

Measurement of Muon Neutrino Quasi-Elastic Scattering on a Hydrocarbon Target at $E_\nu \sim 3.5$ GeV

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We report a study of ν_μ charged-current quasi-elastic events in the segmented scintillator inner tracker of the MINERvA experiment running in the NuMI neutrino beam at Fermilab. The events were selected by requiring a μ^- and low calorimetric recoil energy separated from the interaction vertex. We measure the flux-averaged differential cross-section, $d\sigma/dQ^2$, and study the low energy particle content of the final state. Deviations are found between the measured $d\sigma/dQ^2$ and the expectations of a model of independent nucleons in a relativistic Fermi gas. We also observe an excess of energy near the vertex consistent with multiple protons in the final state.

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Charged-current neutrino quasi-elastic scattering, $\nu_\mu n \rightarrow \mu^- p$, distinguishes neutrino flavor and is valuable for neutrino oscillation experiments at energies near

1 GeV where it is responsible for a large fraction of the total reaction cross-section [1–4]. For free nucleons the scattering process may be described by the stan-

standard theory of weak interactions with the inclusion of nucleon form factors [5]. Electron scattering [6] and neutrino scattering on deuterium [7, 8] determine the most important form factors with good precision [9]. However, neutrino oscillation experiments typically use detectors made of heavier nuclei such as carbon [4, 10], oxygen [11], iron [12], or argon [13, 14] where interactions with nucleons are modified by the nuclear environment. These effects are commonly modeled using a relativistic Fermi gas [15, 16] (RFG) description of the nucleus as quasi-free, independent nucleons with Fermi motion in a uniform binding potential. Neutrino interaction generators [17–19] additionally simulate interactions of final state hadrons inside the target nucleus. The MiniBooNE experiment recently observed that this prescription, utilizing the free deuterium value for the axial form factor, does not accurately describe their measurements of quasi-elastic scattering of neutrinos and antineutrinos on a hydrocarbon target [20, 21].

The RFG approach may be supplemented by accounting for correlations between nucleons within the nucleus. Evidence for these correlations has been observed in electron-nucleus scattering [22]. Processes that produce multiple final state nucleons are thought to lead to enhancements in the cross-section [23–25]. These contributions are modeled using different approaches [26–28] which produce qualitatively similar though not quantitatively identical results. The RFG model may also be replaced by an alternate spectral function (SF) model that calculates the joint probability distribution of scattering off a nucleon of given momentum and binding energy inside a nucleus [29]. These nuclear effects may be significant for oscillation experiments seeking to measure the neutrino mass hierarchy and CP violation [30–32].

In this Letter we report the first study of muon neutrino quasi-elastic interactions at energies between 1.5 and 10 GeV from the MINERvA experiment, which uses a finely segmented scintillator detector at Fermilab to measure muon neutrino interactions on nuclear targets. The analysis technique is similar to the one employed to study the antineutrino reaction [33]. The signal has a μ^- in the final state along with one or more nucleons (typically with a leading proton), and no mesons. We reject events in which mesons are produced by requiring that the hadronic system recoiling against the muon has a low energy. That energy is measured in two spatial regions. The *vertex energy* region corresponds to a sphere around the vertex with a radius sufficient to contain a proton (pion) with 225 (100) MeV kinetic energy. This region is sensitive to low energy protons which could arise from correlations among nucleons in the initial state or final state interactions of the outgoing hadron inside the target nucleus [34]. We do not use the vertex energy in the event selection but study it for evidence of multi-nucleon processes. The *recoil energy* region includes energy depositions outside of the vertex region and is sensitive to

pions and higher energy nucleons. We use the recoil energy to estimate and remove inelastic backgrounds.

The MINERvA detector was exposed to the NuMI neutrino beam at Fermilab, configured for this analysis to produce a beam consisting of $> 95\%$ ν_μ at the peak energy of 3 GeV. The neutrino flux is predicted using a Geant4-based simulation tuned to hadron production data [35] as described in Ref. [33]. This analysis uses data taken between March and July 2010 with 9.42×10^{19} protons on target.

The MINERvA detector consists of a fine-grained scintillator tracker surrounded by electromagnetic and hadronic calorimeters [36]¹. The detector enables reconstruction of the neutrino interaction point, the tracks of outgoing charged particles, and the calorimetric reconstruction of other particles produced in the interaction. MINERvA is located 2 m upstream of the MINOS near detector [12], which is used to reconstruct the momentum and charge of muons. The hadronic energy scale is set using data from through-going muons and a scaled down MINERvA detector exposed to a hadron test beam [36]. The detector's performance is simulated by a Geant4-based hit-level simulation and a readout model tuned to match the data [36]. Event pile-up causes a decrease in the muon track reconstruction efficiency which we studied in both MINERvA and MINOS by projecting tracks found in one of the detectors to the other and measuring the misreconstruction rate. This resulted in a -9.1% (-4.8%) correction to the simulated efficiency for muons with momenta below (above) 3 GeV/c in MINOS. Neutrino interactions in the detector are simulated using the GENIE neutrino event generator [17]. Details of the cross section models and associated parameters are described in Ref. [33].

Event reconstruction and selection for this analysis is nearly identical to that used in the MINERvA antineutrino quasi-elastic measurement [33] with small modifications to account for the likelihood of a leading proton in the final state instead of a neutron. We require events to have a μ^- originating in the 5.57 metric ton fiducial volume and assign remaining energy clusters with > 1 MeV to the recoiling hadronic system. The aforementioned vertex region corresponds to a sphere with 30 g/cm^2 of material centered on the vertex. The recoil system outside the vertex region is required to have ≤ 2 isolated groups of spatially contiguous energy depositions².

The neutrino energy and the square of the four momentum transferred to the nucleus, Q_{QE}^2 , are estimated from the muon momentum and angle using a quasi-elastic hy-

¹ The MINERvA scintillator tracking region is 95% CH and 5% other materials by weight.

² Isolated energy depositions are created directly by the leading proton or by secondary hadronic interactions in the detector.

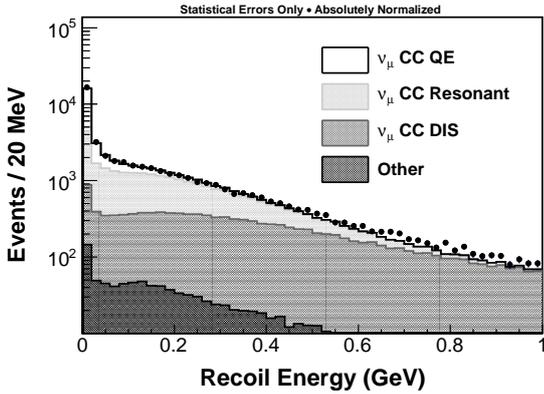


FIG. 1: The measured recoil energy distribution in the data (solid circles) and the predicted composition of signal and background. Backgrounds from baryon resonance production (light grey), continuum/deep-inelastic scattering (dark grey), and other sources (black), such as coherent pion production, are shown.

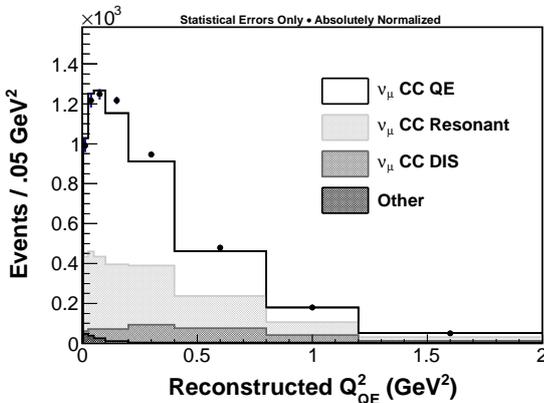


FIG. 2: The measured Q^2_{QE} distributions in the data and the simulation, before correcting for detector resolution and acceptance.

pothesis, as in the antineutrino analysis [33]. The binding energy correction is taken to be +34 MeV instead of +30 MeV used in Ref. [33] due to Coulomb corrections [37], and the proton and neutron masses are interchanged.

Figure 1 shows that the quasi-elastic signal preferentially populates lower recoil energies. However, since the proton's kinetic energy is $\approx Q^2/2M_{\text{neutron}}$ for quasi-elastic scattering, the recoil energy is expected to scale with the momentum transfer as the final state proton becomes increasingly energetic and escapes the vertex region. We account for this by varying a cut on the maximum allowed recoil energy as a function of Q^2_{QE} to assure 95% signal efficiency in each Q^2_{QE} bin.

The background in each Q^2_{QE} bin is estimated from the data by fitting the relative normalizations of signal and background recoil energy distributions whose shapes

Q^2_{QE} (GeV ²)	I	II	III	IV	V	VI	Total
0.0 – 0.025	0.06	0.04	0.02	0.04	0.09	0.03	0.13
0.025 – 0.05	0.06	0.03	0.02	0.03	0.09	0.02	0.12
0.05 – 0.1	0.06	0.03	0.02	0.03	0.09	0.02	0.12
0.1 – 0.2	0.06	0.03	0.03	0.02	0.09	0.02	0.11
0.2 – 0.4	0.05	0.02	0.03	0.03	0.09	0.01	0.11
0.4 – 0.8	0.05	0.03	0.04	0.04	0.09	0.01	0.13
0.8 – 1.2	0.08	0.07	0.07	0.15	0.09	0.02	0.22
1.2 – 2.0	0.12	0.07	0.07	0.16	0.09	0.02	0.24

TABLE I: Fractional systematic uncertainties on $d\sigma/dQ^2_{QE}$ associated with (I) muon reconstruction, (II) recoil reconstruction, (III) neutrino interaction models, (IV) final state interactions, (V) flux and (VI) other sources. The rightmost column shows the total fractional systematic uncertainty due to all sources.

are taken from the simulation. This procedure reduces the relative background prediction by 15% below Q^2_{QE} of 0.8 GeV² and 5% between 0.8 and 2.0 GeV². The purity of the resulting sample ranges from 65% at low Q^2_{QE} to 40% at higher Q^2_{QE} . Figure 2 compares the Q^2_{QE} distribution of the 29,620 events which satisfy the selection criteria to the simulation after rescaling the background according to the fit. The cross-section as a function of Q^2_{QE} is extracted by subtracting the backgrounds, correcting for detector resolution and acceptance, and dividing by the number of neutrons in the fiducial volume ($1.65 \pm 0.02 \times 10^{30}$) and by the flux, as described in Ref. [33]. The total neutrino flux integrated between 1.5 and 10 GeV is estimated by the simulation to be 2.91×10^{-8} /cm² per proton on target³.

The same systematic uncertainties which affect the antineutrino analysis [33] are evaluated in this analysis. Table I shows a summary of systematic uncertainties on $d\sigma/dQ^2_{QE}$. The largest uncertainties on the absolute cross-section come from the neutrino flux and the muon momentum scale. However, the flux uncertainty is largely independent of Q^2_{QE} so comparisons of the shape of $d\sigma/dQ^2_{QE}$ to scattering model predictions are relatively insensitive to knowledge of the flux. The saturation of ionization (dE/dx), parameterized by Birk's law and characterized by a factor of $(1+k_B \times dE/dx)^{-1}$, leads to a recoil reconstruction uncertainty; this uncertainty is negligible for the antineutrino $d\sigma/dQ^2_{QE}$ measurement but is important for the neutrino measurement. By studying stopping proton tracks in the MINERvA test beam detector we estimate $k_B = 0.13 \pm 0.04$ mm/MeV [36], and vary the ionization accordingly in the simulation to propagate the uncertainty.

The vertex energy distribution is sensitive to the multiplicity of low energy charged hadrons in the final state.

³ See Supplemental Material in the Appendix for the flux as a function of energy

Q_{QE}^2 (GeV ²)	Cross-section (10 ⁻³⁸ cm ² /GeV ² /neutron)	Fraction of Cross-section (%)
0.0 – 0.025	0.761 ± 0.035 ± 0.097	2.15 ± 0.10 ± 0.17
0.025 – 0.05	1.146 ± 0.047 ± 0.137	3.24 ± 0.13 ± 0.22
0.05 – 0.1	1.343 ± 0.034 ± 0.156	7.60 ± 0.19 ± 0.50
0.1 – 0.2	1.490 ± 0.028 ± 0.170	16.85 ± 0.32 ± 1.04
0.2 – 0.4	1.063 ± 0.019 ± 0.120	24.06 ± 0.43 ± 1.06
0.4 – 0.8	0.582 ± 0.013 ± 0.074	26.33 ± 0.58 ± 0.85
0.8 – 1.2	0.242 ± 0.014 ± 0.053	10.95 ± 0.64 ± 1.45
1.2 – 2.0	0.097 ± 0.008 ± 0.024	8.81 ± 0.71 ± 1.43

TABLE II: Flux-averaged differential cross-sections and the fraction of the cross-section in bins of Q_{QE}^2 . In each measurement, the first error is statistical and the second is systematic.

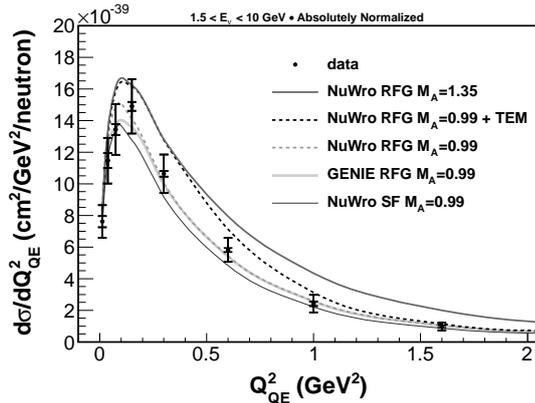


FIG. 3: Neutrino quasi-elastic cross-section as a function of Q_{QE}^2 compared with several different models of the interaction.

Systematic uncertainties on this distribution are evaluated with the same methods used for the cross-section measurement. The largest uncertainties in the distribution come from the detector’s response to protons (constrained by test beam measurements [36]), the Birk’s law constant discussed above, and GENIE’s final state interactions model. The latter is evaluated by varying the underlying model tuning parameters within their systematic uncertainties.

The measured differential cross-section $d\sigma/dQ_{QE}^2$ is shown in Table II and Fig. 3. Integrating over the flux from 1.5 to 10 GeV, we find $\sigma = 0.93 \pm 0.01(\text{stat}) \pm 0.11(\text{syst}) \times 10^{-38} \text{ cm}^2/\text{neutron}^4$. Figures 3 and 4 and Table III compare the data to the RFG model in the GENIE event generator and a set of calculations made with the NuWro generator [19].

Different models of nuclear effects in quasi-elastic scattering lead to significant variations in the shape of

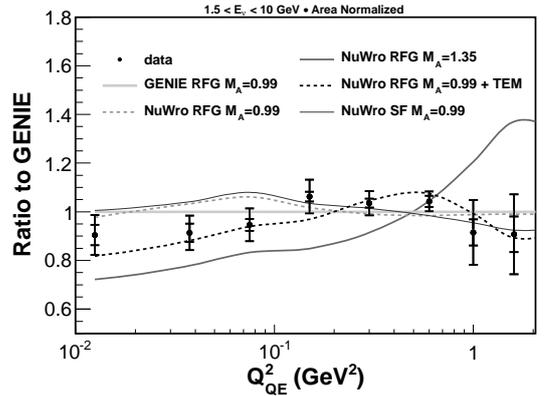


FIG. 4: Ratio between the measured neutrino $d\sigma/dQ_{QE}^2$ shape in Q_{QE}^2 and several different models, where the denominator is the GENIE default quasi-elastic cross-section.

$d\sigma/dQ_{QE}^2$ from the expectation of the RFG model. In particular, correlations between nucleons not considered in the mean field RFG approach are predicted to contribute to the cross-section at neutrino energies below 2 GeV [26–28]. Figure 4 compares the shape of the measured cross section to five different models of the quasi-elastic process on carbon. The GENIE prediction, based on a RFG nuclear model and dipole axial form factor with $M_A = 0.99$ GeV, is taken as a reference; the data and other models are normalized to have the same total cross section across the range shown before forming the ratio. The NuWro calculations utilize an axial-vector form factor parameterized with a dipole form that has one free parameter, the axial mass M_A , and also incorporate different corrections for the nuclear medium. There is little sensitivity to replacement of the Fermi gas with a spectral function (SF) model of the target nucleon energy-momentum relationship [29]. The neutrino data are marginally more compatible, at least in Q_{QE}^2 shape, with a higher axial mass extracted from fits of the Mini-BooNE neutrino quasi-elastic data in the RFG model ($M_A = 1.35 \text{ GeV}/c^2$) [20] than with that extracted from deuterium data ($M_A = 0.99 \text{ GeV}/c^2$). As with the corresponding antineutrino results [33], our data are in best agreement with a transverse enhancement model (TEM) with $M_A = 0.99 \text{ GeV}/c^2$. This model implements an enhancement of the magnetic form factors of bound nucleons that has been extracted from electron-carbon scattering data [25], and is the only one of this type that is applicable at neutrino energies above 2 GeV. Table III shows a comparison using χ^2 values between the measured cross section and the five NuWro models considered.

Observations from electron elastic scattering on carbon suggest that multibody final states are dominated by initial-state np pairs [22]. This could lead to an expectation of final state pp pairs in neutrino quasi-elastic scat-

⁴ See Supplemental Material in Appendix for correlations of the uncertainties among Q_{QE}^2 bins for both the cross-section and shape measurement

NuWro Model	RFG	RFG +TEM	RFG	SF
M_A (GeV/c ²)	0.99	0.99	1.35	0.99
Rate χ^2 /d.o.f.	3.5	2.4	3.7	2.8
Shape χ^2 /d.o.f.	4.1	1.7	2.1	3.8

TABLE III: Comparisons between the measured $d\sigma/dQ_{QE}^2$ (or its shape in Q_{QE}^2) and different models implemented using the NuWro neutrino event generator, expressed as χ^2 per degree of freedom (d.o.f.) for eight (seven) degrees of freedom. The χ^2 computation in the table accounts for significant correlations between the data points caused by systematic uncertainties.

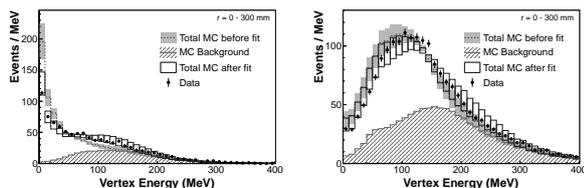


FIG. 5: Reconstructed vertex energy of events passing the selection criteria in the data (points with statistical errors) compared to the GENIE RFG model (shown with systematic errors) for $Q_{QE}^2 < 0.2$ GeV²/c² (left) and for $Q_{QE}^2 > 0.2$ GeV²/c² (right).

tering and nn pairs in the analogous antineutrino channel. The vertex energy measurement, shown in Fig. 5, is sensitive to these effects. These data prefer the addition of a final state proton with less than 225 MeV kinetic energy in $25 \pm 1(\text{stat}) \pm 9(\text{syst})\%$ of the events. The corresponding result in the antineutrino mode [33], in contrast, prefers the *removal* of a final state proton in $10 \pm 1(\text{stat}) \pm 7(\text{syst})\%$ of the events. The systematic uncertainties for the two samples are positively correlated with a correlation coefficient of +0.7, implying that the observed difference is unlikely to be due to one of the systematic uncertainties considered. The systematic uncertainties are primarily from the detector response to protons and uncertainties in reactions in the target nucleus that absorb or create final state protons. Independent of models, elastic and inelastic nucleon reactions which might produce additional final state protons in the neutrino data should have analogous reactions in the anti-neutrino data, and the difference in the two results makes it unlikely that any modification of final state nucleon interactions can explain the discrepancy. Pion FSI processes, especially absorption, would produce more protons in the neutrino reaction and neutrons in the antineutrino reaction, but the associated uncertainties are included in the total systematic errors. The observed patterns in the neutrino and antineutrino channels, combined with the observation that electron quasi-elastic scattering with multinucleon final states in carbon produces primarily final state np pairs, suggests

an initial state of strongly correlated np pairs also may participate in the neutrino quasi-elastic interaction.

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Appendix: Supplementary Material

Q_{QE}^2 (GeV ²) Bins	0.0 – 0.025	0.025 – 0.05	0.05 – 0.1	0.1 – 0.2	0.2 – 0.4	0.4 – 0.8	0.8 – 1.2	1.2 – 2.0
Cross-section in bin (10^{-38} cm ² /GeV ² /neutron)	0.761 ± 0.104	1.146 ± 0.144	1.343 ± 0.160	1.490 ± 0.172	1.063 ± 0.122	0.582 ± 0.075	0.242 ± 0.055	0.097 ± 0.025
Q_{QE}^2 (GeV ²)								
0.0 – 0.025	1.000	0.869	0.882	0.873	0.832	0.690	0.415	0.327
0.025 – 0.05		1.000	0.905	0.917	0.882	0.727	0.457	0.357
0.05 – 0.1			1.000	0.945	0.928	0.751	0.455	0.356
0.1 – 0.2				1.000	0.946	0.788	0.481	0.385
0.2 – 0.4					1.000	0.865	0.600	0.514
0.4 – 0.8						1.000	0.756	0.741
0.8 – 1.2							1.000	0.888
1.2 – 2.0								1.000

TABLE IV: The measurement of the neutrino differential cross-sections in Q_{QE}^2 , their total (statistical and systematic) uncertainties, and the correlation matrix for these uncertainties

Q_{QE}^2 (GeV ²) Bins	0.0 – 0.025	0.025 – 0.05	0.05 – 0.1	0.1 – 0.2	0.2 – 0.4	0.4 – 0.8	0.8 – 1.2	1.2 – 2.0
% of Cross-section in bin	2.15 ±0.20	3.24 ±0.26	7.60 ±0.53	16.85 ±1.09	24.06 ±1.14	26.33 ±1.03	10.95 ±1.58	8.81 ±1.60
Q_{QE}^2 (GeV ²)								
0.0 – 0.025	1.000	0.689	0.712	0.684	0.557	-0.175	-0.585	-0.623
0.025 – 0.05		1.000	0.745	0.770	0.653	-0.211	-0.631	-0.694
0.05 – 0.1			1.000	0.840	0.793	-0.212	-0.736	-0.787
0.1 – 0.2				1.000	0.817	-0.173	-0.780	-0.825
0.2 – 0.4					1.000	-0.129	-0.752	-0.795
0.4 – 0.8						1.000	-0.142	0.060
0.8 – 1.2							1.000	0.760
1.2 – 2.0								1.000

TABLE V: The measurement of the *shape* of the neutrino differential cross-sections for $Q_{QE}^2 < 2.0$ GeV², their total (statistical and systematic) uncertainties, and the correlation matrix for these uncertainties

E_ν in Bin	1.5 – 2	2 – 2.5	2.5 – 3	3 – 3.5	3.5 – 4	4 – 4.5	4.5 – 5	5 – 5.5	
ν_μ Flux (neutrinos/cm ² /POT ($\times 10^{-8}$))	0.310	0.409	0.504	0.526	0.423	0.253	0.137	0.081	
E_ν in Bin	5.5 – 6	6 – 6.5	6.5 – 7	7 – 7.5	7.5 – 8	8 – 8.5	8.5 – 9	9 – 9.5	9.5 – 10
ν_μ Flux (neutrinos/cm ² /POT ($\times 10^{-8}$))	0.055	0.043	0.036	0.031	0.027	0.024	0.021	0.019	0.017

TABLE VI: The calculated muon neutrino flux per proton on target (POT) for the data included in this analysis