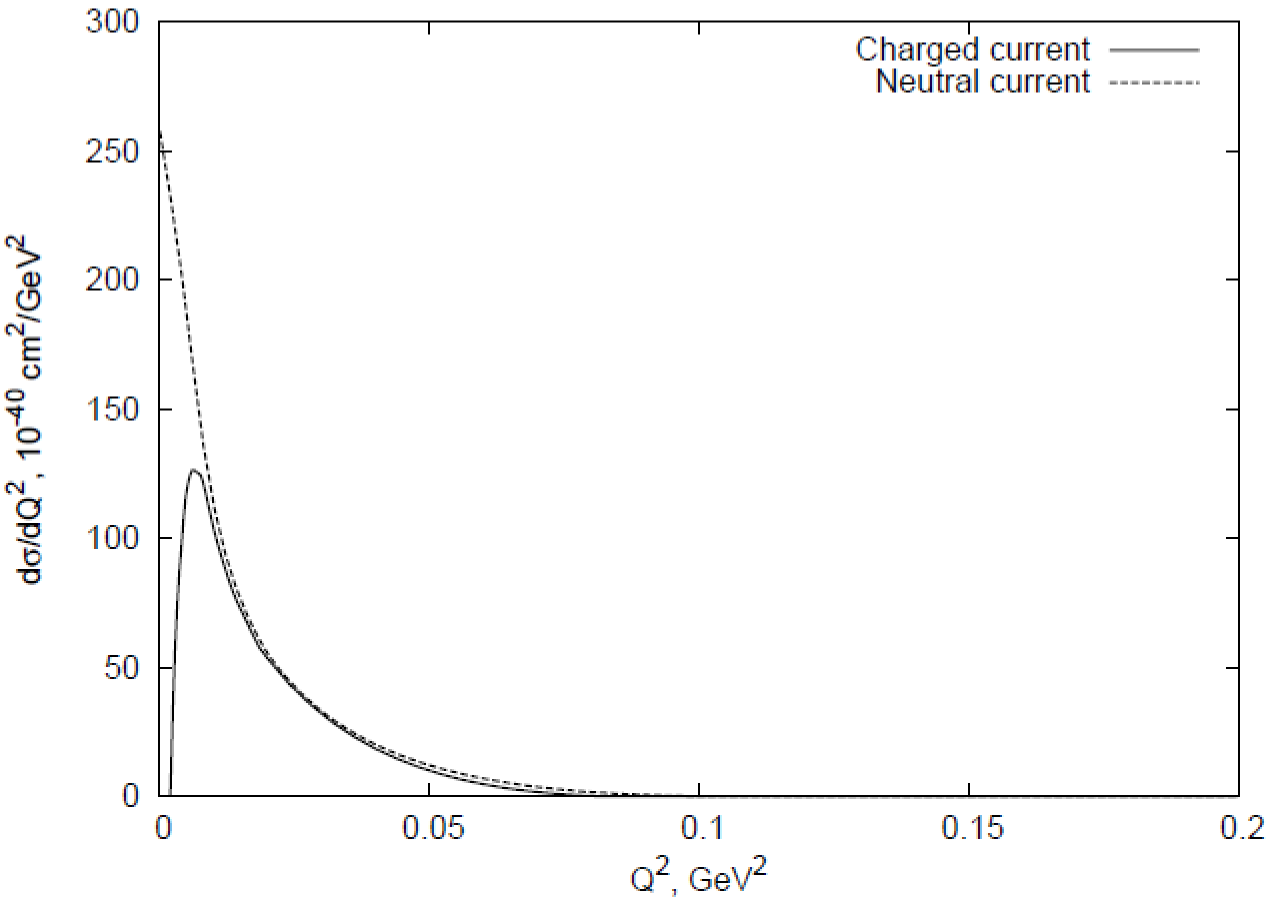
Pion Nucleus data for Carbon

TABLE I. Parameters of the elastic pion nucleus cross sectior model.

ν (GeV)	a (barn/GeV^2)	b (GeV^{-2}
0.210	28.526	159.657
0.228	28.659	147.986
0.260	32.012	129.022
0.290	27.162	101.910
0.320	23.600	90.824
0.340	22.734	90.660
0.370	19.000	83.814
0.400	17.924	84.590
0.420	14.594	73.256
0.766	3.759	49.459
0.864	4.172	58.149
0.942	3.649	56.197

Coherent CC and NC reactions



Summary

- The region of small Q^2 is accurately determined (10% ??). Data accurate to
  a few %.
- Confirmation from the Adler sum rule also provides an estimate for multipion production at larger Q^2.
- C5(A,Q^2) prefers larger value ~ 1.10.
- Comparisons look good but can be improved.
- Antineutrino cross sections much smaller, may help searches for CP violating phase.
- Applications to many other rections possible.

References

1. Lalakulich and Paschos PR D 71 (2006)
2. Paschos and Schalla ,arXiv 11.02.4466
3. Gounaris et al., PR D74,054007(2006)
4. Berger and Sehgal, PR D79,0530(2009)
5. Ji Young Yu et al. Phys.Lett. B574(2003) 232.