An Overview of Neutrino-Hadron Interactions at Low and Intermediate Energies

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April 28, 2011

Fermilab

Precise Neutrino-Hadron Interactions

The need for precise estimates and measurements of the cross sections.

Among the topics to be covered today will be pion production at small Q^2.

Remarks on CP violation.

Coherent pion Production.

The transition from resonaces to DIS.

1.Introduction

- The long base-line experiments require identical comparisons in the near and the far away detectors to decipher differences caused by the oscillations. As an example I show the distortion on the spectrum in the muon energy caused by the oscillation in the SK detector.

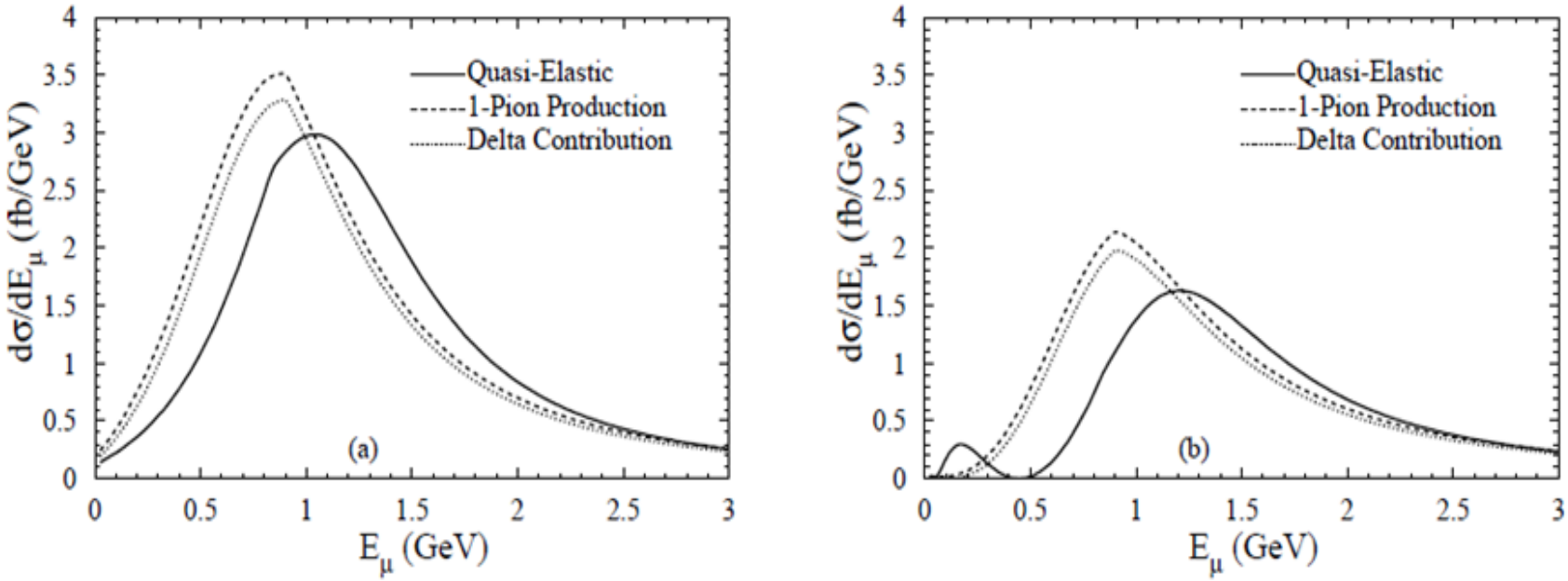


Figure 2: Approximate prediction of the muon energy spectra for the (a) Nearby and (b) SK

$$U_{\rm vac} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

Two quantities to measure

$$P(\overline{\nu}_\mu \to \overline{\nu}_\mu) = 1 - \sin^2(2\overline{\theta}) \sin^2\left(\frac{1.267\Delta\overline{m}^2 L}{E}\right)$$

$$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\overline{\nu}_\mu \to \overline{\nu}_e)} \simeq \frac{\Delta m_{12}^2 L}{4E_\nu} \cdot \frac{\sin 2\theta_{12}}{\sin\theta_{13}} \cdot \delta_{CP}.$$

2. Propreties of neutrino reactions for pion production

- Vector form factors determined by CVC and electroproduction data (Lalakulich+Paschos)

- Matrix elements of the axial current have 4 form factors, out of them two contribute to the divergence.
  (Special Property) : The neutrino reactions at small Q^2 select these two form factors.

Explanation on next page.

$$\left\langle \Delta^{++} \,\middle|\, \mathcal{A}_\mu^+ \,\middle|\, p \right\rangle = \sqrt{3}\, \overline{\psi}_\lambda(p') i g_\mu^\lambda C_5^A(Q^2) u(p) + \sqrt{3} \frac{i f_\pi q_\mu}{q^2 - m_\pi^2} \left\langle \Delta \,\middle|\, j_\pi \,\middle|\, p \right\rangle .$$

$$\begin{aligned}
\epsilon^\mu(\lambda = 0) \left\langle \Delta^{++} \,\middle|\, \mathcal{A}_\mu \,\middle|\, p \right\rangle &= \epsilon^\mu(\lambda = 0) i \sqrt{3} \overline{\psi}_\mu(p') C_5^A(Q^2) u(p) \\
&\approx i\sqrt{3} \frac{q^\mu}{\sqrt{Q^2}} \overline{\psi}_\mu(p') C_5^A(Q^2) u(p) + \mathcal{O}\left(\frac{Q^2}{\nu^2}\right) \\
&\approx -\frac{f_\pi \sqrt{2}}{\sqrt{Q^2}} A(\pi^+ p \to \Delta^{++})
\end{aligned}$$

2. Pion production at small Q^2

- Cross Section with zero helicity and longitudinal polarization. The formula is exact without transverse polarizations.

$$\frac{d\sigma^A}{dQ^2 d\nu} = \frac{G_F^2 |V_{ud}|^2}{2\pi} \frac{1}{4\pi} \frac{\nu}{E_\nu^2} \frac{f_\pi^2}{Q^2} \left\{ \tilde{L}_{00} + 2\tilde{L}_{l0} \frac{m_\pi^2}{Q^2 + m_\pi^2} + \tilde{L}_{ll} \left(\frac{m_\pi^2}{Q^2 + m_\pi^2} \right)^2 \right\} \sigma(\pi^+ p \to X^{++}). \tag{12}$$

$$\tilde{L}_{00} = 4\left[\frac{[Q^2(2E_\nu - \nu) - \nu m_\mu^2]^2}{Q^2(Q^2+\nu^2)} - Q^2 - m_\mu^2\right]$$

$$W_2^A(Q^2,\nu) = \frac{2f_\pi^2}{\pi}\frac{\nu}{Q^2+\nu^2}\sigma(\pi^+ p \to X^{++})$$

Values for Ev=1.0 GeV and Q^2= 0.010GeV^2

ν [GeV]	W [GeV]	$\sigma(\pi^+ p)$ [mb]	$\sigma(\pi^- p)$ [mb]	$\frac{\nu f_\pi^2 \tilde{L}_{00}}{E_\nu Q^2}$ [GeV]
0.20	1.118	16	12	0.329
0.25	1.159	77	30	0.240
0.30	1.199	189	67	0.171
0.35	1.238	175	63	0.119
0.40	1.275	95	37	0.079
0.45	1.311	60	28	0.047
0.50	1.347	42	26	0.022
0.55	1.381	31	28	0.002

Factor infront for Paschos+ Schalla vs Berger Sehgal

E=1 GeV	Q^2 =0.010GeV^2				Q^2 =0.10 GeV^2	
v GeV	Q^2 /min	Pasc+sch	BS		Pasc+Sch	BS
0.20	0.0028	.329	.215		.150	.189
0.25	0.0037	.240	.161		.140	.163
0.30	0.0048	.171	.122		.130	.136
0.35	0.0061	.119	.095		.110	.117
0.40	0.0074	.079	.073		.095	.097
0.45	0.0091	.047	.056		.085	.078
0.50	0.0112	.022			.070	.064
0.55	0.136				.058	.051

Alternative general formula

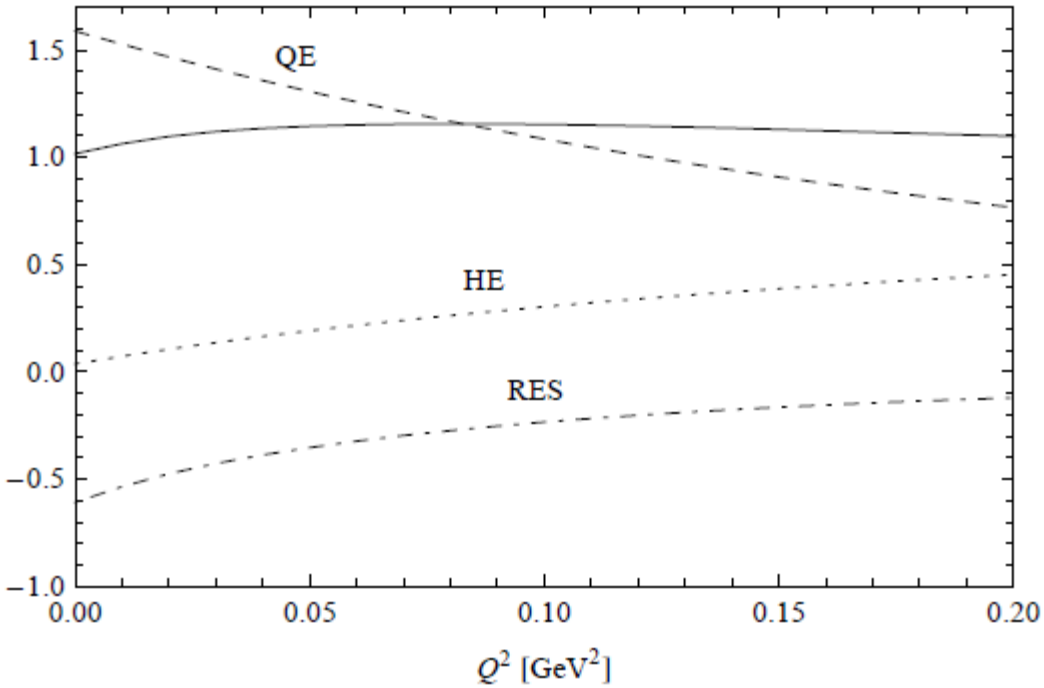
$$\frac{d\sigma}{dQ^2 dW} = \frac{G^2}{4\pi}\cos^2\theta_C \frac{W}{m_N E^2}\Big\{\mathcal{W}_1(Q^2 + m_\mu^2)$$
$$+ \frac{\mathcal{W}_2}{m_N^2}\Big[2(k\cdot p)(k'\cdot p) - \frac{1}{2}m_N^2(Q^2 + m_\mu^2)\Big]$$
$$- \frac{\mathcal{W}_3}{m_N^2}\Big[Q^2 k\cdot p - \frac{1}{2}q\cdot p(Q^2 + m_\mu^2)\Big]$$
$$+ \frac{\mathcal{W}_4}{m_N^2}m_\mu^2\frac{(Q^2 + m_\mu^2)}{2} - 2\frac{\mathcal{W}_5}{m_N^2}m_\mu^2(k\cdot p)\Big\}$$

2a. Adler sum rule

$$W_2^A(Q^2,\nu) = \frac{2f_\pi^2}{\pi}\frac{\nu}{Q^2+\nu^2}\sigma(\pi^+ p \to X^{++})$$

$$\left[g_A(Q^2)\right]^2 + \frac{2f_\pi^2}{\pi}\int_{\nu_{th}}^{\infty} \mathrm{d}\nu \frac{\nu}{Q^2+\nu^2}\left[\sigma^{\pi^+ n}(\nu) - \sigma^{\pi^+ p}(\nu)\right] = 1.$$

Comparison with Adler sum rule



The difference gives how big is the high energy contribution

- 1- $\left[g_A(Q^2)\right]^2 + \frac{2f_\pi^2}{\pi}\int_{\nu_{th}}^{\infty} \mathrm{d}\nu \frac{\nu}{Q^2+\nu^2}\left[\sigma^{\pi^+ n}(\nu) - \sigma^{\pi^+ p}(\nu)\right]$

The difference gives how big is the high energy contribution (from A. Bodek)

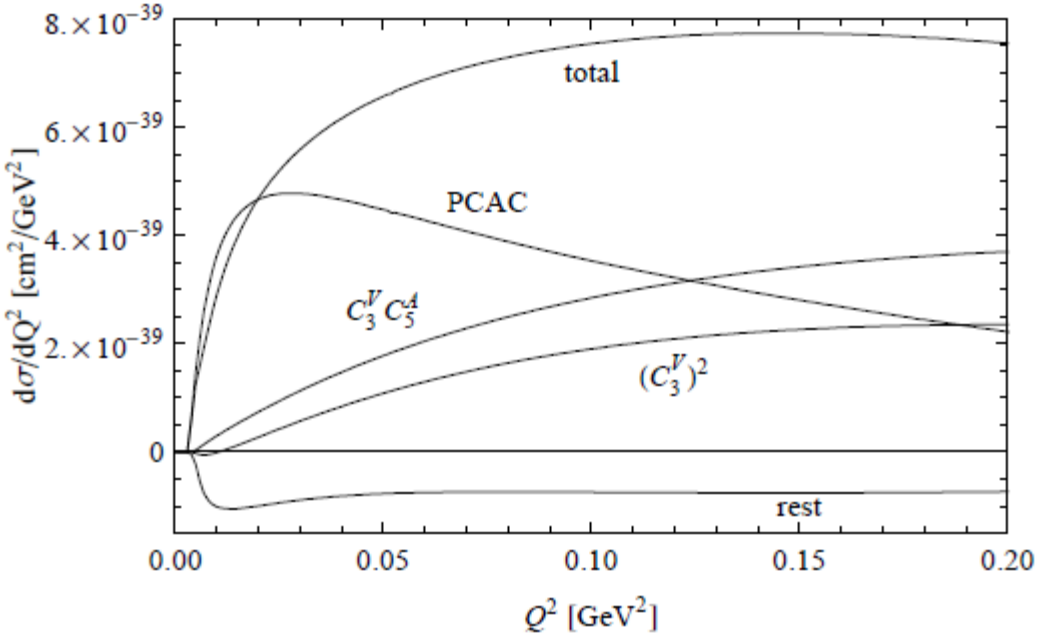
-

0	0
0.05	0.075067224
0.10	0.152807688
0.15	0.236130089
0.20	0.304015692
0.25	0.378436883
0.30	0.433828477
0.35	0.487216568
0.40	0.540360896
0.45	0.575967877

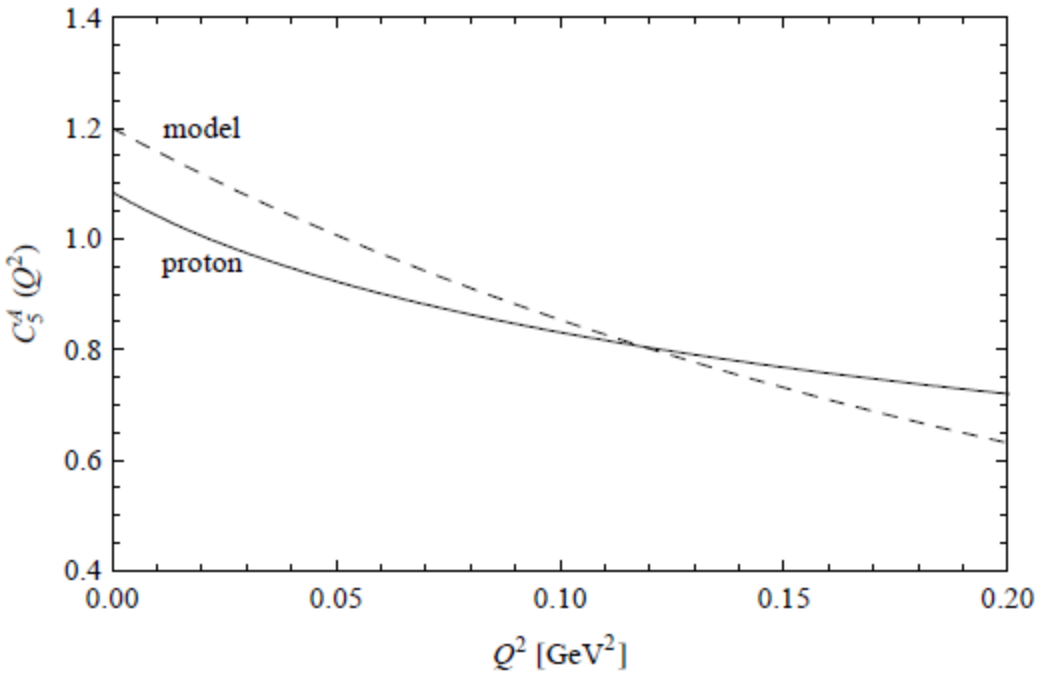
Estimates from Arie Bodek

Q2	axial Integral of d-u above W>1.4
0	0
0.05	0.075067224
0.10	0.152807688
0.15	0.236130089
0.20	0.304015692
0.25	0.378436883
0.30	0.433828477
0.35	0.487216568
0.40	0.540360896
0.45	0.575967877
0.50	0.614372485
0.55	0.645610486
0.60	0.67523349
0.65	0.70076928

Vector, Axial, Interference



Determine C5(A, Q^2)



2b. Compare experiments

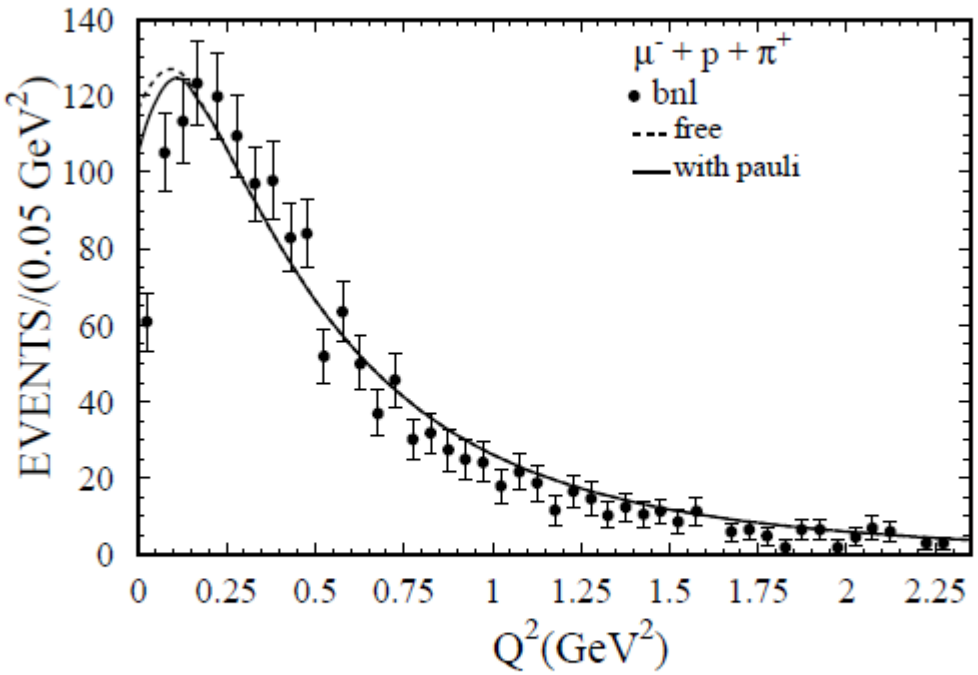
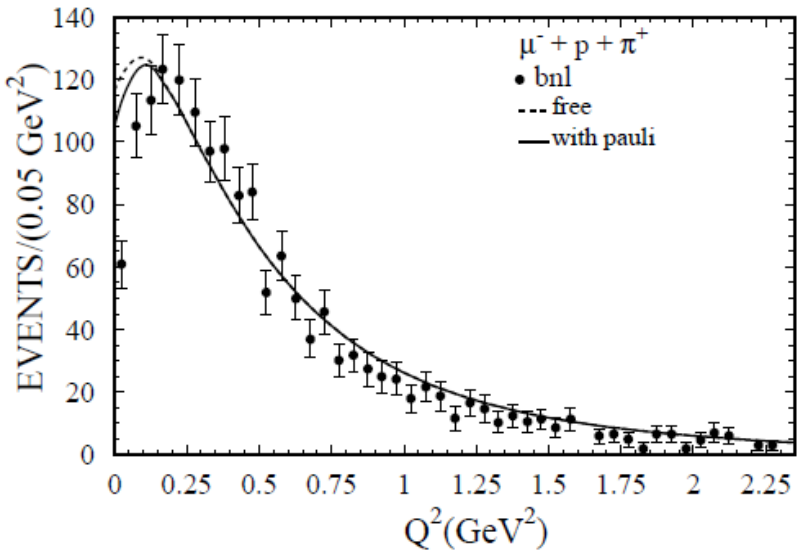


Figure 6: Q^2-spectrum of the process $\nu p \to \mu^- p \pi^+$.

Brookhaven exp.



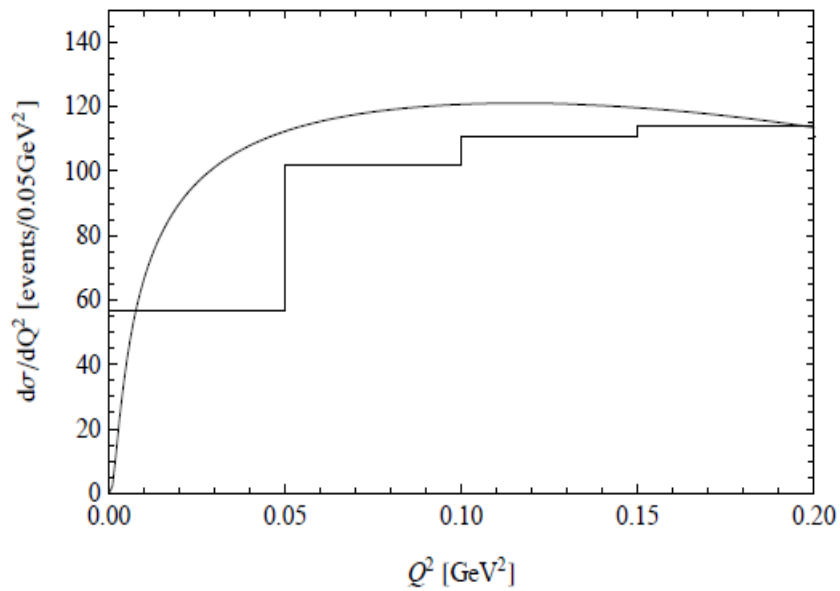
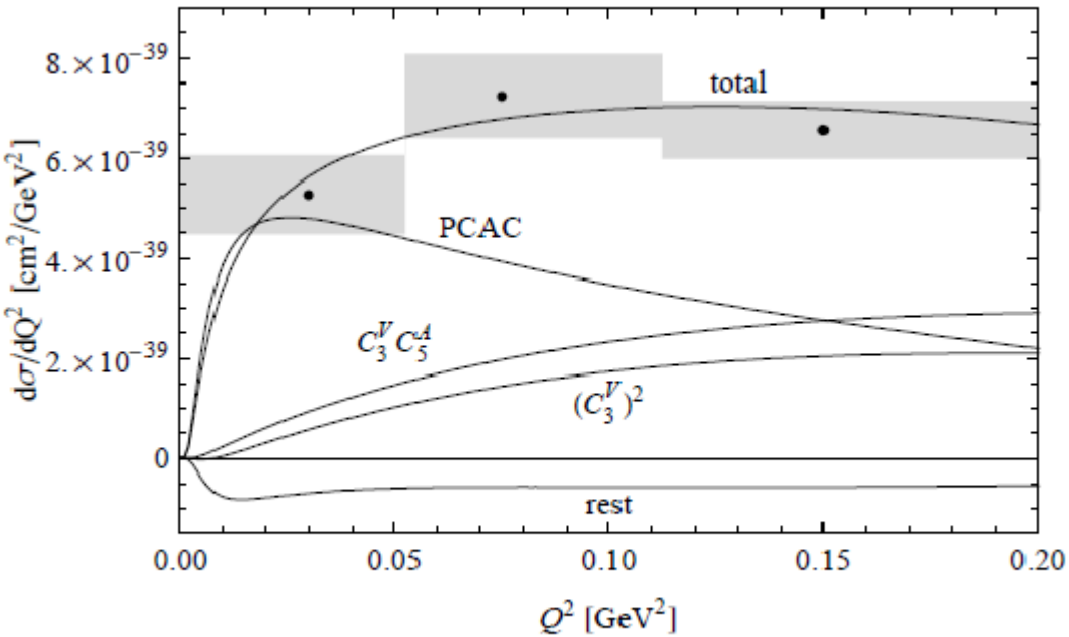
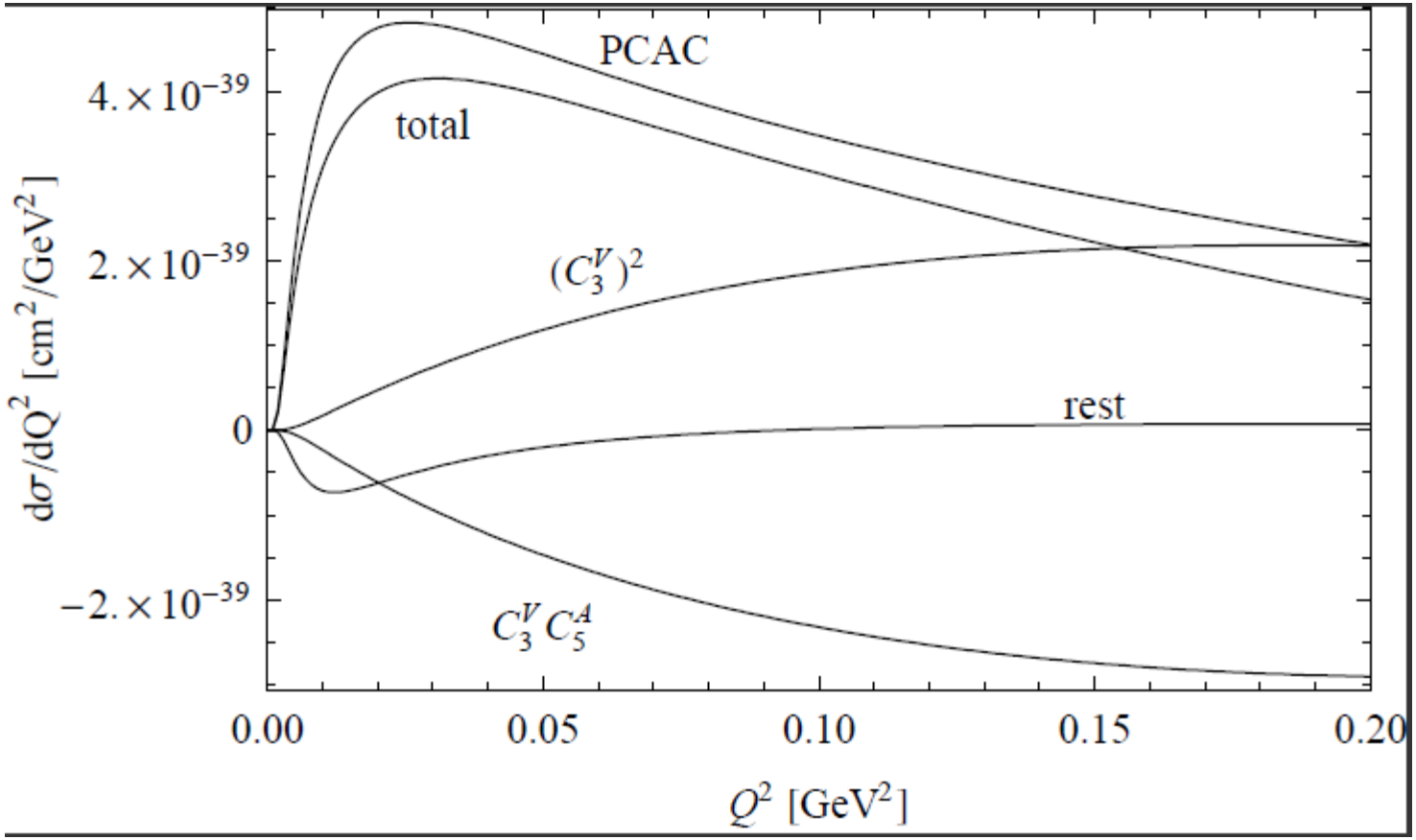


Figure 6: Q^2-spectrum of the process $\nu p \to \mu^- p \pi^+$.

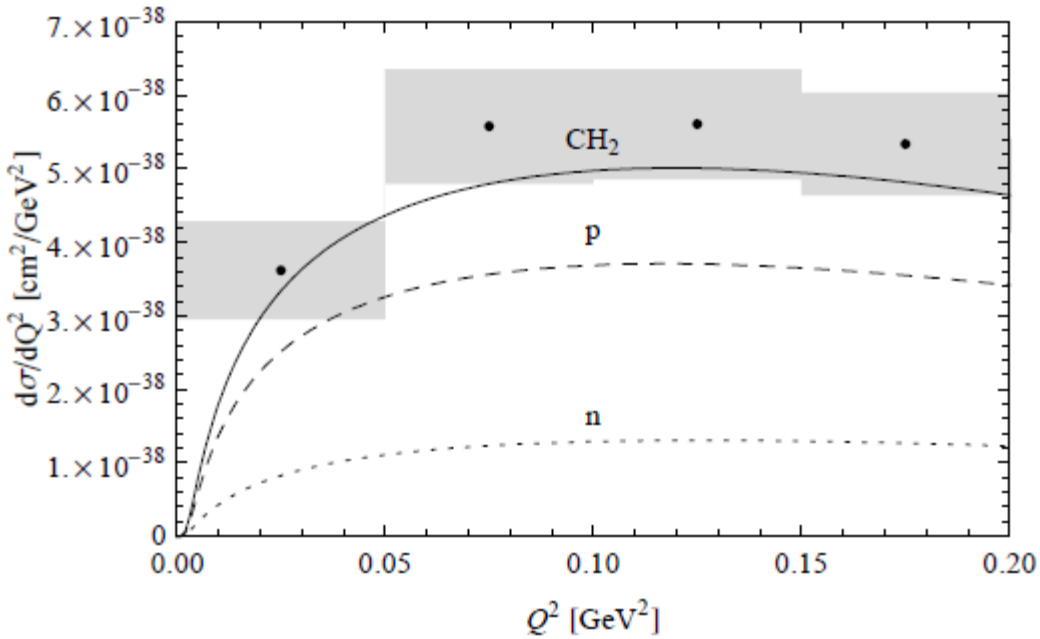
Argonne data



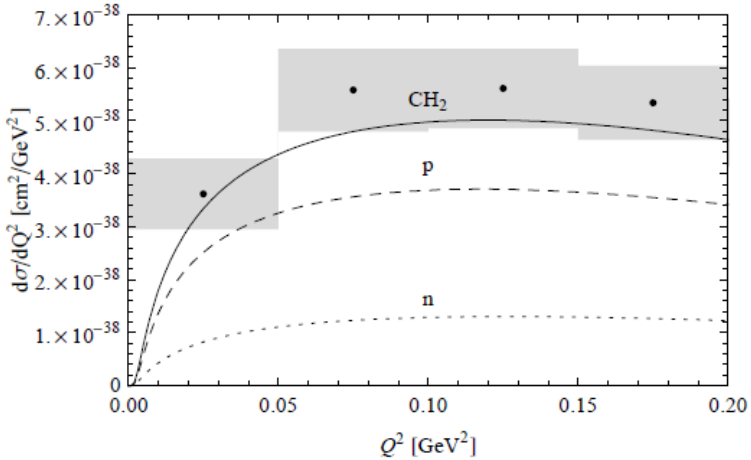
Argonne antineutrino

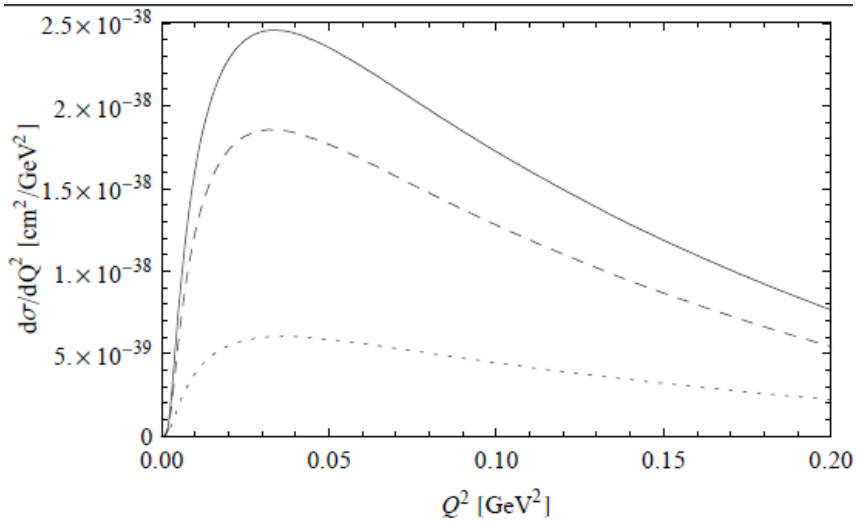


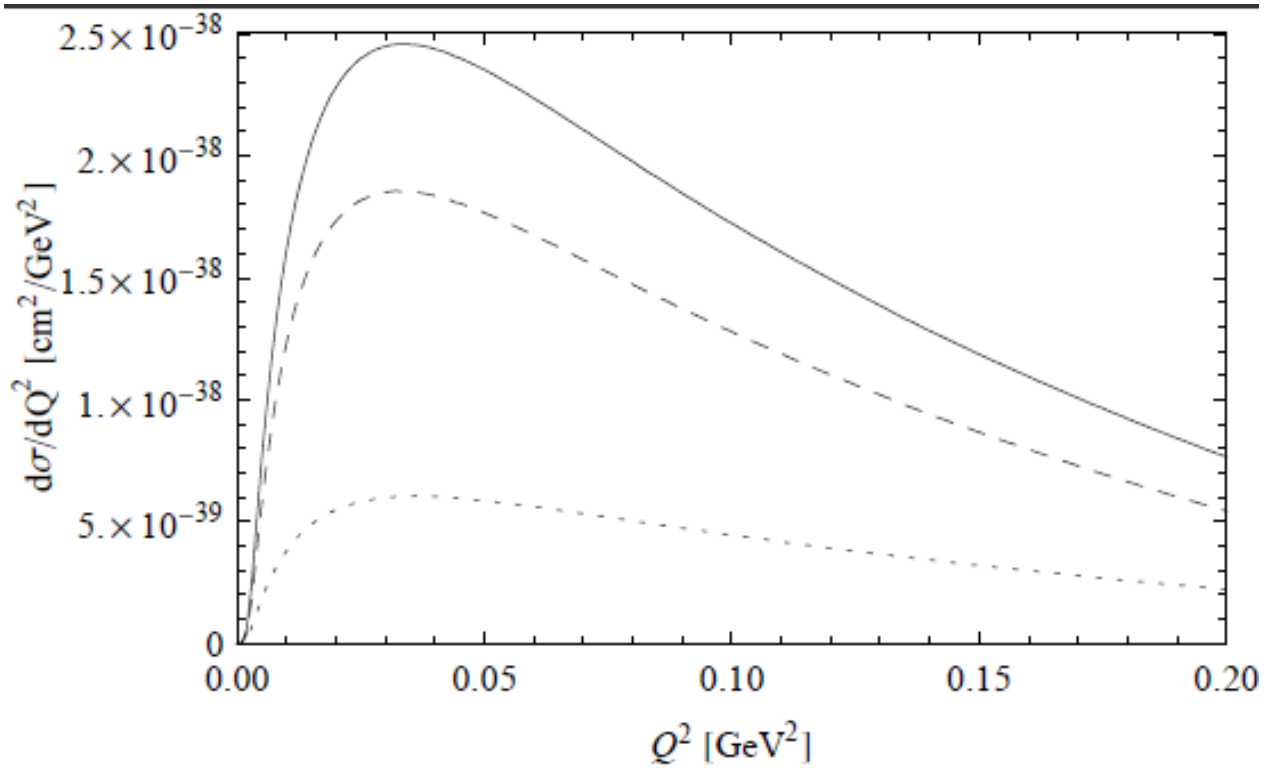
MiniBoone data



neutrino vs antineutrino





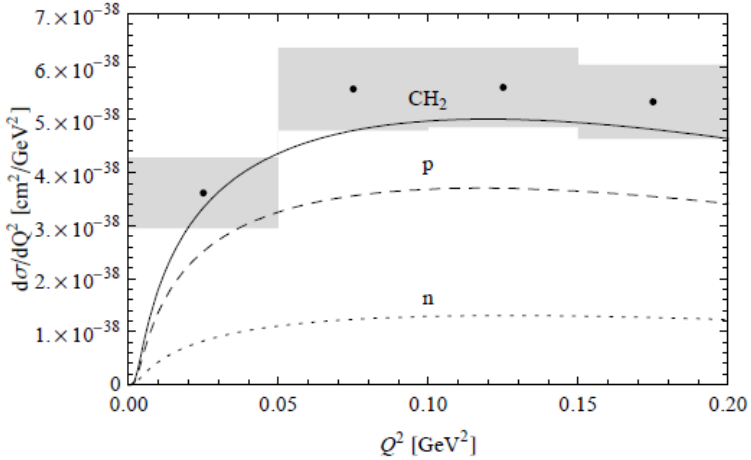

2.5×10^−38
2.×10^−38
1.5×10^−38
1.×10^−38
5.×10^−39
0
dσ/dQ^2 [cm^2/GeV^2]
0.00
0.05
0.10
0.15
0.20
Q^2 [GeV^2]

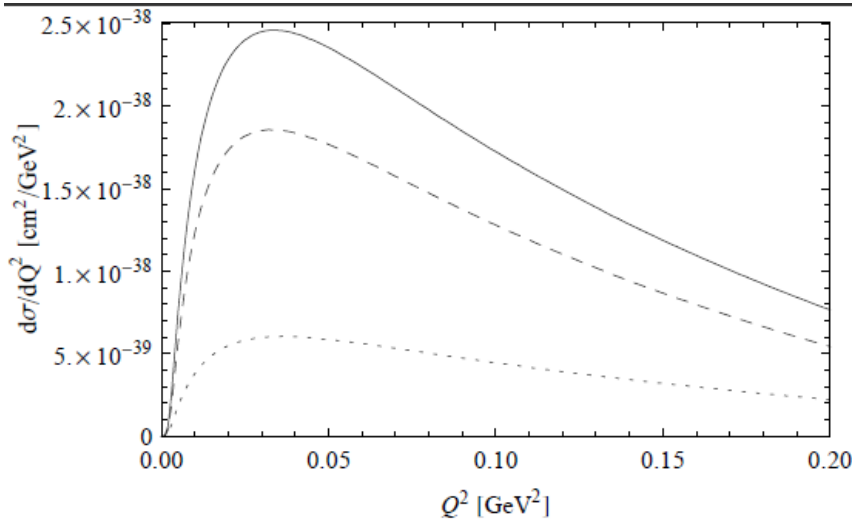
3. CP violation in oscillations

- The cross section depends on two functions

$d\sigma/dp^3 = F(c,s,\Delta m^2) + G(c,s,\Delta m^2)\sin\delta$

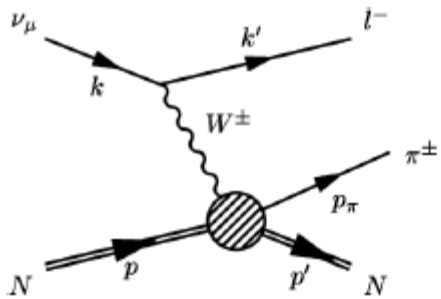
F and G ~ 10E-4 and comparable


7.×10^-38
6.×10^-38
5.×10^-38
4.×10^-38
3.×10^-38
2.×10^-38
1.×10^-38
0
dσ/dQ^2 [cm^2/GeV^2]
CH2
p
n
0.00
0.05
0.10
0.15
0.20
Q^2 [GeV^2]


2.5×10^-38
2.×10^-38
1.5×10^-38
1.×10^-38
5.×10^-39
0
dσ/dQ^2 [cm^2/GeV^2]
0.00
0.05
0.10
0.15
0.20
Q^2 [GeV^2]

4. Coherent Pion Production

- The process occurs for small Q^2 <0.15 Gev^2



Feynman diagram of the charged current reaction.

$$\left(\frac{Q^2 + m_\pi^2}{2\nu}\right)^2 \leq |t| \leq \infty$$

$$\frac{d\sigma_\pi}{dt} = a \exp[-b|t|]$$

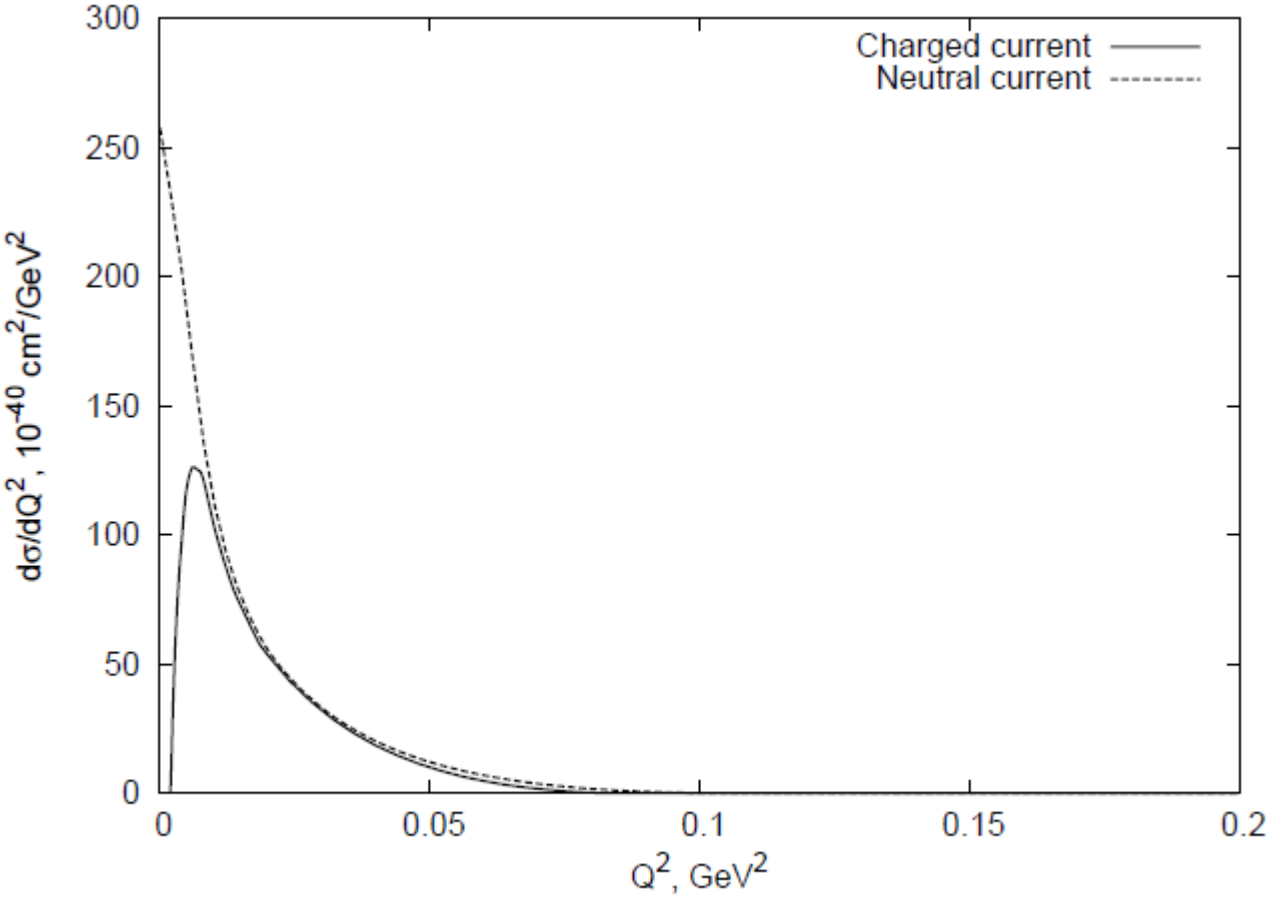
Same formula with nuclear cross section

$$\frac{d\sigma^A}{dQ^2 d\nu} = \frac{G_F^2 |V_{ud}|^2}{2\pi} \frac{1}{4\pi} \frac{\nu}{E_\nu^2} \frac{f_\pi^2}{Q^2} \left\{ \tilde{L}_{00} + 2\tilde{L}_{l0} \frac{m_\pi^2}{Q^2 + m_\pi^2} + \tilde{L}_{ll} \left(\frac{m_\pi^2}{Q^2 + m_\pi^2} \right)^2 \right\} \sigma(\pi^+ p \to X^{++}). \quad (12)$$

TABLE I. Parameters of the elastic pion nucleus cross sectior model.

ν (GeV)	a (barn/GeV^2)	b (GeV^{-2}
0.210	28.526	159.657
0.228	28.659	147.986
0.260	32.012	129.022
0.290	27.162	101.910
0.320	23.600	90.824
0.340	22.734	90.660
0.370	19.000	83.814
0.400	17.924	84.590
0.420	14.594	73.256
0.766	3.759	49.459
0.864	4.172	58.149
0.942	3.649	56.197

Coherent CC and NC reactions



Summary

- The region of small Q^2 is accurately determined (10% ??). Data accurate to a few %.
- Confirmation from the Adler sum rule, which also provides estimate for multipion production at larger Q^2.
- C5(A,Q^2) prefers larger value ~ 1.10.
- Comparisons look good but cna be improved.
- Antineutrino cross sections much smaller, may help searches for CP phase.
- Applications to many other rections possible.

References

1. Lalakulich and Paschos PR D 71 (2006)
2. Paschos and Schalla ,arXiv 11.024466
3. Gounaris et al PR D74,054007(2006)
4. Berger and Sehgal PR D79,053003(09)