

Nuclear Effects in Neutrino Quasi-Elastic Scattering

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Motivations

- QE scattering dominates neutrino cross sections in the energy range relevant to neutrino oscillation experiments ($E_\nu=200$ MeV - 2.5GeV).
- Important source of systematic uncertainties is related to treatment of nuclear model dependence of QE cross sections.
- We study QE neutrino-nuclear scattering using:
 - (*) Relativistic Fermi Gas Model (RFGM)
 - (*) Independent Particle Shell Model (IPSM) with Nucleon-Nucleon (NN) correlation effects
 - (*) Plane-Wave Impulse Approximation (PWIA) with realistic description of the nuclear ground state
 - (*) Final state interactions (FSI) effects within Relativistic Distorted-Wave Impulse Approximation (RDWIA)

Formalism of the quasi-elastic scattering

We consider lepton charged-current (CC) QE exclusive

$$l(k_i) + A(p_A) \rightarrow l'(k_f) + N(p_x) + B(p_B),$$

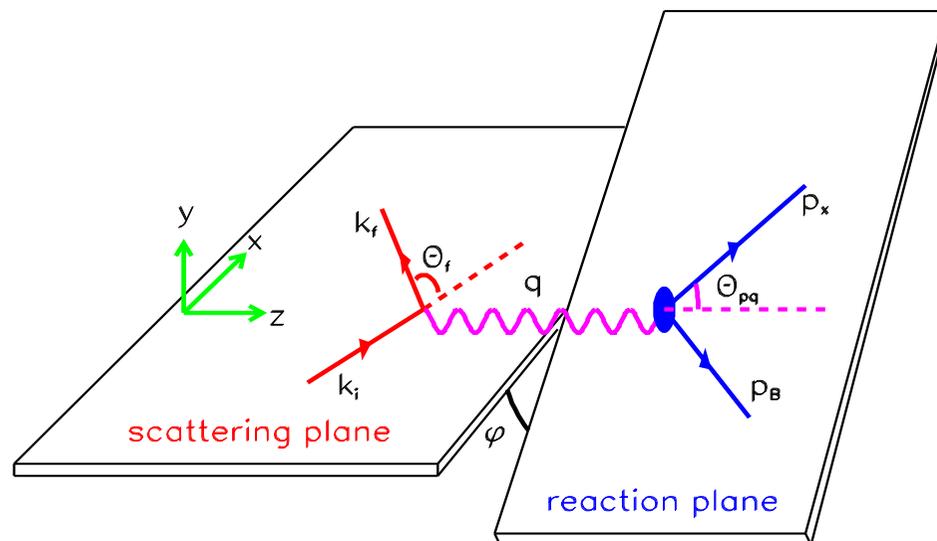
and inclusive

$$l(k_i) + A(p_A) \rightarrow l'(k_f) + X$$

scattering off nuclei, where

- ★ l is incident lepton (e/ν_μ), l' is scattered lepton (e/μ), $k_i = (\varepsilon_i, \mathbf{k}_i)$ and $k_f = (\varepsilon_f, \mathbf{k}_f)$ are initial and final lepton momenta
- ★ $p_A = (\varepsilon_A, \mathbf{p}_A)$, and $p_B = (\varepsilon_B, \mathbf{p}_B)$ are the initial and final target momenta, $p_x = (\varepsilon_x, \mathbf{p}_x)$ is ejectile nucleon momentum
- ★ $q = (\omega, \mathbf{q})$ is momentum transfer carried by the virtual photon (W-boson), and $Q^2 = -q^2 = \mathbf{q}^2 - \omega^2$ is photon (W-boson) virtuality
- ★ m , m_A and m_B are masses of nucleon, target and recoil nucleus, respectively. The missing momentum and energy are defined by $\mathbf{p}_m = \mathbf{p}_x - \mathbf{q}$, $\varepsilon_m = m + m_B - m_A$

A. QE lepton-nucleus cross section



Kinematic definitions for $A(l, l'N)B$ reactions.

A1. Cross Section for exclusive reactions

In the lab frame the differential cross section for exclusive electron (σ^{el}) and (anti)neutrino (σ^{cc}) CC scattering can be written as

$$\frac{d^6 \sigma^{el}}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{|\mathbf{p}_x| \varepsilon_x \varepsilon_f \alpha^2}{(2\pi)^3 \varepsilon_i Q^4} L_{\mu\nu}^{(el)} \mathcal{W}^{\mu\nu(el)}$$

$$\frac{d^6 \sigma^{cc}}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{|\mathbf{p}_x| \varepsilon_x |\mathbf{k}_f| G^2 \cos^2 \theta_C}{(2\pi)^5 \varepsilon_i 2} L_{\mu\nu}^{(cc)} \mathcal{W}^{\mu\nu(cc)},$$

where Ω_f is the solid angle for the lepton momentum, Ω_x is the solid angle for the ejectile nucleon momentum, $\alpha \simeq 1/137$ is the fine-structure constant, $G \simeq 1.16639 \times 10^{-11} \text{ MeV}^{-2}$ is the Fermi constant, θ_C is the Cabibbo angle ($\cos \theta_C \approx 0.9749$).

The lepton tensor can be written as the sum of symmetric $L_S^{\mu\nu}$ and antisymmetric $L_A^{\mu\nu}$ tensors

$$L^{\mu\nu} = L_S^{\mu\nu} + L_A^{\mu\nu}$$

$$L_S^{\mu\nu} = 2 \left(k_i^\mu k_f^\nu + k_i^\nu k_f^\mu - g^{\mu\nu} k_i k_f \right)$$

$$L_A^{\mu\nu} = h 2i \epsilon^{\mu\nu\alpha\beta} (k_i)_\alpha (k_f)_\beta,$$

where h is $+1$ for positive lepton helicity and -1 for negative lepton helicity, $\epsilon^{\mu\nu\alpha\beta}$ is the antisymmetric tensor. For the scattering of unpolarized incident electrons $L^{\mu\nu(el)}$ only has symmetric part and (anti)neutrino tensor $L^{\mu\nu(cc)}$ involves both, symmetric and antisymmetric parts.

The electromagnetic and the weak CC hadronic tensors, $\mathcal{W}_{\mu\nu}^{(el)}$ and $\mathcal{W}_{\mu\nu}^{(cc)}$, are given by bilinear products of the transition matrix elements of the nuclear electromagnetic or CC operator $J_\mu^{(el)(cc)}$ between the initial nucleus state $|A\rangle$ and the final state $|B_f\rangle$ as

$$\mathcal{W}_{\mu\nu}^{(el)(cc)} = \sum_f \langle B_f, p_x | J_\mu^{(el)(cc)} | A \rangle \langle A | J_\nu^{(el)(cc)\dagger} | B_f, p_x \rangle \delta(\varepsilon_A + \omega - \varepsilon_x - \varepsilon_{B_f}),$$

where the sum is taken over undetected states.

If only a single discrete state or narrow resonance of the target is excited it is possible to integrate over the peak in missing energy and obtain a fivefold differential cross section of the form

$$\frac{d^5\sigma^{el}}{d\varepsilon_f d\Omega_f d\Omega_x} = R \frac{|\mathbf{p}_x| \tilde{\varepsilon}_x \varepsilon_f \alpha^2}{(2\pi)^3 \varepsilon_i Q^4} L_{\mu\nu}^{(el)} \mathcal{W}^{\mu\nu(el)}$$

$$\frac{d^5\sigma^{cc}}{d\varepsilon_f d\Omega_f d\Omega_x} = R \frac{|\mathbf{p}_x| \tilde{\varepsilon}_x |\mathbf{k}_f| G^2 \cos^2 \theta_c}{(2\pi)^5 \varepsilon_i 2} L_{\mu\nu}^{(cc)} \mathcal{W}^{\mu\nu(cc)},$$

where R is a recoil factor

$$R = \int d\varepsilon_x \delta(\varepsilon_x + \varepsilon_B - \omega - m_A) = \left| 1 - \frac{\tilde{\varepsilon}_x \mathbf{p}_x \cdot \mathbf{p}_B}{\varepsilon_B \mathbf{p}_x \cdot \mathbf{p}_x} \right|^{-1},$$

$\tilde{\varepsilon}_x$ is solution to equation $\varepsilon_x + \varepsilon_B - m_A - \omega = 0$, where $\varepsilon_B = \sqrt{m_B^2 + \mathbf{p}_B^2}$, $\mathbf{p}_B = \mathbf{q} - \mathbf{p}_x$.

It is also useful to define a reduced cross section

$$\sigma_{red} = \frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} / K \sigma_{lN},$$

where $K^{el} = Rp_x \varepsilon_x / (2\pi)^3$ and $K^{cc} = Rp_x \varepsilon_x / (2\pi)^5$ are phase-space factors for electron and neutrino scattering, and σ_{lN} is corresponding elementary cross section for the lepton scattering from moving free nucleon $l(k_i) + N(p_m) \rightarrow l'(k_f) + N(p_x)$.

A2. Hadronic tensor for exclusive reaction

- A general structure of the hadronic tensor can be established basing on requirements of Lorentz invariance, parity and time reversal symmetries and current conservation.
- **Lorentz invariance** = this tensor must be constructed from three linearly independent four-vectors q , p_x , and p_A , the scalars that can be constructed from them, the second-rank metric tensor $g_{\mu\nu}$, and completely antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$.
- **Symmetries**: generally, due to the final state interaction effects the time reversal symmetry does not constraint the form of the nuclear tensor for the exclusive reactions [A.Picklesimer et al. (1985) and (1987)],

A2.1 Hadronic tensor for exclusive reaction in electron scattering

For electron scattering [A.Picklesimer et al. (1985), T.W.Donnely et al. (1986)] (taking into account current and parity conservation)

$$W^{\mu\nu(el)} = W_S^{\mu\nu(el)} + W_A^{\mu\nu(el)},$$

$$W_S^{\mu\nu(el)} = W_1^{(el)} \tilde{g}^{\mu\nu} + W_2^{(el)} \tilde{p}_x^\mu \tilde{p}_x^\nu + W_3^{(el)} \tilde{p}_A^\mu \tilde{p}_A^\nu + W_4^{(el)} (\tilde{p}_x^\mu \tilde{p}_A^\nu + \tilde{p}_x^\nu \tilde{p}_A^\mu),$$

$$W_A^{\mu\nu(el)} = W_5^{(el)} (\tilde{p}_x^\mu \tilde{p}_A^\nu - \tilde{p}_x^\nu \tilde{p}_A^\mu),$$

where

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2},$$

$$\tilde{p}_x^\mu = p_x^\mu + \frac{p_x \cdot q}{Q^2} q^\mu,$$

$$\tilde{p}_A^\mu = p_A^\mu + \frac{p_A \cdot q}{Q^2} q^\mu.$$

In the chosen coordinate system the result of contraction of the electron and nuclear response tensors reduces to the form

$$L_{\mu\nu}^{(el)} W_S^{\mu\nu(el)} = 4\varepsilon_i \varepsilon_f \cos^2 \frac{\theta}{2} (V_L R_L^{(el)} + V_T R_T^{(el)} + V_{LT} R_{LT}^{(el)} \cos \phi + V_{TT} R_{TT}^{(el)} \cos 2\phi)$$

where

$$V_L = Q^4 / \mathbf{q}^4,$$

$$V_T = \frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2},$$

$$V_{LT} = \frac{Q^2}{\mathbf{q}^2} \left(\frac{Q^2}{\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right),$$

$$V_{TT} = \frac{Q^2}{2\mathbf{q}^2}$$

are the electron coupling coefficients and

$$R_L^{(el)} = W^{00(el)},$$

$$R_T^{(el)} = W^{xx(el)} + W^{yy(el)},$$

$$R_{LT}^{(el)} \cos \phi = - \left(W^{0x(el)} + W^{x0(el)} \right),$$

$$R_{TT}^{(el)} \cos 2\phi = W^{xx(el)} - W^{yy(el)}$$

are four independ nuclear response functions $R = f(Q^2, \omega, \mathbf{p}_x, \theta_{pq})$.

A2.2 Hadronic tensor for exclusive reaction in neutrino scattering

In weak interactions the weak current and parity are not conserved. For this reason the hadronic tensor has additional independent Lorentz structures and a general nuclear tensor can be written as

$$\begin{aligned}
W^{\mu\nu(cc)} &= W_S^{\mu\nu} + W_A^{\mu\nu}, \\
W_S^{\mu\nu} &= W_1 g^{\mu\nu} + W_2 q^\mu q^\nu + W_3 p_x^\mu p_x^\nu + W_4 p_A^\mu p_A^\nu + W_5 (p_x^\mu q^\nu + p_x^\nu q^\mu) \\
&\quad + W_6 (p_A^\mu q^\nu + p_A^\nu q^\mu) + W_7 (p_x^\mu p_A^\nu + p_x^\nu p_A^\mu), \\
W_A^{\mu\nu} &= W_8 (p_x^\mu q^\nu - p_x^\nu q^\mu) + W_9 (p_A^\mu q^\nu - p_A^\nu q^\mu) + W_{10} (p_x^\mu p_A^\nu - p_x^\nu p_A^\mu) \\
&\quad + W_{11} \epsilon^{\mu\nu\tau\rho} q_\tau p_{x\rho} + W_{12} \epsilon^{\mu\nu\tau\rho} q_\tau p_{A\rho} + W_{13} \epsilon^{\mu\nu\tau\rho} p_{x\tau} p_{A\rho}.
\end{aligned}$$

The result of contraction of the lepton and nuclear tensors can be written as

$$\begin{aligned}
L_{\mu\nu}^{(cc)} W^{\mu\nu(cc)} &= L_{\mu\nu}^S W_S^{\mu\nu} + L_{\mu\nu}^A W_A^{\mu\nu} = 2\varepsilon_i \varepsilon_f \{ v_0 R_0 + v_T R_T + v_{TT} R_{TT} \cos 2\phi + v_{zz} R_{zz} \\
&\quad + (v_{xz} R_{xz} - v_{0x} R_{0x}) \cos \phi - v_{0z} R_{0z} + h [v_{yz} (R'_{yz} \sin \phi + R_{yz} \cos \phi) \\
&\quad - v_{0y} (R'_{0y} \sin \phi + R_{0y} \cos \phi) - v_{xy} R_{xy}] \},
\end{aligned}$$

where v_i are neutrino coupling coefficients and

$$\begin{aligned}
 R_0 &= W_S^{00}, \\
 R_T &= W_S^{xx} + W_S^{yy}, \\
 R_{TT} \cos 2\phi &= W_S^{xx} - W_S^{yy}, \\
 R_{0z} &= W_S^{0z} + W_S^{z0}, \\
 R_{zz} &= W_S^{zz}, \\
 R_{0x} \cos \phi &= W_S^{0x} + W_S^{x0}, \\
 R_{xz} \cos \phi &= W_S^{xz} + W_S^{zx}, \\
 R_{xy} &= i (W_A^{xy} - W_A^{yx}), \\
 R'_{yz} \sin \phi + R_{yz} \cos \phi &= i (W_A^{yz} - W_A^{zy}), \\
 R'_{0y} \sin \phi + R_{0y} \cos \phi &= i (W_A^{0y} - W_A^{y0})
 \end{aligned}$$

are ten independ response functions which describe the weak properties of hadronic system.

A2.3 Cross sections in terms of response functions

The exclusive lepton scattering cross sections can be written in terms of response functions as

$$\frac{d^5 \sigma^{el}}{d\varepsilon_f d\Omega_f d\Omega_x} = \frac{|\mathbf{p}_x| \tilde{\varepsilon}_x}{(2\pi)^3} \sigma_M R (V_L R_L^{(el)} + V_T R_T^{(el)} + V_{LT} R_{LT}^{(el)} \cos \phi + V_{TT} R_{TT}^{(el)} \cos 2\phi),$$

$$\begin{aligned} \frac{d^5 \sigma^{cc}}{d\varepsilon_f d\Omega_f d\Omega_x} &= \frac{|\mathbf{p}_x| \tilde{\varepsilon}_x}{(2\pi)^5} G^2 \cos^2 \theta_c \varepsilon_f |\mathbf{k}_f| R \{ v_0 R_0 + v_T R_T + v_{TT} R_{TT} \cos 2\phi + v_{zz} R_{zz} \\ &+ (v_{xz} R_{xz} - v_{0x} R_{0x}) \cos \phi - v_{0z} R_{0z} + h [v_{yz} (R'_{yz} \sin \phi + R_{yz} \cos \phi) \\ &- v_{0y} (R'_{0y} \sin \phi + R_{0y} \cos \phi) - v_{xy} R_{xy}] \}, \end{aligned}$$

where

$$\sigma_M = \frac{\alpha^2 \cos^2 \theta / 2}{4\varepsilon_i^2 \sin^4 \theta / 2}$$

is the Mott cross section.

A3.1 Inclusive reactions with electron

In the inclusive reactions only the outgoing lepton is detected and the differential cross sections can be written as

$$\frac{d^2\sigma^{el}}{d\varepsilon_f d\Omega_f} = \frac{\varepsilon_f \alpha^2}{\varepsilon_i Q^4} L_{\mu\nu}^{(el)} \overline{W}^{\mu\nu(el)},$$

$$\frac{d^2\sigma^{cc}}{d\varepsilon_f d\Omega_f} = \frac{1}{(2\pi)^2} \frac{|\mathbf{k}_f|}{\varepsilon_i} \frac{G^2 \cos^2 \theta_c}{2} L_{\mu\nu}^{(cc)} \overline{W}^{\mu\nu(cc)},$$

where $\overline{W}^{\mu\nu}$ is inclusive hadronic tensor.

For inclusive lepton scattering there are only two linearly independent four vectors q and p_A . The time reversal symmetry reduces the number of Lorenz structures in hadronic tensor and we have

$$\overline{W}_S^{\mu\nu(el)} = \overline{W}_1^{(el)} \tilde{g}^{\mu\nu} + \overline{W}_2^{(el)} p_A^\mu p_A^\nu + \overline{W}_3^{(el)} (q^\mu p_A^\nu + q^\nu p_A^\mu).$$

Then the inclusive electron scattering cross sections reduce to

$$\frac{d^2\sigma^{el}}{d\varepsilon_f d\Omega_f} = \sigma_M (V_L R_L^{(el)} + V_T R_T^{(el)}),$$

A3.2 Inclusive reactions with neutrino

The CC weak nuclear tensor for inclusive neutrino scattering can be written as the sum

$$\begin{aligned}\overline{W}^{\mu\nu(cc)} &= \overline{W}_S^{\mu\nu} + \overline{W}_A^{\mu\nu}, \\ \overline{W}_S^{\mu\nu} &= \overline{W}_1 g^{\mu\nu} + \overline{W}_2 q^\mu q^\nu + \overline{W}_3 p_A^\mu p_A^\nu + \overline{W}_4 (p_A^\mu q^\nu + p_A^\nu q^\mu), \\ \overline{W}_A^{\mu\nu} &= \overline{W}_5 \epsilon^{\mu\nu\tau\rho} q_\tau p_{A\rho}\end{aligned}$$

Then the inclusive neutrino scattering cross sections reduce to

$$\frac{d^2\sigma^{cc}}{d\varepsilon_f d\Omega_f} = \frac{G^2 \cos^2 \theta_c}{(2\pi)^2} \varepsilon_f |\mathbf{k}_f| (v_0 R_0 + v_T R_T + v_{zz} R_{zz} - v_{0z} R_{0z} - h v_{xy} R_{xy}),$$

where the response functions R_i depend only on Q^2 and ω .

B. Nuclear current

A commonly used approach to describe the lepton-nucleon scattering is the **Impulse Approximation (IA)**, in which the incoming lepton incoherently interacts with bound nucleon. The nuclear current is written as the **sum of single-nucleon currents**.

- The single-nucleon electromagnetic vertex function for a **free nucleon** can be represented by any of three operators [T. de Forest (1983), C.R. Chinn et al (1992)]

$$\Gamma^\mu = G_M(Q^2)\gamma^\mu - \frac{P^\mu}{2m}F_M^{(el)}(Q^2) \quad (CC1),$$

$$\Gamma^\mu = F_V^{(el)}(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_M^{(el)}(Q^2) \quad (CC2),$$

$$\Gamma^\mu = \frac{P^\mu}{2m}F_V^{(el)}(Q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}G_M(Q^2) \quad (CC3),$$

which are related by the **Gordon identity**. Here $\sigma^{\mu\nu} = i[\gamma^\mu\gamma^\nu]/2$, $P = p_m + p_x$, $F_V^{(el)}$ and $F_M^{(el)}$ are the Dirac and Pauli nucleon form factors.

- For electron scattering off nuclei, CC1, CC2 and CC3 representations of the electromagnetic vertex function are **not equivalent** because the bound nucleons are off shell ($p_m^2 \neq 0$).

Theoretical uncertainties:

- ★ What expression for the Γ^μ should be used?
Most calculations use the **CC2** electromagnetic vertex function.
- ★ The vertex Γ^μ should be extrapolated to off-shell region. How ?
To this end we employ de Forest prescription

$$\tilde{\Gamma}^\mu = F_V^{(el)}(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{\tilde{q}_\nu}{2m}F_M^{(el)}(Q^2),$$

where $\tilde{q} = (\varepsilon_x - \tilde{E}, \mathbf{q})$ and the nucleon energy $\tilde{E} = \sqrt{m^2 + (\mathbf{p}_x - \mathbf{q})^2}$ is placed on shell. We use the **MMD** approximation [P.Mergell et al (1996)] of the nucleon form factors. The **Coulomb gauge** is assumed for the single-nucleon current, *i.e.* $J_3 = (\omega/\mathbf{q})J_0$.

- The single-nucleon charged current has $V-A$ structure $J^{\mu(cc)} = J_V^\mu + J_A^\mu$. For a free nucleon vertex function $\Gamma^{\mu(cc)} = \Gamma_V^\mu + \Gamma_A^\mu$ we use **CC2 vector current vertex function**

$$\Gamma_V^\mu = F_V(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_M(Q^2)$$

and the **axial current vertex function**

$$\Gamma_A^\mu = F_A(Q^2)\gamma^\mu\gamma_5 + F_P(Q^2)q^\mu\gamma_5.$$

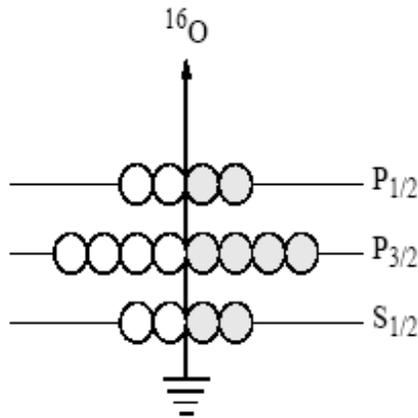
- We use de Forest prescription for off-shell extrapolation of $\Gamma^{\mu(cc)}$. Similar to electromagnetic current, Coulomb gauge is applied for the vector current J_V (the hypothesis of conserved vector current).
- The axial F_A and pseudoscalar F_P form factors in the dipole approximation are parameterized as

$$F_A(Q^2) = \frac{F_A(0)}{(1 + Q^2/M_A^2)^2}, \quad F_P(Q^2) = \frac{2mF_A(Q^2)}{m_\pi^2 + Q^2},$$

where $F_A(0) = 1.267$, m_π is the pion mass, and $M_A \simeq 1.032$ GeV is the axial mass.

Model

In Independent Particle Shell Model (IPSM) the model space for $^{16}\text{O}(l, l' N)$ consists of $1s_{1/2}$, $1p_{3/2}$, and $1p_{1/2}$ nucleon-hole states in ^{15}N and ^{15}O nuclei, for total of 6 states. The $1s_{1/2}$ state is regarded as a discrete state.



Shell occupancy:

$$S(P_{1/2})=0.7$$

$$S(P_{3/2})=0.66$$

$$S(S_{1/2})=1$$

Average occupancy of nuclear shells $\bar{S}=0.75$
(supported by recent JLab measurement)

Missing Energy

Neutron: $E_m(1p_{1/2})=15.7$ MeV, $E_m(1p_{3/2})=21.2$ MeV, $E_m(1s_{1/2})=42.9$ MeV

Proton: $E_m(1p_{1/2})=12.1$ MeV, $E_m(1p_{3/2})=18.4$ MeV, $E_m(1s_{1/2})=40.1$ MeV

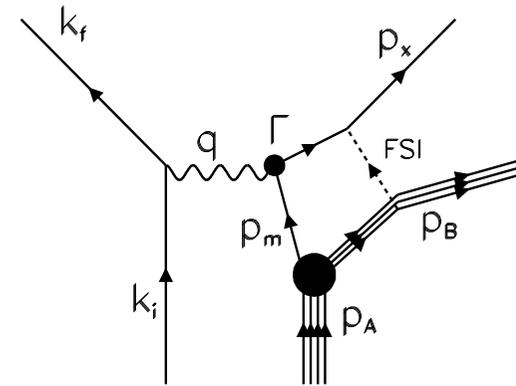
In our studies we compare the results of calculations within different models and approximations: relativistic distorted-wave impulse approximation (RDWIA), plane-wave impulse approximation (PWIA) and the Fermi gas model.

RDWIA

In RDWIA a nuclear matrix elements of the single-nucleon current leaving the residual nucleus in asymptotic channel α can be written in terms of the matrix elements between the bound nucleon wave functions and the distorted wave in the final state

$$\begin{aligned} \langle p, B_\alpha | J^\mu | A \rangle &= \sum_{\beta\gamma m_b m'_b} c_{\beta\gamma} \int d^3r \exp(i\mathbf{t} \cdot \mathbf{r}) \langle \bar{\psi}_{\alpha\beta}^{(-)} | \mathbf{r} m_b \rangle \\ &\times \langle \mathbf{r} m_b | \tilde{\Gamma}^\mu | \mathbf{r} m'_b \rangle \langle \mathbf{r} m'_b | \phi_{\beta\gamma} \rangle, \end{aligned}$$

where $c_{\beta\gamma}$ is a parentage coefficient, $\langle \mathbf{r} m_b | \tilde{\Gamma}_{\lambda\lambda'}^\mu | \mathbf{r} m'_b \rangle$ is a 2×2 current spin matrix,



$$\langle \mathbf{r} m'_b | \phi_{\beta\gamma} \rangle = \begin{pmatrix} F_{\beta\gamma m'_b}(\mathbf{r}) \\ iG_{\beta\gamma m'_b}(\mathbf{r}) \end{pmatrix}$$

is the bound state overlap wave function and

$$\langle \bar{\psi}_{\alpha\beta}^{(-)} | \mathbf{r} m_b \rangle = N_\alpha \begin{pmatrix} \chi_{\alpha\beta m_b}^{(-)*}(\mathbf{r}) \\ -i\zeta_{\alpha\beta m_b}^{(-)*}(\mathbf{r}) \end{pmatrix}$$

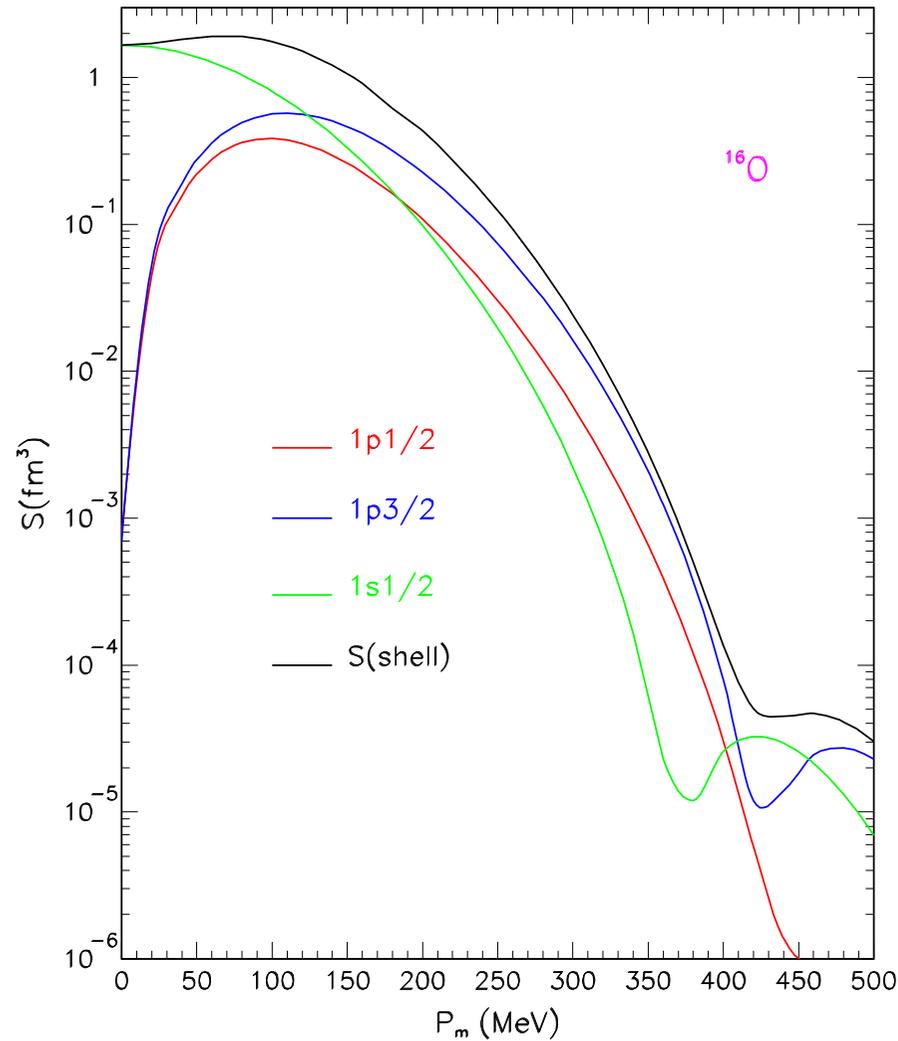
is the Dirac adjoint of time-reversed distorted waves.

Both wave functions are assumed to satisfy local single-nucleon Dirac equations.

The bound nucleon waves functions, obtained as solutions of a Dirac equation, derived within a relativistic mean field approach, from Lagrangian containing σ , ω and ρ mesons. The missing momentum distribution is determined by the wave functions in momentum space

$$P_\beta(p_m) = \frac{|c_\beta|^2}{2\pi^2} \left(|\tilde{F}_\beta(p_m)|^2 + |\tilde{G}_\beta(p_m)|^2 \right).$$

In this work NLSH bound-nucleon wave function [M.M. Sharma et al (1993)] are used in numerical analysis with normalization factors $S_\alpha = |c_\alpha|^2$ relative to full occupancy of ^{16}O : $S(1p_{3/2}) = 0.66$, $S(1p_{1/2}) = 0.7$ [K.G. Fissum et al (2004) (JLab)] and $S(1s_{1/2}) = 1$.



Momentum distribution for the NLSH models. The NLSH wave functions predict binding energies, single-particle energies, and a charge radius for ^{16}O which are all in good agreement with the data.

Outgoing state wave functions

The distorted wave functions are evaluated using a relativized Schrödinger equation for upper components of Dirac wave functions. The system of two coupled first-order Dirac equations

$$[\boldsymbol{\alpha} \cdot \mathbf{p} + \beta(m + S)] \psi = (E - V)\psi,$$

where

$$\psi(\mathbf{r}) = \begin{pmatrix} \psi_+(\mathbf{r}) \\ \psi_-(\mathbf{r}) \end{pmatrix}$$

is the four-component Dirac spinor, can be reduced to a single second-order Schrödinger equation

$$\left[\nabla^2 + k^2 - 2\mu \left(U^C + U^{LS} \mathbf{L} \cdot \boldsymbol{\sigma} \right) \right] \xi = 0,$$

where ξ is a two-component Pauli spinor. Here k is the relativistic wave number, μ is the reduced mass of the scattering state,

and

$$\begin{aligned}
 U^C &= \frac{E}{\mu} \left[V + \frac{m}{E} S + \frac{S^2 - V^2}{2E} \right] + U^D, \\
 U^D &= \frac{1}{2\mu} \left[-\frac{1}{2r^2 D} \frac{d}{dr} (r^2 D') + \frac{3}{4} \left(\frac{D'}{D} \right)^2 \right], \\
 U^{LS} &= -\frac{1}{2\mu r} \frac{D'}{D}, \\
 D &= 1 + \frac{S - V}{E + m}.
 \end{aligned}$$

$D(r)$ is known as the Darwin nonlocality factor and U^C and U^{LS} are the central and spin-orbit potentials. The upper and lower components of the Dirac wave functions are then obtained using

$$\begin{aligned}
 \psi_+ &= D^{1/2} \xi, \\
 \psi_- &= \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m + S - V} \psi_+
 \end{aligned}$$

We use the LEA program [J.J. Kelly (1995)] for numerical calculation of the distorted wave functions with EDAD1 SV relativistic optical potential [E. Cooper (1993)].

Plane-Wave Impulse Approximation (PWIA)

In the PWIA the final state interaction between the outgoing nucleon and the residual nucleus is neglected. In nonrelativistic PWIA the knockout cross section has a factorized form

$$\frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} = K \sigma_{ex} P(E, \mathbf{p}), \quad K^{el} = R \frac{p_x \varepsilon_x}{(2\pi)^3}, \quad K^{cc} = R \frac{p_x \varepsilon_x}{(2\pi)^5}$$

K are the phase-space factors, R is recoil factor, σ_{ex} is the half-off-shell cross section for scattering of a lepton by moving nucleon. The nuclear spectral function P can be written as

$$P(E, \mathbf{p}) = \sum_f \left| \langle B_f | a(\mathbf{p}) | A \rangle \right|^2 \delta(E - \varepsilon_m).$$

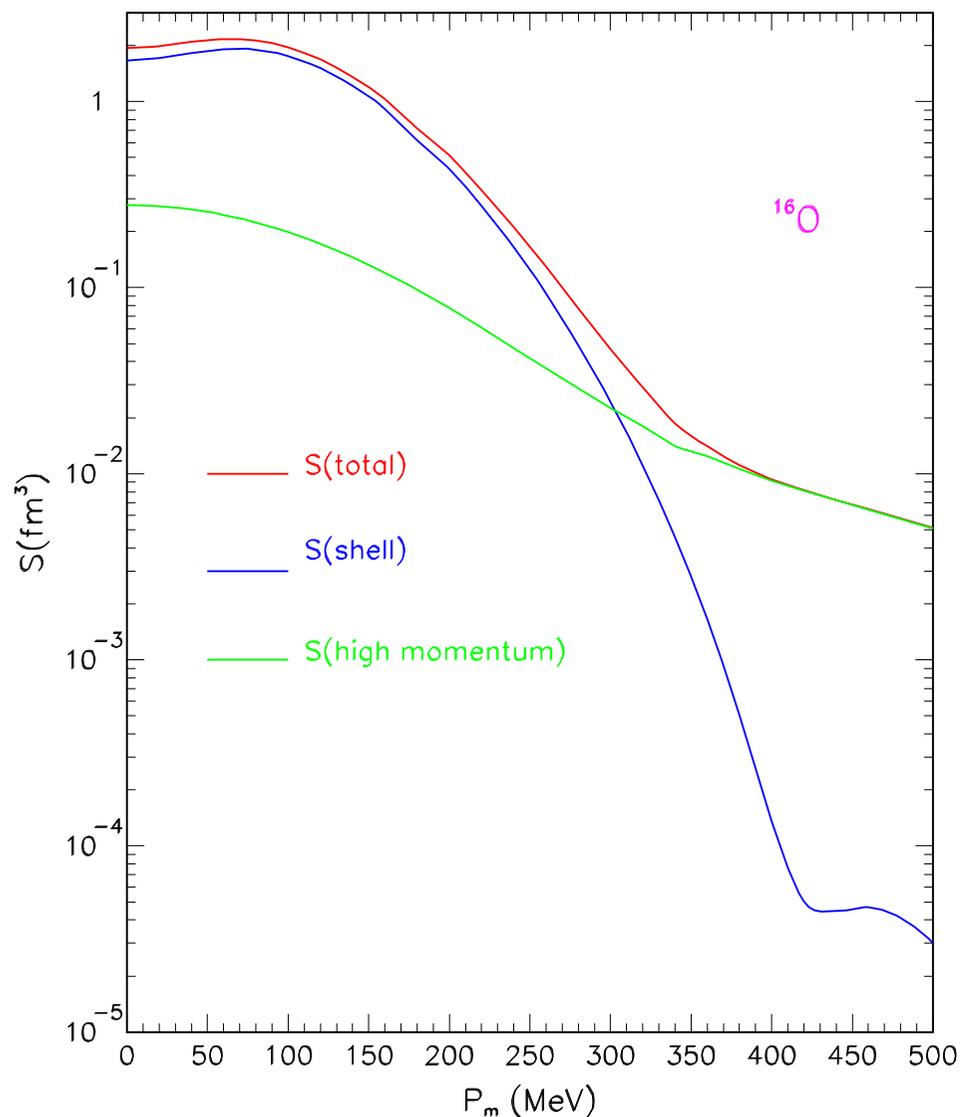
and represents the probability of removing a nucleon with momentum \mathbf{p} and energy E from the nuclear target A and leaving the residual nucleus B in the state B_f .

In the nuclear shell model the momentum distribution $P_\beta(\mathbf{p})$ for an orbit β is related to the upper component of the corresponded bound-state wave function as

$$P_\beta(\mathbf{p}) = \frac{c_\beta^2}{2\pi^2} \left| \tilde{F}_\beta(\mathbf{p}) \right|^2 \quad \text{and} \quad \int d^3p P_\beta(\mathbf{p}) = |c_\beta|^2.$$

NN-correlations in nuclear ground state

- According to JLab data [K. Fissum (2004)] the occupancy of the IPISM orbitals of ^{16}O is approximately 75% on average. We assume that the missing strength can be attributed to the short-range NN-correlations in the ground state.
- We consider a phenomenological model which incorporates high-energy and high-momentum component P_{HM} due to NN-correlations [C.Ciofi degli Atti et al (1996), S.Kulagin et al (2006)].
- In our calculations the spectral function P_{HM} incorporates 25% of the total normalization of the spectral function.



Momentum distribution for the NLSH models and high-momentum component of the NN-correlation. The occupancy of the orbitals of ^{16}O is 75% on average and the spectral function P_{HM} incorporates 25% of the total normalization of the spectral function.

Fermi gas model

In the RFGM the nucleons are described as a system of quasi-free nucleons. This model takes into account the Fermi motion of bound nucleon, Pauli blocking factor and relativistic kinematics. The Fermi gas model provides a simplest form of the spectral function which is given by

$$P_{FG}(E, |\mathbf{p}|) = \frac{3}{4\pi p_F^3} \Theta(p_F - |\mathbf{p}|) \Theta(|\mathbf{p} + \mathbf{q}| - p_F) \times \delta[(\mathbf{p}^2 + m^2)^{1/2} - \varepsilon - E],$$

where p_F is the Fermi momentum and ε is effective binding energy, introduced to account of nuclear binding. For oxygen we use $p_F=250$ MeV/c and $\varepsilon=27$ MeV.

The RFGM does not account nuclear shell structure, FSI effect, and the presence of NN-correlations.

Inclusive and total cross sections

- In exclusive channels nucleon flux is absorbed due to FSI. Flux absorption is described by imaginary part of optical potential.
- For inclusive reaction (sum over all channels) probability flux must conserve. Therefore we calculate the contribution to inclusive and total cross sections for nuclear shells using only the real part of optical potential.
- We also apply FSI correction to the contribution from the high-momentum part of nuclear spectral function

$$\frac{d^3\sigma}{d\varepsilon_f d\Omega_f} = \underbrace{\left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{RDWIA}}_{\text{nuclear shells}} + \Lambda(\varepsilon_f, \Omega_f) \underbrace{\left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{HM}}_{\text{NN correlations}},$$

where

$$\Lambda(\varepsilon_f, \Omega_f) = \underbrace{\left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{RDWIA}}_{\text{nuclear shells}} / \underbrace{\left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{PWIA}}_{\text{nuclear shells}},$$

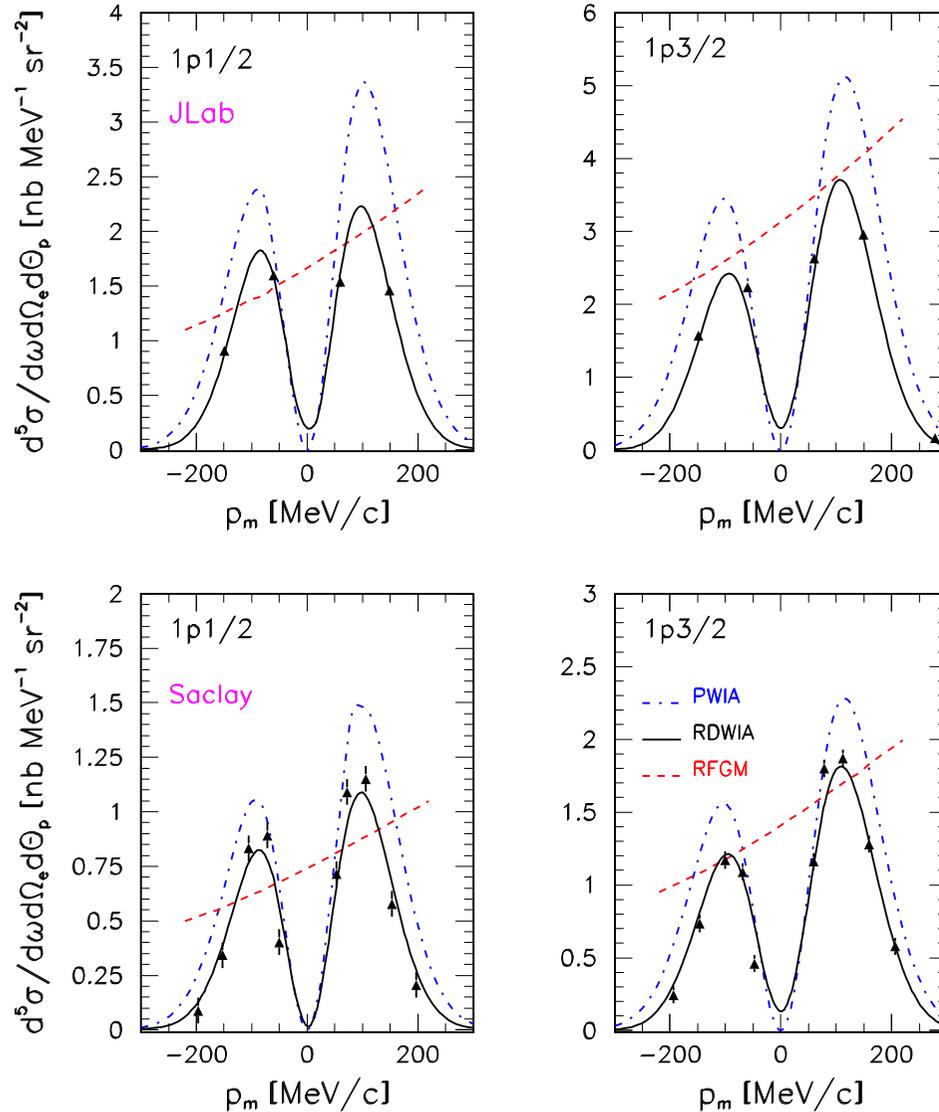
describes the relative strength of FSI.

More details can be found in [Kulagin et al (2006) and Butkevich et al(2007)].

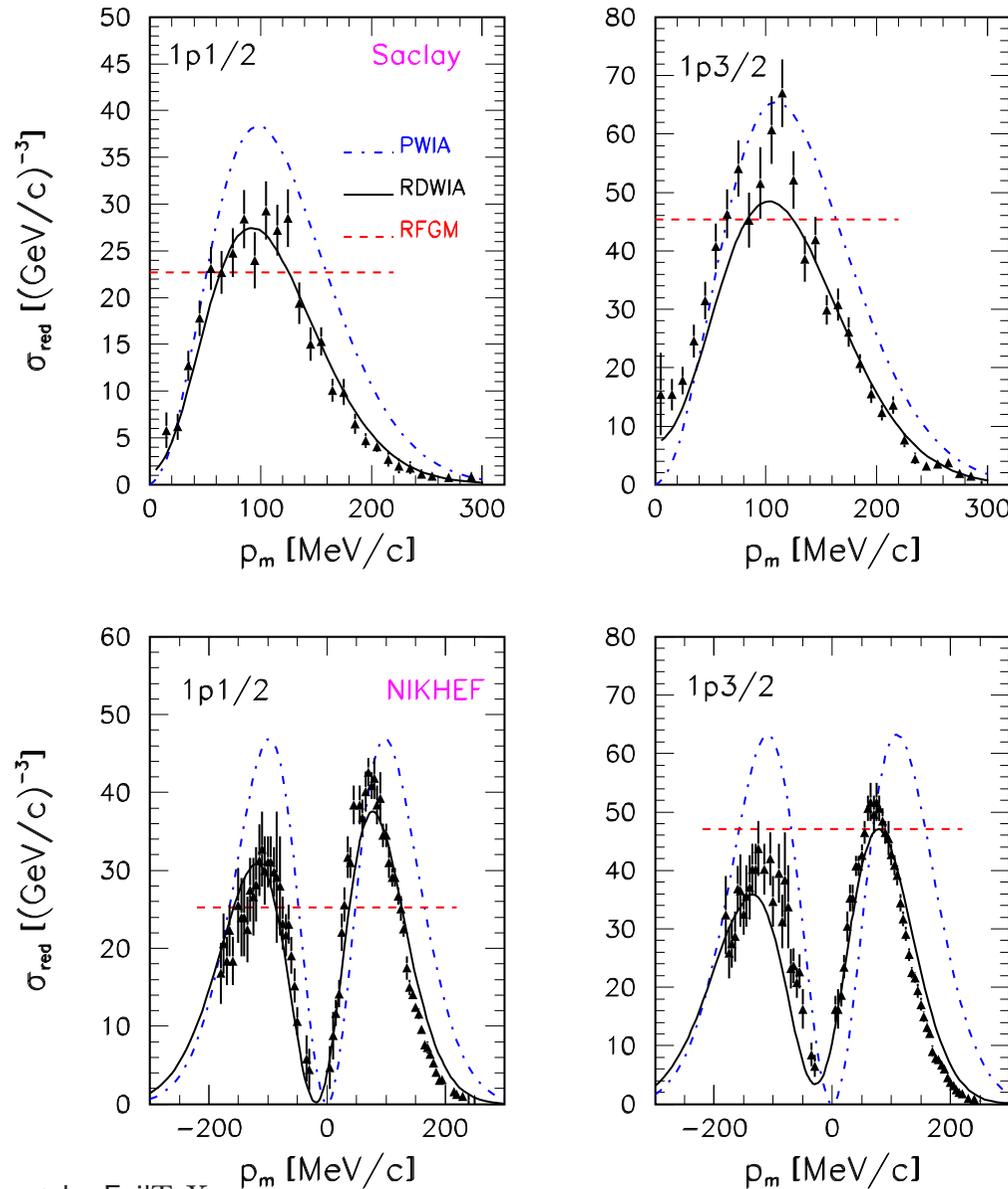
Results

The LEA code for nucleon knockout by electron scattering was adopted in this work for neutrino interactions. This code was successfully tested against $A(e, e'p)$ data:

- JLab - J.J. Kelly(2005), J.Gao(2000), K. Fissum (2004)
- Saclay - L.Chinitz(1991), M.Bernhein(1982)
- NIKHEF - M.Leuschner(1994)

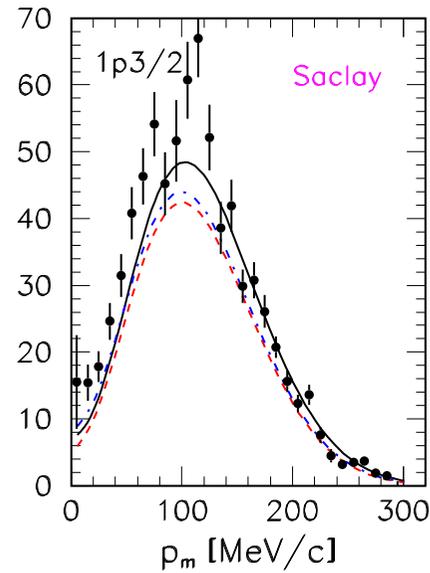
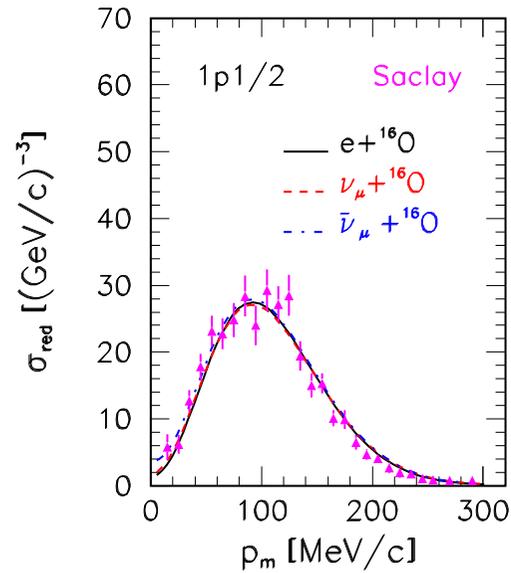


Measured differential exclusive cross-section data for the removal of protons from 1p-shell of ^{16}O as a function of missing momentum. The upper panels show JLab data for electron beam energy $E_{beam}=2.442$ GeV, proton kinetic energy $T_p=427$ MeV, and $Q^2=0.8$ GeV². The lower panels show Saclay data for $E_{beam}=580$ MeV, $T_p=160$ MeV, and $Q^2=0.3$ GeV². The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations. Negative values of p_m correspond to $\phi = \pi$ and positive ones to $\phi = 0$.

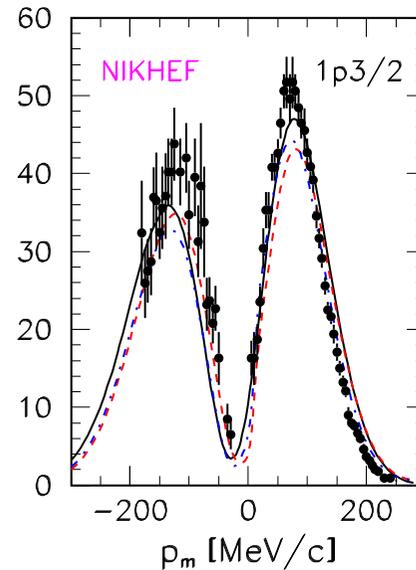
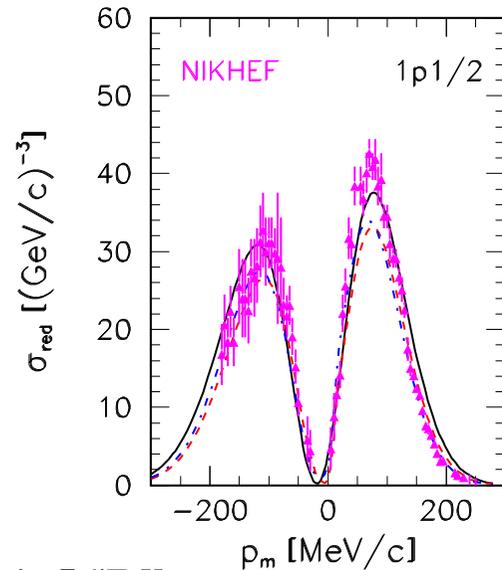


Measured reduced exclusive cross-section data for the removal of protons from 1p-shell of ^{16}O as a function of missing momentum. The upper panels show Saclay data for electron beam energy $E_{\text{beam}}=500$ MeV, proton kinetic energy $T_p=100$ MeV, and $Q^2=0.3$ GeV^2 . The lower panels show NIKHEF data for $E_{\text{beam}}=521$ MeV, $T_p=96$ MeV, Q^2 is varied. The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations.

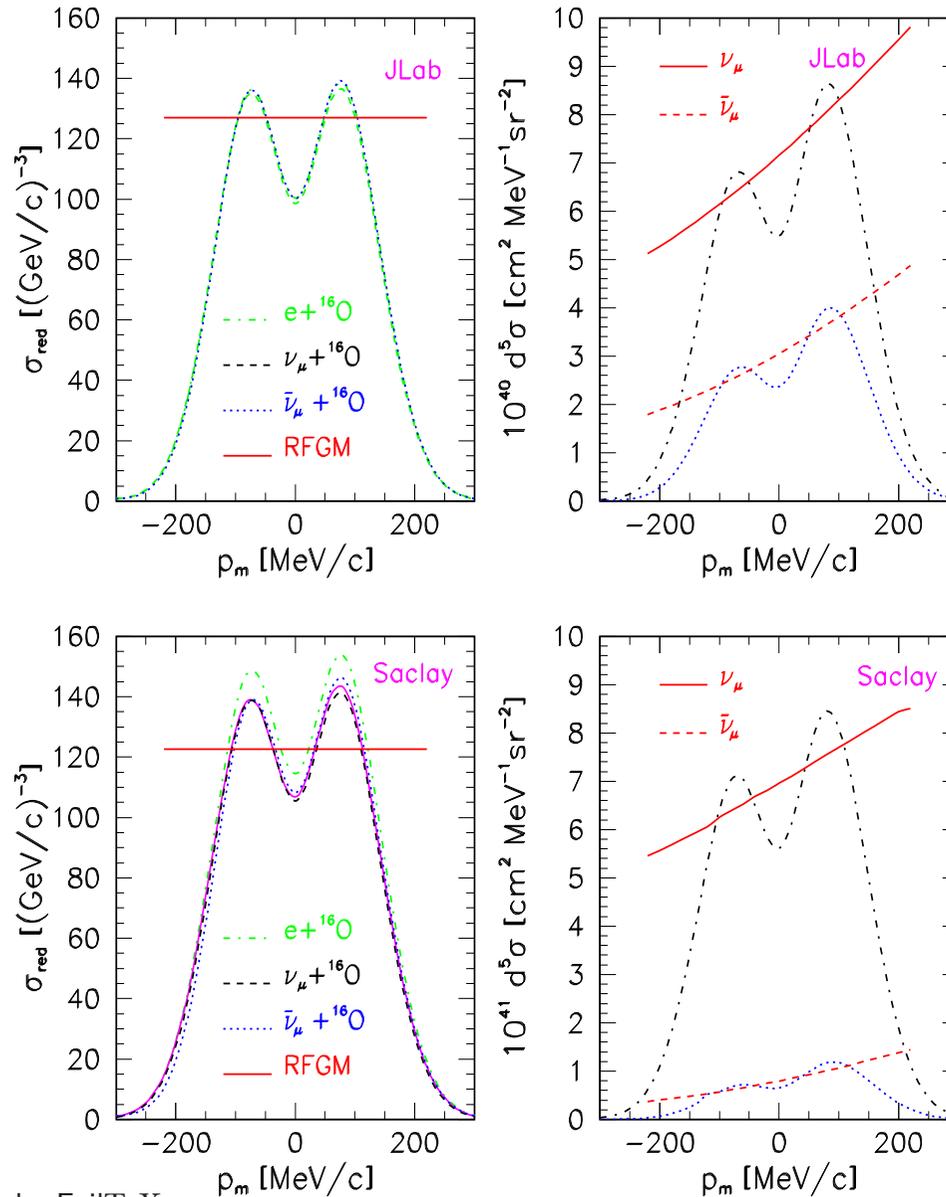
The RFGM predictions are completely off of the exclusive data.



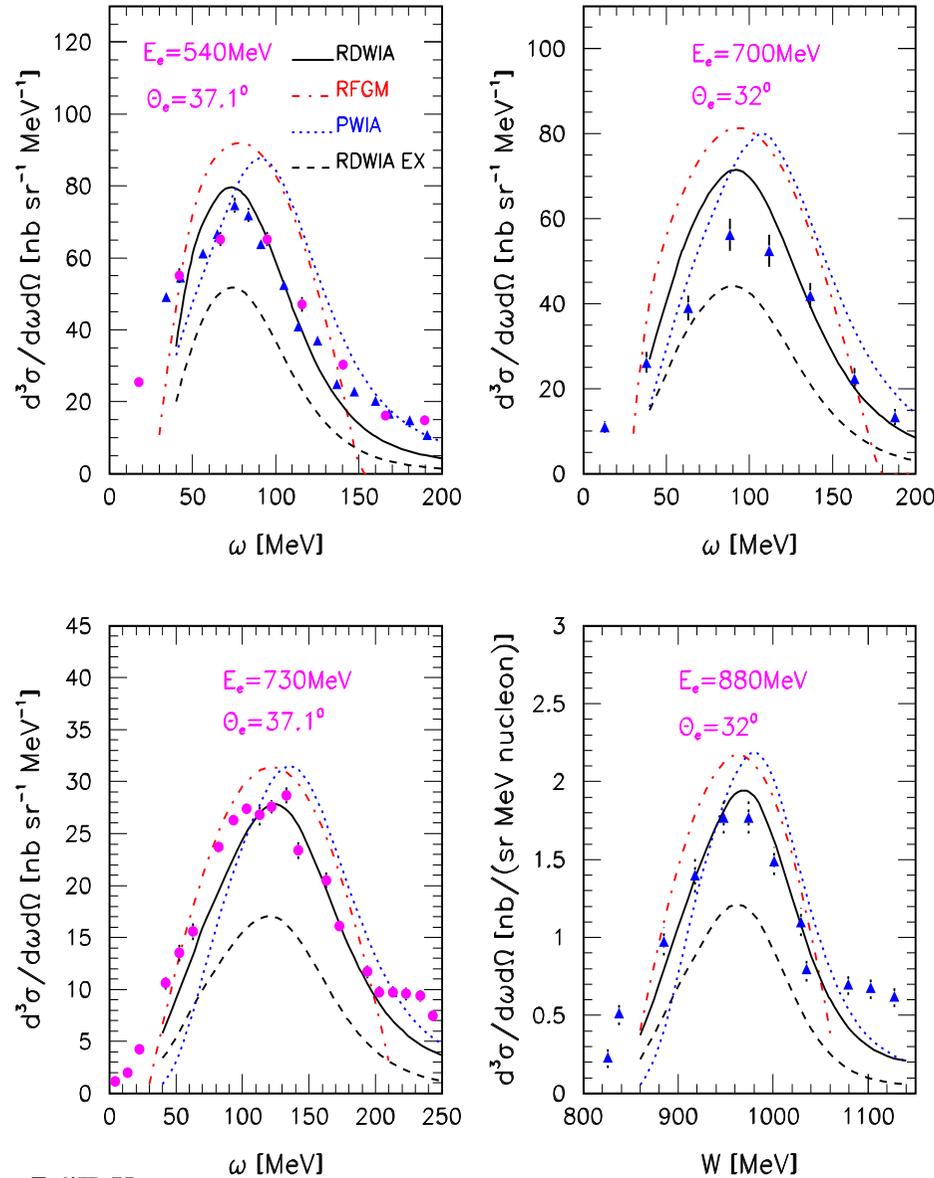
Comparison of the RDWIA electron, neutrino and antineutrino reduced cross sections for the removal of nucleons from 1p-shell of ^{16}O for Saclay (upper panels) and NIKHEF (lower panels) kinematics as functions of p_m . The solid line is electron while the dashed and dashed-dotted lines are respectively neutrino and antineutrino cross sections.



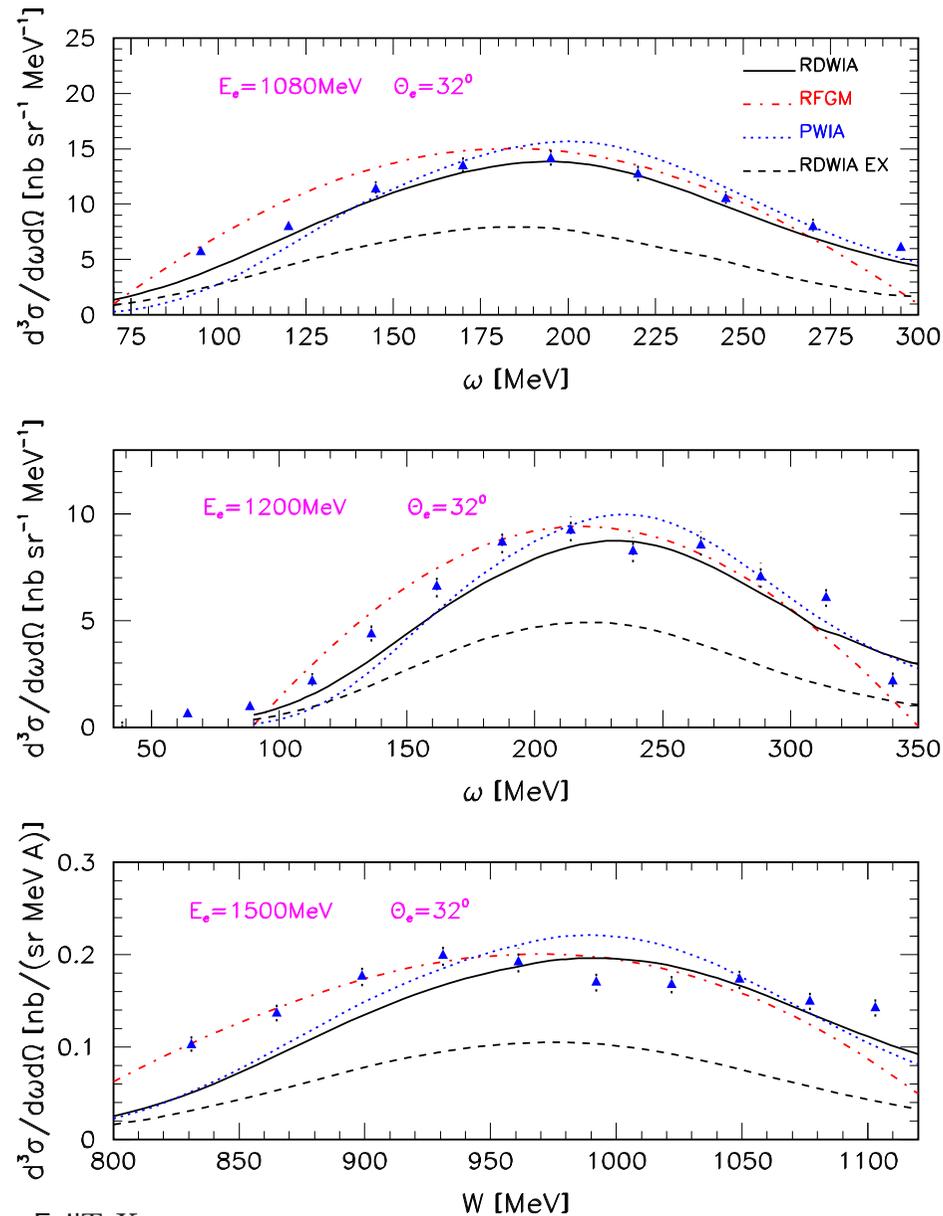
At the maximum electron cross section are higher (less than 10%) than (anti)neutrino ones.



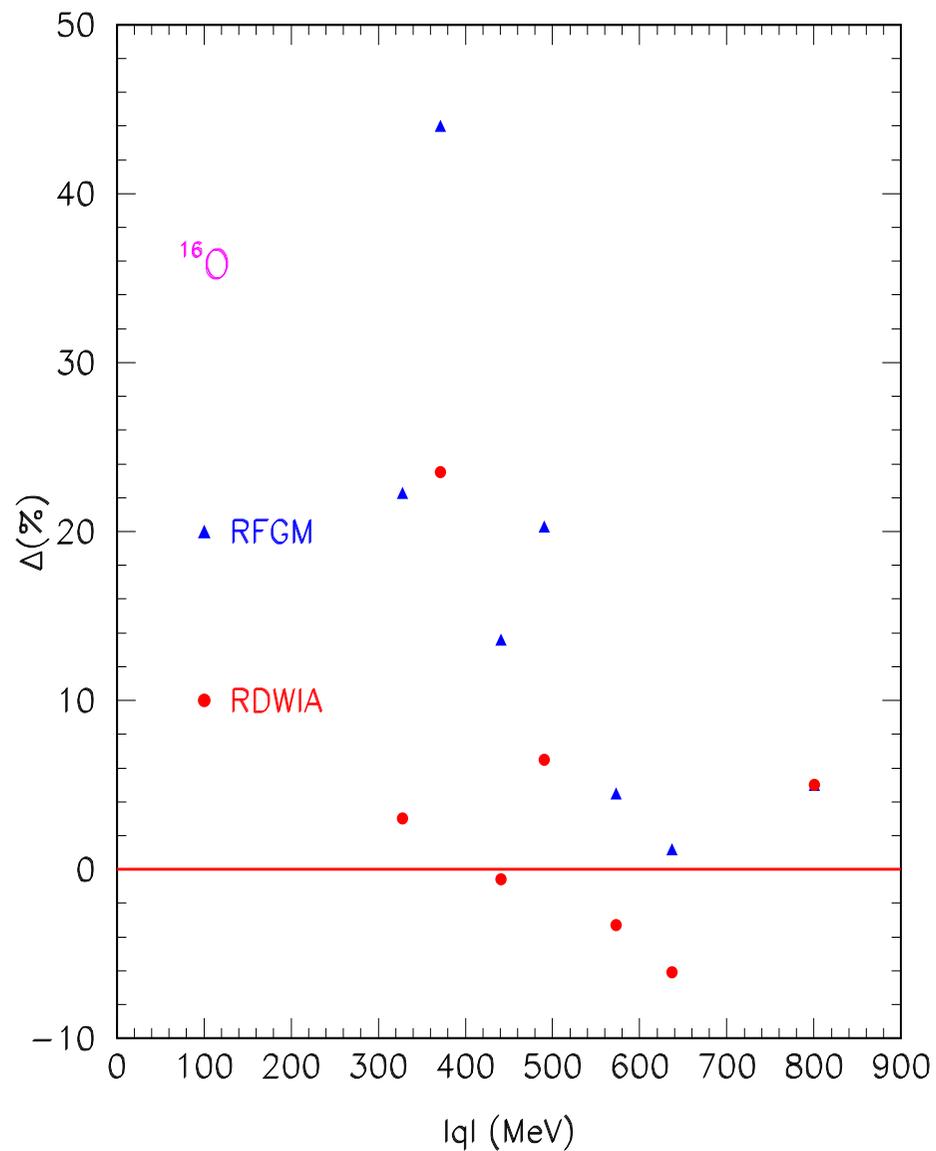
Comparison of the RDWIA and the RFGM calculations for electron, neutrino and antineutrino reduced and differential results for the removal of nucleons from 1p- and 1s-shells of ${}^{16}\text{O}$. The dashed-dotted line is the RDWIA calculation for electron scattering while the dashed and dotted lines are respectively for neutrino and antineutrino scattering. The solid line on the left panels shows the RFGM result while the solid and dashed lines on the right panels are respectively neutrino and antineutrino cross sections calculated in the Fermi gas model. The dashed-dotted and dotted lines on right panels are respectively neutrino and antineutrino cross sections calculated in the RDWIA.



Inclusive cross section versus the energy transfer ω or invariant mass W (lower-right panel) for electron scattering on ^{16}O . The data are from SLAC (filled circles) and Frascati (filled triangles). SLAC data are for electron beam energy $E_e = 540, 730$ MeV and scattering angle $\theta = 37.1^\circ$. Frascati data are for $E_e = 540$ MeV and $\theta = 37.1^\circ$, $E_e = 700, 880$ MeV and $\theta = 32^\circ$. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.

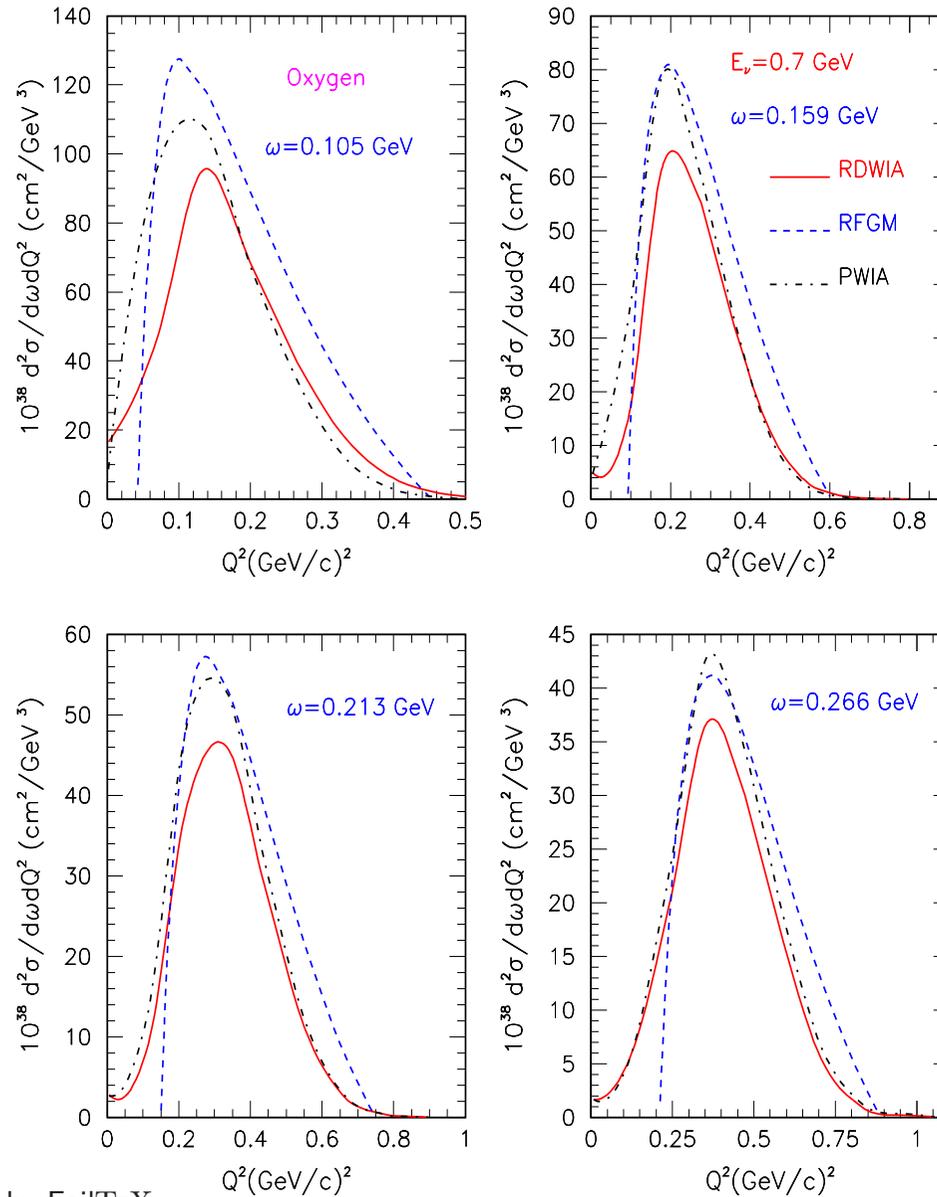


Inclusive cross section versus the energy transfer ω or invariant mass W (lower panel) for electron scattering on ^{16}O . The data are from Frascati for electron beam energy $E_e = 1080, 1200, \text{ and } 1500$ MeV and scattering angle $\theta = 32^\circ$. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with a complex optical potential.

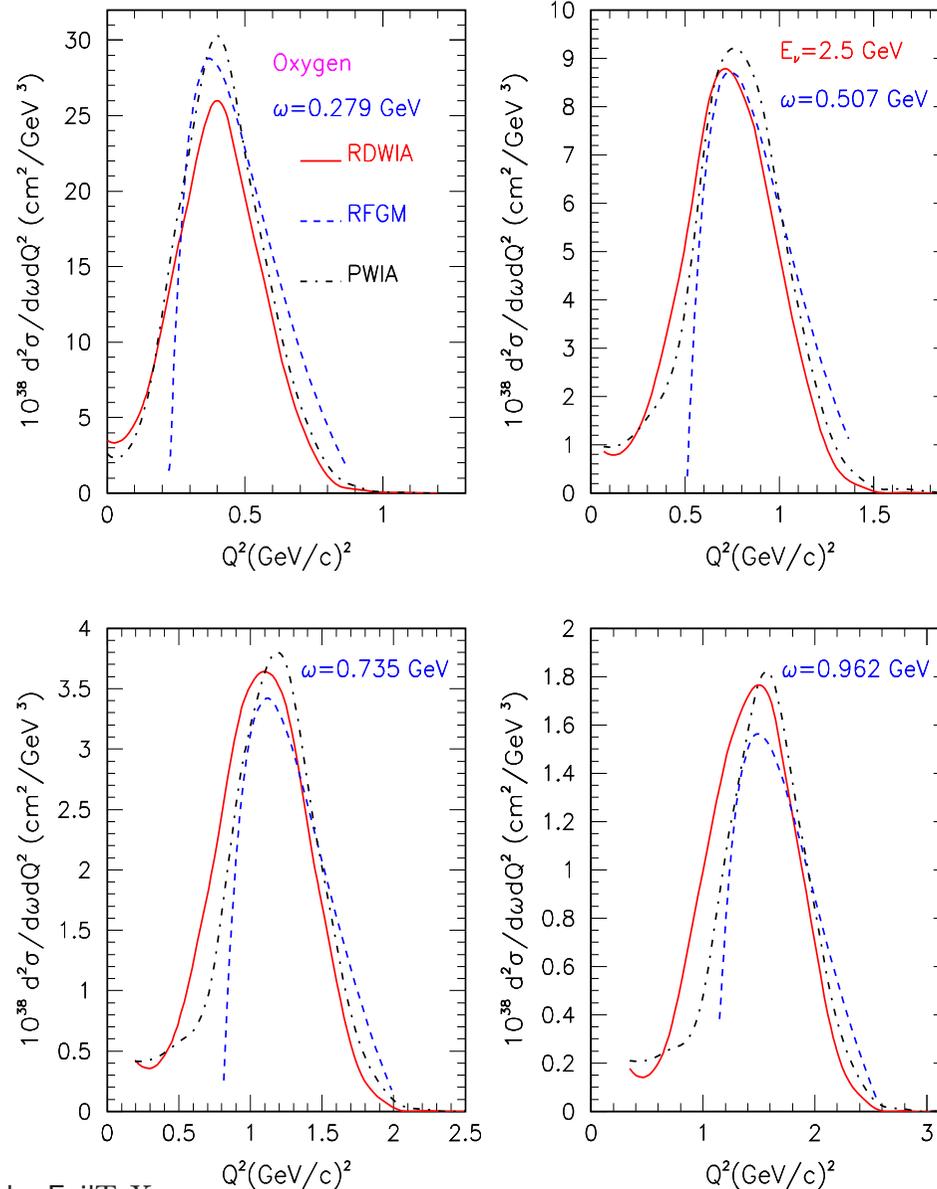


Differences between calculated and measured values of the inclusive cross sections at maximum as a function of three-momentum transfer.

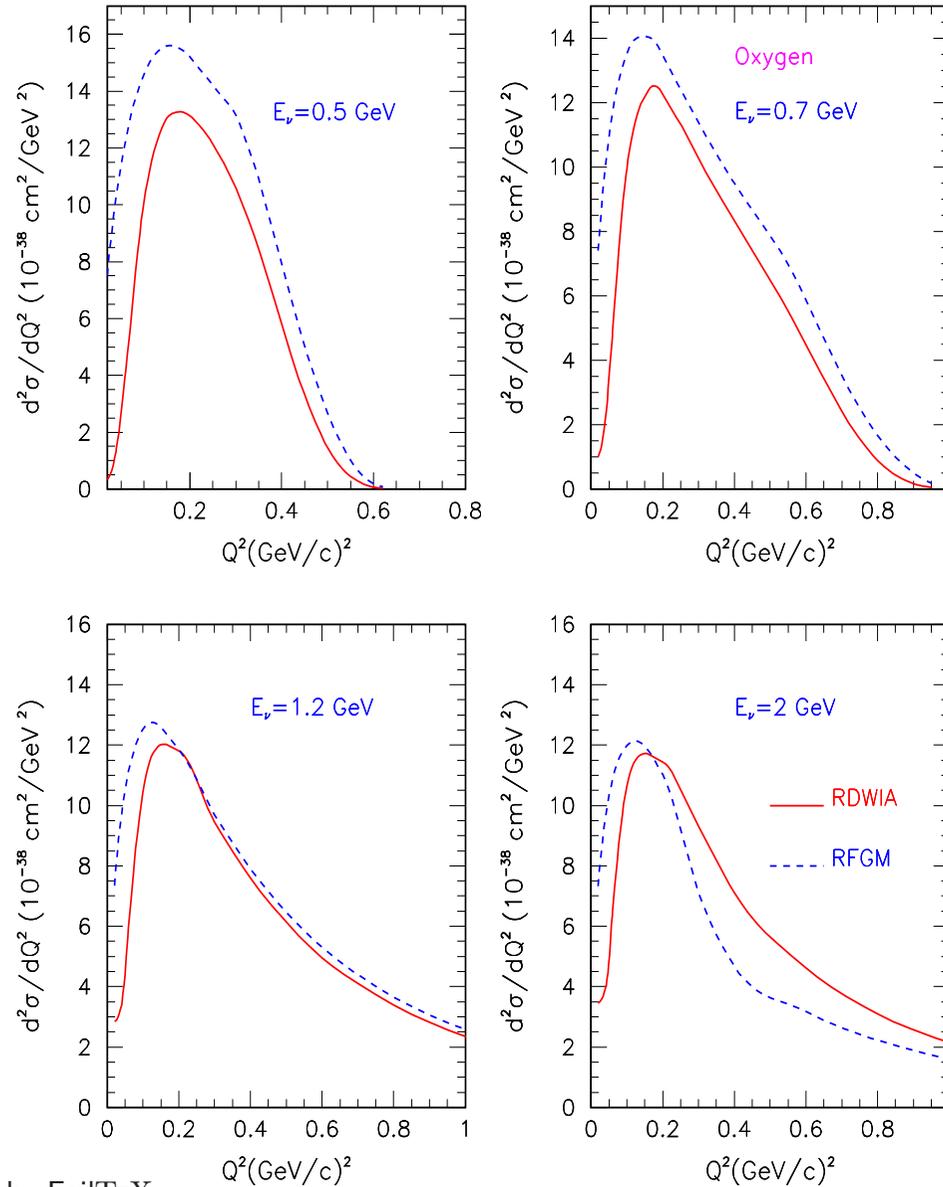
Note that: $Q^2 = q^2 - \omega^2$



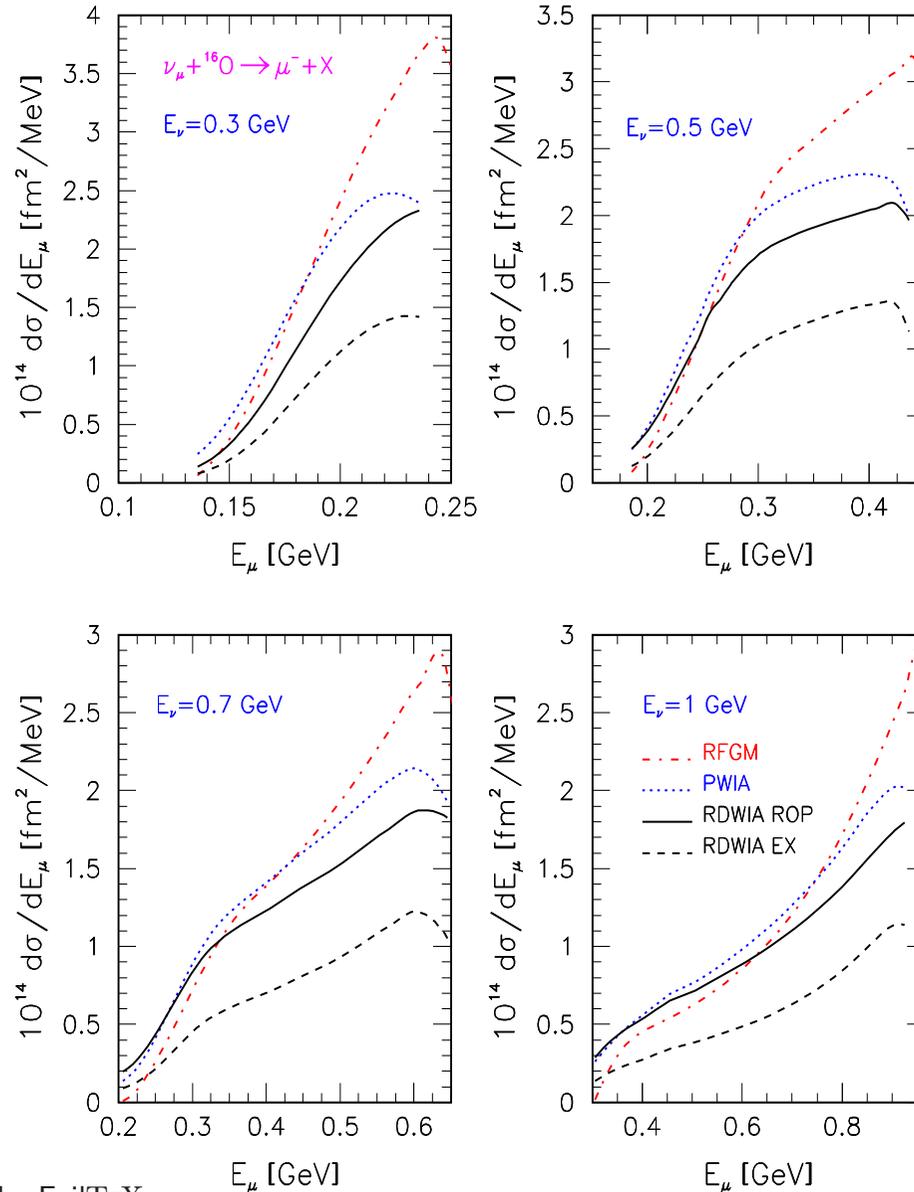
Inclusive cross section $d^3\sigma/d\omega dQ^2$ versus Q^2 for neutrino scattering on ¹⁶O with incoming energy $E_\nu = 0.7$ GeV and for the four values of energy transfer: $\omega = 0.105, 0.158, 0.213$ and 0.266 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.



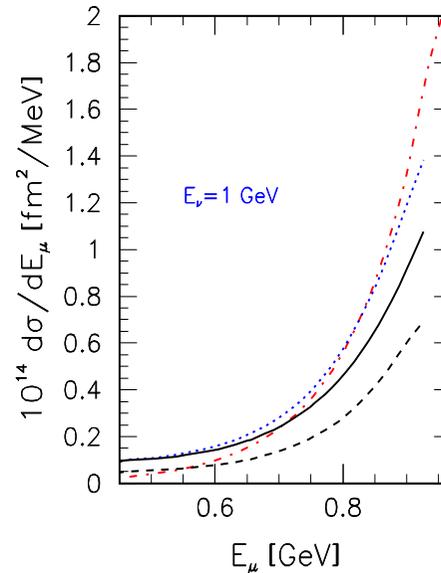
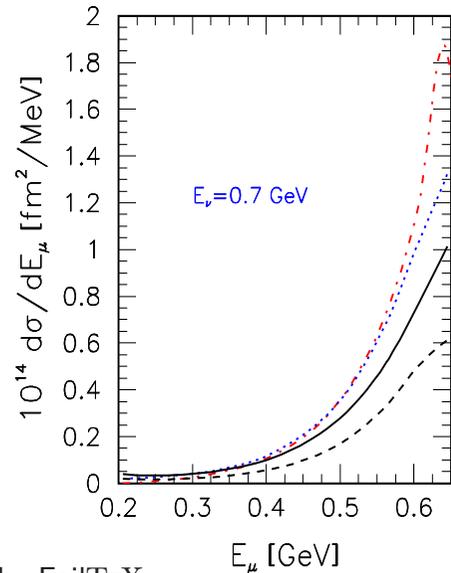
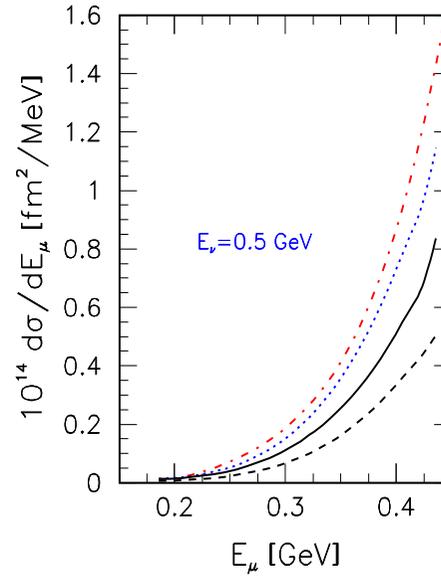
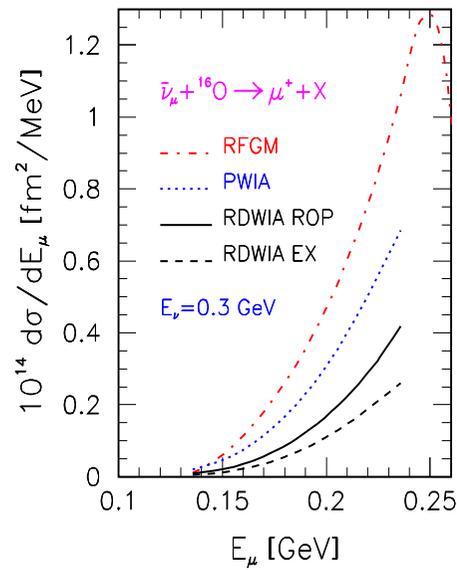
Inclusive cross section $d^3\sigma/d\omega dQ^2$ versus Q^2 for neutrino scattering on ^{16}O with incoming energy $E_\nu = 2.5$ GeV and for the four values of energy transfer: $\omega = 0.279, 0.507, 0.735$ and 0.962 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.



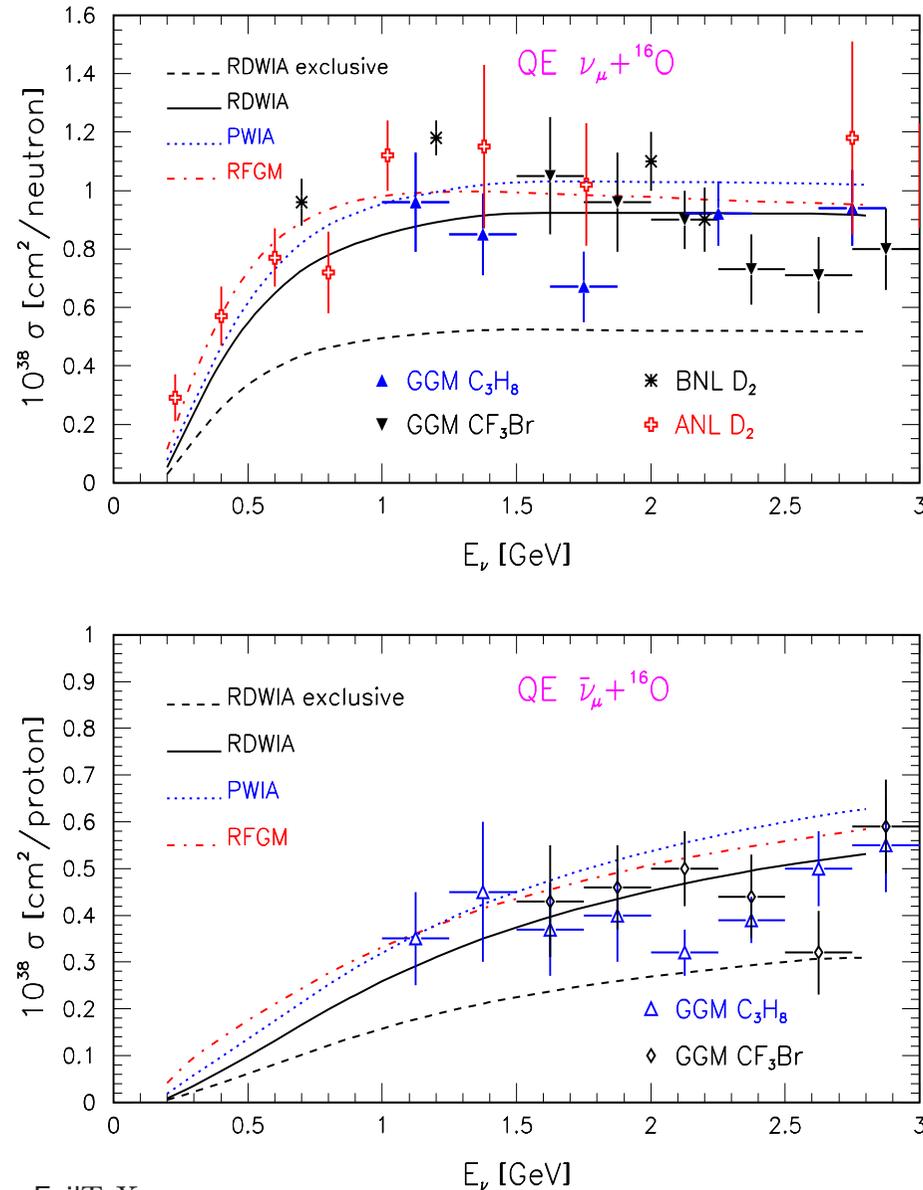
Inclusive cross section $d^2\sigma/dQ^2$ versus Q^2 for the four values of incoming neutrino energy : $E_\nu = 0.5, 0.7, 1.2$ and 2 GeV . The solid line is the RDWIA calculation while the dashed lines are the RFGM calculations.



Inclusive cross section versus the muon energy for neutrino scattering on ¹⁶O and for the four values of incoming neutrino energy: $E_\nu = 0.3, 0.5, 0.7$ and 1 GeV. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.



Inclusive cross section versus the muon energy for antineutrino scattering on ${}^{16}\text{O}$ and for the four values of incoming neutrino energy: $E_\nu = 0.3, 0.5, 0.7$ and 1 GeV. The solid line is the RDWIA calculation while the dotted and dashed lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.



Total cross section for CC QE scattering of muon neutrino (upper panel) and antineutrino (lower panel) on ${}^{16}\text{O}$ as a function of incoming (anti)neutrino energy. The solid and dashed lines are respectively the RDWIA results with the real and complex optical potential. The dashed-dotted and dotted lines are respectively the RFGM and PWIA results. Data points for different targets are from ANL, BNL, and GGM

Summary

QE CC $\nu(\bar{\nu})^{16}\text{O}$ cross sections were studied in different approaches.

- In **RDWIA** the reduced exclusive cross sections for $\nu(\bar{\nu})$ scattering are similar to those of electron scattering and in a good agreement with data.
- The inclusive and total cross sections were calculated neglecting the imaginary part of relativistic optical potential and taking into account the effect of NN-correlations in the target ground state and tested against $^{16}\text{O}(e, e')$ scattering data.
- **FSI** effect reduces the inclusive cross sections at low Q^2 and energy transfer compared to **PWIA** and **RFGM**. This effect decreases with neutrino energy.
- **FSI** effect reduces the total cross section for about 30% for $E_\nu = 200$ MeV compared to **PWIA** and decreases with neutrino energy down to 10% at 1 GeV.

- Effect of NN-correlations further reduces the total cross section for about 15% for $E_\nu = 200$ MeV. This effect decreases with neutrino energy, down to 8% at 1 GeV.
- The Fermi gas model was tested against $e^{16}\text{O}$ data:
 - (★) RFGM fails completely when compared to exclusive cross section data.
 - (★) In the peak region RFGM overestimates the value of inclusive cross section at low momentum transfer ($|\mathbf{q}| < 500$ MeV/c). The discrepancy with data is about 20% at $|\mathbf{q}| = 300$ MeV/c and decreases as momentum transfer increases.
- For total neutrino cross sections RFGM result is about 15% higher than the RDWIA predictions at $E_\nu \sim 1$ GeV.
- Our results show that nuclear-model dependence of the inclusive and total cross sections weakens with neutrino energy but still remains significant for energy $E_\nu \lesssim 1$ GeV.