Diffractive neutrino-production of pions

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Minerva, Fermilab,
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Outline

Historical overview

- Process & kinematics
- Adler relation and beyond

Evaluation in color dipole model

- Evaluation on the proton target
- Evaluation on the nuclear target

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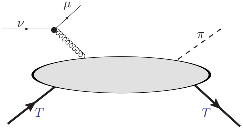
Kinemaitcs

- Diffractive pion production, $\nu T \to \mu \pi T$



$$E_\nu = \frac{p \cdot k_\nu}{m_N}, \nu = \frac{p \cdot q_W}{M}, y = \frac{p \cdot q_W}{p \cdot k}$$

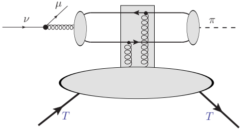
  - T is either proton or nucleus
  - neutrino may be ν_μ, ν_e

$$Q^2 = -q_W^2 = 4E_\nu (E_\nu - \nu) \sin^2 \frac{\theta}{2} + \mathscr{O}\left(m_l^2\right)$$

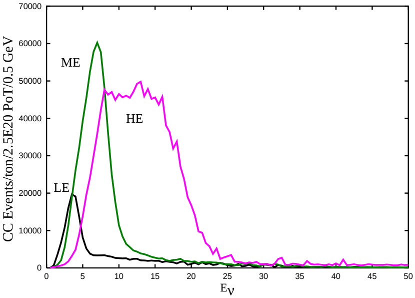
$$t = (p' - p)^2 = \Delta^2 = t_{min} - \Delta_\perp^2$$

Kinemaitcs

- Diffractive pion production, $\nu T \to \mu \pi T$



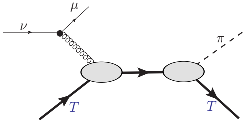
- Diffractive kinematics, energy $\nu \gg \nu_{min} \sim (Q^2 + m_\pi^2) R_A$ *Minerva@Fermilab*:



- High statistics
- *Differential* cross-sections are measured
- Proton and nuclear targets (C, Fe, Pb)

Kinemaitcs

- Diffractive pion production, $\nu T \to \mu \pi T$
- Diffractive kinematics, energy $\nu \gg \nu_{min} \sim (Q^2 + m_\pi^2) R_A$
- In the small-ν dominant contribution comes from resonances
  MiniBooNE, SciBooNE, ...
  More on this in talk by E. Paschos



Adler relation & PCAC

- PCAC Hypothesis: Opeartor relation which connects the operators,

$$\partial_\mu A_\mu \sim m_\pi^2 \phi_\pi(x)$$

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- For the case of small $q^2 \approx m_l^2 \approx 0$ and $k_\mu \sim q_\mu$, so lepton tensor may be cast to the form

$$L_{\mu\nu} = 2\frac{E_\nu\left(E_\nu - \nu\right)}{\nu^2} q_\mu q_\nu + \mathcal{O}\left(q^2\right) + \mathcal{O}\left(m_l^2\right)$$

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- So the cross-section may be evaluated using the PCAC hypothesis (*S. Adler, 1966*)
$$\left.\frac{d\sigma_{\nu T \to lF}}{d\nu dQ^2}\right|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T \to F}$$

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Rein-Sehgal corrections

- Rein, Sehgal offered an absorptive correction factor

$$\left.\frac{d\sigma_{\nu T\to lF}}{d\nu dQ^2}\right|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu}\sigma_{\pi T\to F} F_{abs}$$

$$F_{abs} = e^{-\langle x\rangle/\lambda},$$
$$\lambda^{-1} = \sigma_{in}\langle\rho_A\rangle$$
$$\langle x\rangle \approx \frac{3}{4}R_A$$

which takes into account nuclear effects.

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  which takes into account nuclear effects.
- More accurate treatment should take into account the nuclear effects in the Gribov-Glauber framework

Why Adler relation needs improvements ?

- In real measurements, we have $q^2 \neq 0$, so Adler contribution for longitudinal part requires extrapolation (maybe up to a few GeV^2).
  - $$Q^2 = 4E_\nu(E_\nu - \nu)\sin^2\frac{\theta}{2} + \mathcal{O}\left(m_l^2\right),$$
    so for $E_\nu \sim 20$ GeV, $\nu \sim 10$ GeV, selection $Q^2 \lesssim 0.1$ GeV^2 requires $\theta \lesssim 2.2\times 10^{-2}$rad.
  - Phenomenological approach: extrapolate with formfactor
    $$\sim \frac{m_A^2}{Q^2+m_A^2},$$
    $m_A \sim 1$GeV
    - Does not explain the value of m_A, e.g. why $m_A \neq m_\pi$?

Why Adler relation needs improvements ?

- In real measurements, we have $q^2 \neq 0$, so Adler contribution for longitudinal part requires extrapolation (maybe up to a few GeV^2).
- In addition, we have contributions from transverse part and from the vector part ($\mathcal{O}\left(q^2\right)$ for small q^2)
  - For vector current:

$$\frac{d^2\sigma_{\nu p\to lF}}{d\nu\, dQ^2} = \frac{G^2}{4\pi^2}\frac{|\vec{q}|}{E_\nu}\frac{Q^2}{1-\varepsilon}\left[\sigma_\perp^V\left(\nu, Q^2\right) + \varepsilon\sigma_{||}^V\left(\nu, Q^2\right)\right];$$

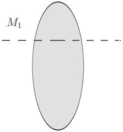
similar expression for $\sigma_\perp^A\left(\nu, Q^2\right)$

Absorptive corrections

Different structure for elastic scattering and meson production

- For elastic meson scatterig they have a form

$$\sigma_{el}^{\pi A} \sim \int d^2 b \left(1 - \exp\left(-\frac{1}{2}\sigma_{\pi N} T_A(b)\right)\right);$$



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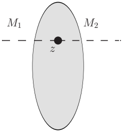
$$\sigma_{el}^{\pi A} \sim \int d^2 b \left(1 - \exp\left(-\frac{1}{2}\sigma_{\pi N} T_A(b)\right)\right);$$

- For diffractive meson production they have a form

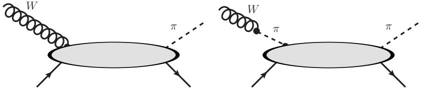
$$\sigma_{\pi A \to MA} \sim \int dz \rho_A(b,z) \exp\left(-\frac{1}{2}\sigma_{\pi N} \int_{-\infty}^{z} d\zeta \rho_A(\zeta)\right)$$
$$\times \exp\left(-\frac{1}{2}\sigma_{MN} \int_{z}^{\infty} d\zeta \rho_A(\zeta)\right)$$
$$\sim \int d^2 b \left[\frac{\exp\left(-\frac{1}{2}\sigma_{\pi N} T_A(b)\right) - \exp\left(-\frac{1}{2}\sigma_{MN} T_A(b)\right)}{\sigma_{\pi N} - \sigma_{MN}}\right]$$



Black disk limit

Adler relation is inconsistent with black disk limit: consider single-pion production,

$$\underbrace{\left.\frac{d\sigma_{\nu T\to l\pi T}}{d\nu\, dQ^2}\right|_{Q^2=0}}_{\text{off-forward diffraction, } W\to\pi} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu-\nu}{E_\nu \nu} \underbrace{\sigma_{\pi T\to\pi T}}_{\text{elastic scattering, } \pi\to\pi}$$



$$\sim \frac{q_\mu}{q^2-m_\pi}$$

(pions are suppressed by lepton mass)

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$\sim 2\pi R$ | $\sim \pi R^2$

Energy dependence in limit $s\to\infty$:

$\sim \ln s$ | $\sim \ln^2 s$

Black disk limit

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$\sim 2\pi R$

$\sim \pi R^2$

Rein-Sehgal factor $\sim \exp(-const\, A^{1/3})$, does not solve the discrepancy

PCAC vs. pion dominance

Adler relation: replace W with π for $Q^2 = 0$

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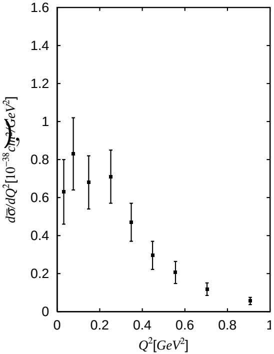
Pion dominance model:

$$T_\mu(\dots) \sim \frac{q_\mu}{q^2 - m_\pi^2} + T_\mu^{non-pion}(\dots),$$

but lepton currents are conserved, so

$$q_\mu L_{\mu\nu} = \mathscr{O}(m_l)$$



$\Rightarrow$contribution of pions is zero *Barish et. al, 1979*

$\Rightarrow$contribution of non-pions should exactly match the contribution of pions

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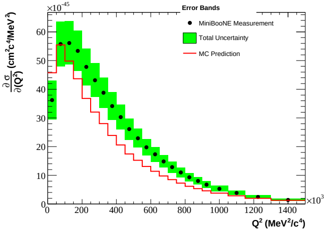

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Chiral symmetry & Adler relation

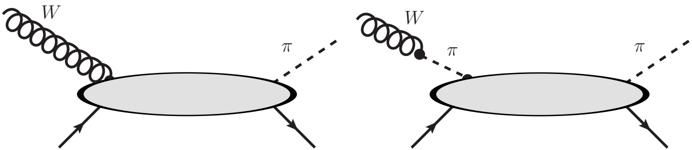


Figure: W may couple directly to quarks in the target or via intermediate pion

$$\mathscr{L}_2 \approx \frac{F^2}{2}\left(\partial_\mu \vec{\phi} - \vec{a}_\mu\right)^2 + \mathscr{O}\left(m, \phi^3, a^3, a^2\phi, \dots\right),$$

$$\mathscr{L}_{\pi N}^{(1)} \approx \bar{\Psi}\left(i\gamma_\mu \partial_\mu + m_N - i\frac{g_A}{4}\gamma_\mu \gamma_5 \left(\vec{a}_\mu - \partial_\mu \vec{\phi}\right)\right)\Psi + \mathscr{O}\left(m, \phi^3, a^3, a^2\phi, \dots\right).$$

$$T_\mu^{(a\to\pi)} = T_{\pi\pi}(p,q)\left(\frac{q_\mu q_\nu}{q^2 - m_\pi^2} - g_{\mu\nu}\right) P_\nu(p, \Delta),$$

Chiral symmetry & color dipole

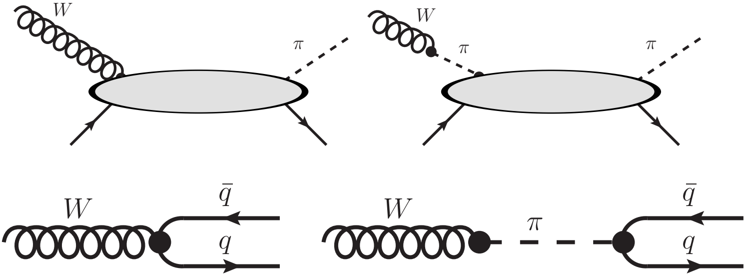
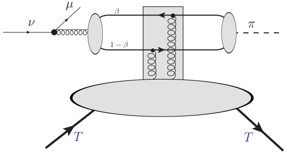


Figure: Relation between couplings $\pi\bar{q}q$, $W\bar{q}q, W\pi$ gurantees that the amplitude remains transverse

$$T_{\mu}^{(a\to\pi)} = T_{\pi\pi}(p,q)\left(\frac{q_\mu q_\nu}{q^2 - m_\pi^2} - g_{\mu\nu}\right) P_\nu\,(p,\Delta)\,,$$

Color dipole and neutrino-proton interactions

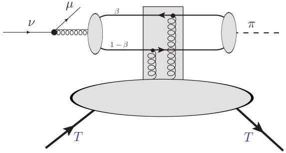


The amplitude has a form

$$\mathscr{A}^{aT\to\pi T}=\int d\beta\, d\beta' d^2r d^2r'\bar{\Psi}_\pi\left(\beta',r'\right)\mathscr{A}_T^d\left(\beta',r';\beta,r\right)\Psi_a\left(\beta,r\right),$$

- $\mathscr{A}_T^d\left(\beta',r';\beta,r\right)$ universal object, depends only on the target T. (We'll discuss it on the next slides)
- $\bar{\Psi}_\pi, \Psi_a$ are the distribution amplitudes of the initial and final states

Color dipole and neutrino-proton interactions



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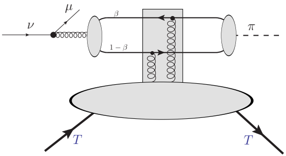
- Earlier applications of color dipole model:
  - Formulated for photon-proton and proton-nuclear processes (vector current)
  - Applications to processes with neutrinos (vector current)
    - electroweak DVCS (*Machado 2007*)
    - charm/heavy meson production (*Fiore, Zoller 2009; Gay Ducati, Machado 2009*)
    - electroweak DIS (*Fiore, Zoller 2005; Gay Ducati*

Color dipole and neutrino-proton interactions



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- Earlier applications of color dipole model:
  - Formulated for photon-proton and proton-nuclear processes (vector current)
  - Applications to processes with neutrinos (vector current)
- We are going to use color dipole for description of the axial current

Extension from vector to axial current

Extension of effective models from vector to axial current is unsafe due to massless pions.
Example: extension of Generalized Vector meson Dominance (GVMD) leads to Piketty-Stodolsky paradox:

$$\sigma_{\pi p \to \pi p} \neq \sigma_{\pi p \to a_1 p}$$

- VMD does not work for axial current, dominant contributions comes from multimeson states $(\rho\pi, \pi\pi\pi, \ldots)$ (*Belkov, Kopeliovich, 1986*)

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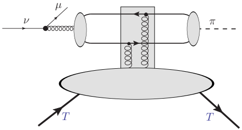
- In color dipole there is no such problems because in the intermediate states we work in quark-gluon basis, not in hadronic like in GVMD

Dipole scattering amplitude (I)

- Universal (depends on the target)

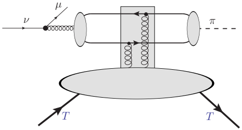


Dipole scattering amplitude (I)

- Universal (depends on the target)
- In the small-r limit behaves like

$$\mathscr{A}^d(\beta, r) \sim r^2$$

(color transparency)



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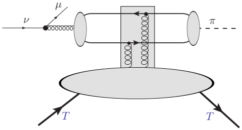

- Its evolution is described by BK equation (*Balitsky 1995; Kovchegov 1999*)

$$\frac{\partial U(x_\perp, y_\perp)}{\partial \mu} = \int d^2 z_\perp K_0(x_\perp, y_\perp; z_\perp)\left(U(x_\perp, z_\perp) + U(z_\perp, y_\perp) - U(x_\perp, y_\perp) + U(x_\perp, z_\perp)\, U(z_\perp, y_\perp)\right),$$

$U(x_\perp, y_\perp)$- dipole propagator,
$\sim \mathscr{A}^d\left(\beta, r = y_\perp - x_\perp, b = \frac{x_\perp + y_\perp}{2}\right)$; $\mu \sim \ln(1/x)$.

$$K_0(x_\perp, y_\perp; z_\perp) \sim \frac{\alpha_s}{2\pi} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} + \mathscr{O}\left(\alpha_s^2\right)$$

The equation is *nonlinear*, solution is numerical

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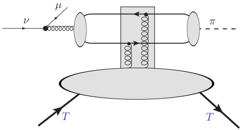

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- There are soft contribiutions, which correspond to large-size dipoles.

Dipole scattering amplitude (II)

- GBW parameterization (*Golec-Biernat, Wusthoff 1998*)

$$\sigma(x,r) = \sigma_0(x)\left(1-\exp\left(-\frac{r^2}{4R_0^2(x)}\right)\right),$$

$$R_0(x) \sim \left(\frac{x}{x_0}\right)^{\lambda/2}$$

Free parameters fixed from fits to DIS, photoabsorption, ...

Has built-in scaling, but for $Q^2 = 0$ $x_B \equiv 0 \Rightarrow$ this parameterization is not applicable.

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- KST parameterization (*Kopeliovich, Schafer, Tarasov 2000*, ...)

$$\sigma(s,r)=\sigma_0(s)\left(1-\exp\left(-\frac{r^2}{4R_0^2(s)}\right)\right),$$

$$R_0(s)\sim\left(\frac{s_0}{s_1+s}\right)^{\lambda/2}$$

Good for low-Q^2, but no scaling for large Q^2

Dipole scattering amplitude (II)

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$$R_0(x) \sim \left(\frac{x}{x_0}\right)^{\lambda/2}$$

- Modified GBW with

$$\tilde{x} = \frac{Q^2 + m^2}{s},$$

$m^2 \sim 1$ GeV^2, has built-in scaling for large Q^2, for low-Q^2 depends only on energy

Dipole scattering amplitude (II)

- b-SAT Parameterization(*Iancu, Itakura, Munier 2004*, ...)

$$\sigma(x,r)=\sigma_0\times\begin{cases}\mathcal{N}_0\left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s+\frac{1}{k\lambda\ln(1/x)}\ln\frac{2}{rQ_s}\right)}, & rQ_s\le 2\\ 1-\exp\left(-A\ln^2\left(BrQ_s\right)\right), & rQ_s>2\end{cases}$$

$$Q_s(x)\sim\left(\frac{x_0}{x}\right)^{\lambda/2}\exp\left(-\frac{b^2}{2C}\right)$$

Based on numerical solution of the BK equation

Distribution amplitudes from the Instanton Vacuum Model

Why IVM ?

- The model is valid for low virtualities Q^2

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Distribution amplitudes from the Instanton Vacuum Model

Why IVM ?

- The model is valid for low virtualities Q^2
- The model has built-in chiral symmetry
- Effective action :

$$S=\int d^4x\left(2\Phi^\dagger(x)\Phi(x)-\bar\psi\left(\hat p+\hat v+\hat a\gamma_5-m-c\bar L f\otimes\Phi\cdot\Gamma_m\otimes fL\right)\psi\right),$$

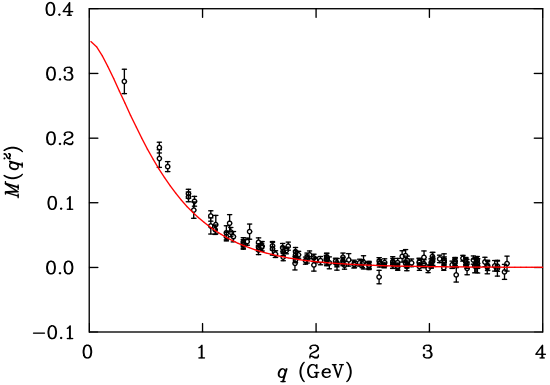
  - may be rewritten as NJL with nonlocal interactions (nonlocality from instanton shape)
  - has only two parameters (average instanton size $\rho\sim 1/600MeV$ and average distance $R\sim 1/200MeV$), but reproduces the low-energy constants in chiral lagrangian with reasonable precision.

Some results from the instanton vacuum model

- Degrees of freedom: mesons and *constituent* quarks

Some results from the instanton vacuum model

- Degrees of freedom: mesons and *constituent* quarks
- The quarks have dynamical mass



Data points: P. Bowman et. al., 2004 (lattice), curve: D.Diakonov et. al., 1986 (IVM). **No fitting !**

Quark propagator

$$S(p) = \frac{1}{\hat{p} + iM(p)}$$

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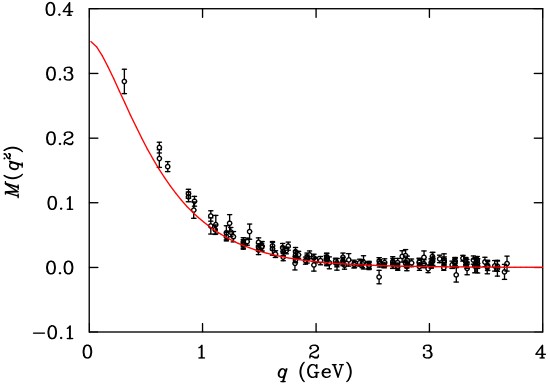
Some results from the instanton vacuum model

- Degrees of freedom: mesons and *constituent* quarks
- The quarks have dynamical mass



- All quark-meson vertices are nonlocal

Some results from the instanton vacuum model

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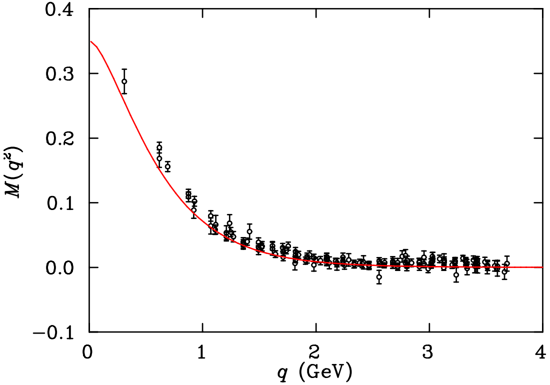

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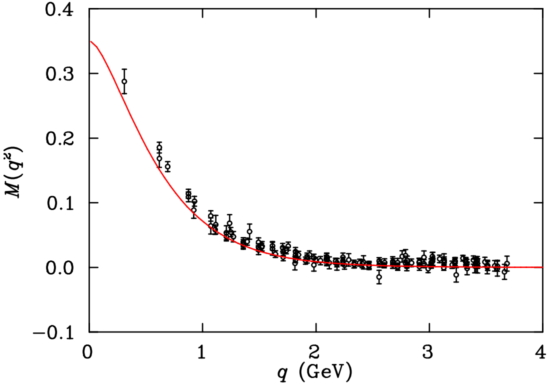

- All quark-meson vertices are nonlocal
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- All scale-dependent quantities (condensates, DAs etc.) are given at the scale $1/\rho$ ∼600 MeV

Some results from the instanton vacuum model

- Degrees of freedom: mesons and *constituent* quarks
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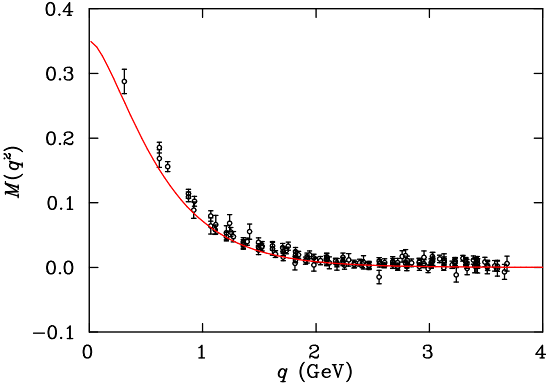


- All quark-meson vertices are nonlocal
- Formal expansion parameter $1/N_c$
- All scale-dependent quantities (condensates, DAs etc.) are given at the scale $1/\rho \sim$600 MeV
- For large momenta pQCD is restored

Distribution amplitudes of pion

Pion distribution amplitudes (P. Ball *et al*, 2006)

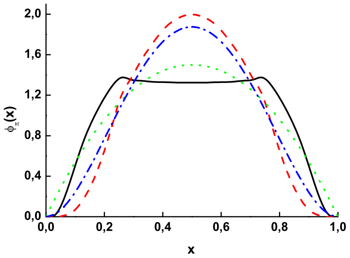
$$\left\langle 0 \left| \bar{\psi}(y)\gamma_\mu\gamma_5\psi(x) \right| \pi(q) \right\rangle = if_\pi \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \times$$
$$\times \left(p_\mu \phi_{2;\pi}(u) + \frac{1}{2}\frac{z_\mu}{(p\cdot z)}\psi_{4;\pi}(u) \right),$$

$$\langle 0 | \bar{\psi}(y)\gamma_5\psi(x) | \pi(q) \rangle = -if_\pi \frac{m_\pi^2}{m_u+m_d}\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \phi_{3;\pi}^{(p)}(u).$$

$$\left\langle 0 \left| \bar{\psi}(y)\sigma_{\mu\nu}\gamma_5\psi(x) \right| \pi(q) \right\rangle = -\frac{i}{3} f_\pi \frac{m_\pi^2}{m_u+m_d}\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \times$$
$$\times \frac{1}{p\cdot z}\left(p_\mu z_\nu - p_\nu z_\mu\right)\phi_{3;\pi}^{(\sigma)}(u),$$

Distribution amplitude of pion

- Most "popular" leading twist contribution is $\phi_{2;\pi}\left(x;\mu^2\right)$



(*A. Dorokhov 2002*)

- We have to take into account all the DAs in order not to kill the chiral symmetry

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Distribution amplitudes of axial meson

Axial DAs (K.-C. Yang 2007)

$$\begin{aligned}\langle 0|\bar{\psi}(y)\gamma_\mu\gamma_5\psi(x)|A(q)\rangle &= if_A m_A \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \times \\ &\times \left(p_\mu \frac{e^{(\lambda)}\cdot z}{p\cdot z}\Phi_{||}(u) + e_\mu^{(\lambda=\perp)} g_\perp^{(a)}(u) - \frac{1}{2} z_\mu \frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_A^2 g_3(u)\right),\end{aligned}$$

$$\langle 0|\bar{\psi}(y)\gamma_\mu\psi(x)|A(q)\rangle = -if_A m_A \varepsilon_{\mu\nu\rho\sigma} e_\nu^{(\lambda)} p_\rho z_\sigma \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \frac{g_\perp^{(v)}(u)}{4}$$

$$\begin{aligned}\langle 0|\bar{\psi}(y)\sigma_{\mu\nu}\gamma_5\psi(x)|A(q)\rangle &= f_A^\perp \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \left(e_{[\mu}^{(\lambda=\perp)} p_{\nu]}\Phi_\perp(u)\right. \\ &+ \left.\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_A^2 p_{[\mu} z_{\nu]} h_{||}^{(t)}(u) + \frac{1}{2} e_{[\mu}^{(\lambda)} z_{\nu]} \frac{m_A^2}{p\cdot z} h_3(u)\right),\end{aligned}$$

$$\langle 0|\bar{\psi}(y)\gamma_5\psi(x)|A(q)\rangle = f_A^\perp m_A^2 e^{(\lambda)}\cdot z \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \frac{h_{||}^{(p)}(u)}{2}.$$

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

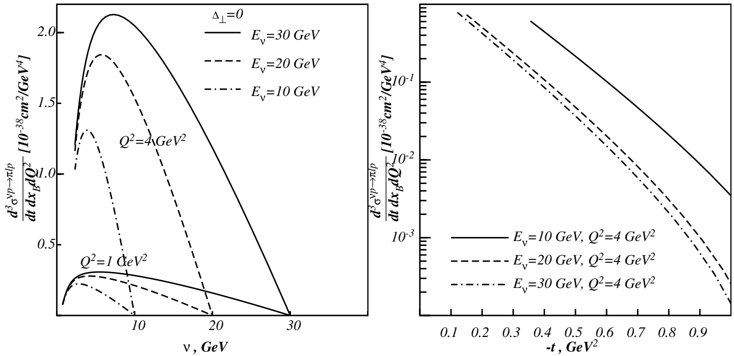

Figure: Differential cross-section $d\sigma/d\nu dt dQ^2$ for different neutrino energies E_ν.

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Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

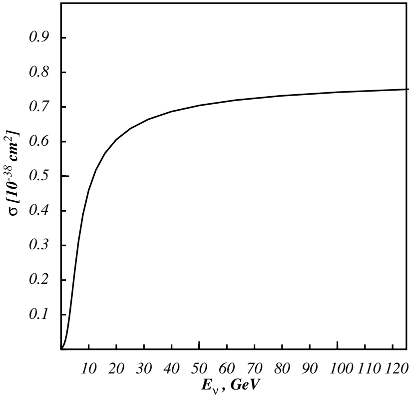

Figure: Total cross-section as a function of the neutrino energy E_ν.

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Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

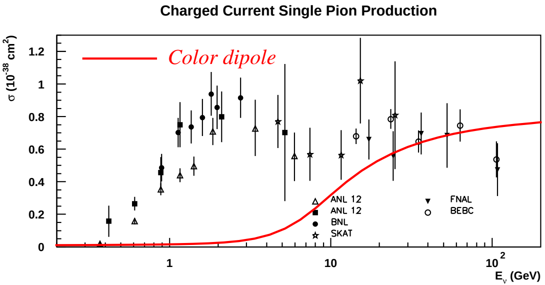


Figure: Total cross-section as a function of the neutrino energy E_ν.*Compilation of experimental data from Minerva proposal, 2004*

Agreement for energies $E_\nu > 10$ GeV, problem for $E_\nu < 10$ GeV

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

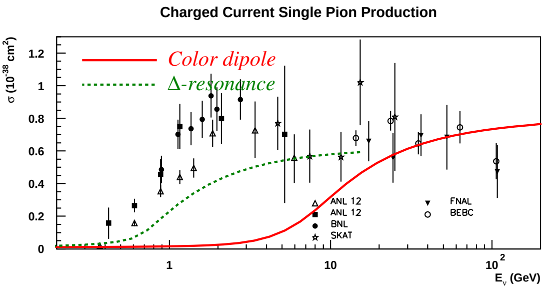


Figure: Total cross-section as a function of the neutrino energy E_ν. *Compilation of experimental data from Minerva proposal, 2004*

Low-energy region is dominated by $\Delta(1232)$

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

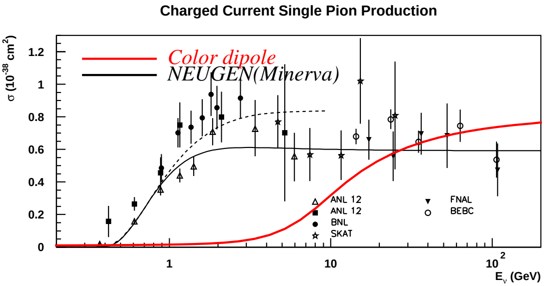


Figure: Total cross-section as a function of the neutrino energy E_ν. *Compilation of experimental data from Minerva proposal, 2004*

Difference between NEUGEN and color dipole: cross-section is slowly growing for high energies

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

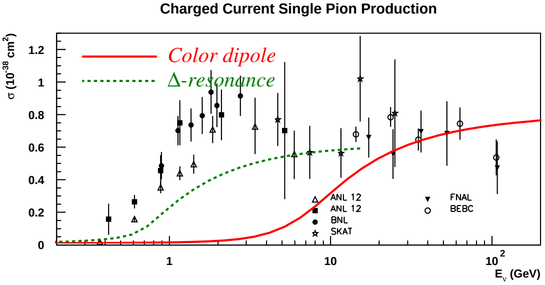
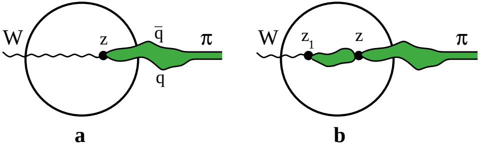


Figure: Total cross-section as a function of the neutrino energy E_ν. *Compilation of experimental data from Minerva proposal, 2004*

Total cross-section is not very informative, it is dominated by Δ-resonance, differential cross-section would give much more information

Coherent neutrino-nuclear scattering

- We use an approach suggested in *(B. Kopeliovich, A. Schäfer, A. Tarasov, 2000)*



Equation of motion of dipole in the nuclear media:

$$i\frac{\partial G(z_2, r_2; z_1, r_1)}{\partial z_2} =$$

$$= \left\{ -\frac{\Delta_{r_2}}{2\nu\alpha(1-\alpha)} - i\sigma_{dN}(r)\rho_A(z_2, r_2) \right\} G(z_2, r_2; z_1, r_1)$$

- Dipoles can change size during propagation in media

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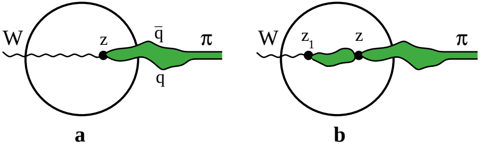

- Two different coherence lengths: coherence length of the pion and effective axial meson

$$l_c^\pi = \frac{2\nu}{m_\pi^2 + Q^2}, \quad l_c^a = \frac{2\nu}{m_a^2 + Q^2}.$$

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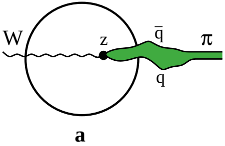

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- For large Q^2, $l_c^\pi \approx l_c^a$, so this case is similar to photon-nuclear processes, we have only two regimes: $l_c \gg R_A$ and $l_c \ll R_A$.

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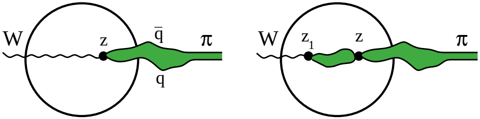

a **b**

- Two different coherence lengths: coherence length of the pion and effective axial meson

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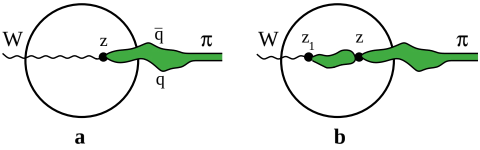
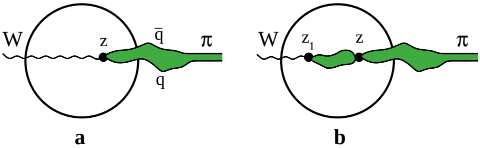
- For large Q^2, $l_c^{\pi} \approx l_c^{a}$, so this case is similar to photon-nuclear processes, we have only two regimes: $l_c \gg R_A$ and $l_c \ll R_A$.
- For small $m_{\pi}^2 \lesssim Q^2 \ll m_a^2$ the two scales are essentially different, $l_c^{a} \ll l_c^{\pi}$, so there are three regimes depending on relations between R_A and l_c^{a}, l_c^{π}.

Coherent neutrino-nuclear scattering (contd.)



- $l_c^a \ll l_c^\pi \ll R_A$: small energy, nuclear effects are due to Fermi motion (EMC-effect; and this is not diffractive production).

Coherent neutrino-nuclear scattering (contd.)



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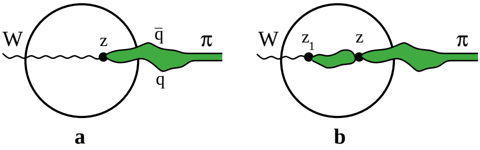
Coherent neutrino-nuclear scattering (contd.)



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- $l_c^a \ll R_A \ll l_c^\pi$: moderate energy, nuclear effects are present; Adler relation is valid for small Q^2, $\sigma \sim A^{2/3}$
- $R_A \ll l_c^a \ll l_c^\pi$: absorptive corrections are large, Adler relation is not valid even for $Q^2 = 0$, $\sigma \sim A^{1/3}$

Result for the $\nu A \to l\pi^+ A$ differential cross-section

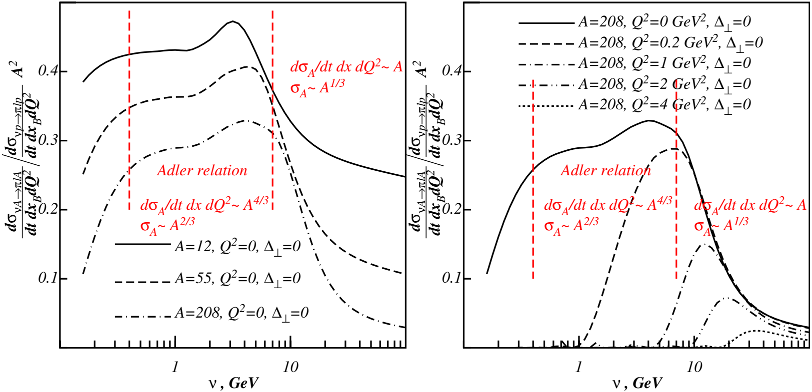


Figure: Ratio of cross-sections on the nucleus and proton.

Adler relation (AR) is valid for energies < 10 GeV; for high energies ($\nu \geq 10$ GeV) AR is broken due to shadowing

Is this behaviour model dependent ?

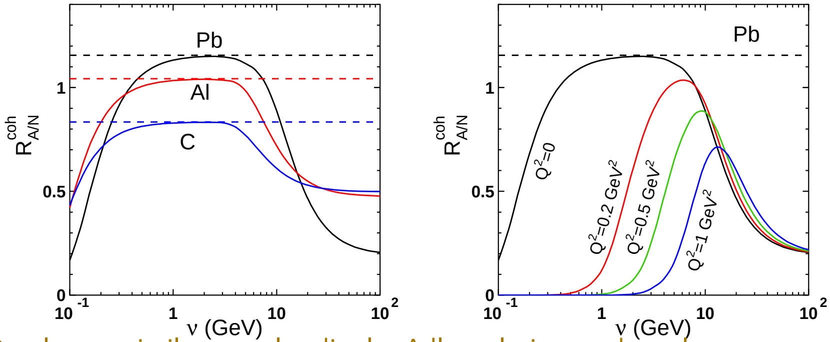
Cross-check in a simple model:

- Assume Adler relation works for nucleon for $Q^2 = 0$
- Use for extrapolation to nonzero Q^2 factor

$$\sim \frac{m_A^2}{m_A^2 + Q^2}$$

- Evaluate the shadowing in standard Gribov-Glauber approach

Result for the $\nu A \to l\pi^+ A$ differential cross-section



Results are similar to color dipole, Adler relation works only in the region $\nu \leq 10$ GeV

Conclusion

- We have shown that the Adler relation cannot always be correct for the neutrino-nuclear processes. For energies $\nu \gtrsim 10$ GeV–shadowing (absorptive) corrections are important

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- We have shown that the Adler relation cannot always be correct for the neutrino-nuclear processes. For energies $\nu \gtrsim 10$ GeV–shadowing (absorptive) corrections are important
- We evaluated the results in color dipole model not using Adler relation for the neutrino-proton and neutrino-nuclear collisions. Our results are valid in the region $\nu \geq (Q^2 + m_\pi^2) R_A$

- Thank You for your attention !

Kinematics

- Vector current contribution to kinematics:

$$\frac{d^2\sigma_{\nu p\to lF}}{d\nu\, dQ^2} = \frac{G^2}{4\pi^2}\frac{|\vec{q}|}{E_\nu}\frac{Q^2}{1-\varepsilon}\left[\sigma_{\perp}^{V}\left(\nu, Q^2\right) + \varepsilon\sigma_{||}^{V}\left(\nu, Q^2\right)\right];$$

$$\varepsilon = \frac{4E_\nu\left(E_\nu-\nu\right)-Q^2}{4E_\nu\left(E_\nu-\nu\right)+Q^2+2\nu^2}$$

$$\sigma_{||,\perp}^{V}\left(\nu, Q^2\right) = \frac{\varepsilon_\mu^{||,\perp}(q)\varepsilon_\nu^{||,\perp}(q)V_{\mu\nu}(q)}{\sqrt{\nu^2+q^2}}$$