Hadronic nature of neutrinos

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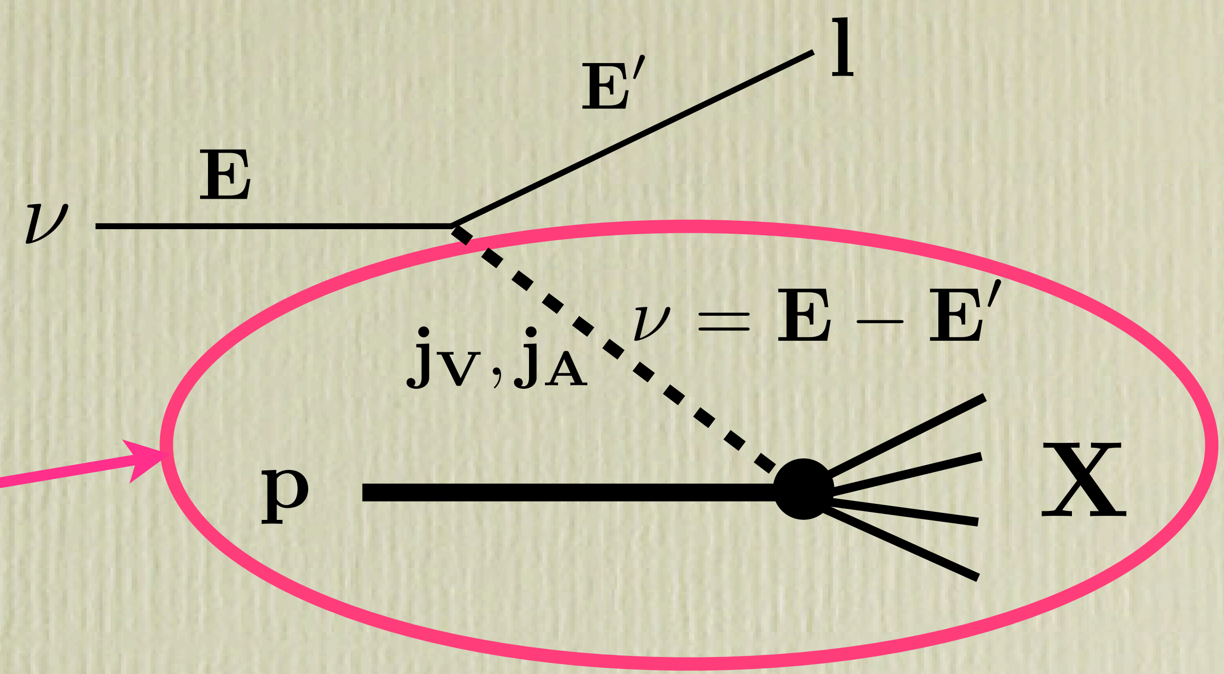
Outline

- Similarities and differences between axial and vector currents
- Goldberger-Treiman relation: pion dominance
- PCAC for high-energy neutrino scattering. PCAC - Adler relation
- Incurable Piketti-Stodolsky puzzle.
- Breakdown and restoration of the Adler relation

Strongly interacting neutrino

$$\nu + \mathbf{p} \rightarrow \mathbf{l} + \mathbf{X}$$

$$\mathbf{M} = \frac{\mathbf{G}}{\sqrt{\mathbf{2}}}\, \mathbf{l}_\mu (\mathbf{V}_\mu + \mathbf{A}_\mu)$$



$$\mathbf{l}_\mu = \bar{\mathbf{l}}(\mathbf{k'})\, \gamma_\mu (\mathbf{1} + \gamma_\mathbf{5})\, \nu(\mathbf{k})$$

$$\left|\overline{\mathbf{M}}\right|^\mathbf{2} = \frac{\mathbf{G^2}}{\mathbf{2}}\, \mathbf{L}_{\mu\nu}\; (\mathbf{V}_{\mu\nu} + \mathbf{A}_{\mu\nu} + \mathbf{W}_{\mu\nu})$$

$$\mathbf{L}_{\mu\nu} = \mathbf{k}_\mu \mathbf{k}'_\nu + \mathbf{k}_\nu \mathbf{k}'_\mu - (\mathbf{k}\mathbf{k'})\mathbf{g}_{\mu\nu} + \mathbf{i}\epsilon_{\mu\nu\sigma\rho}\mathbf{k}_\rho \mathbf{k}'_\sigma$$

Strongly interacting neutrino

$$\mathbf{L}_{\mu\nu}(\mathbf{Q}^2 = 0) = 2\,\frac{\mathbf{E}_\nu(\mathbf{E}_\nu - \nu)}{\nu^2}\,\mathbf{q}_\mu \mathbf{q}_\nu$$

$$\mathbf{q}_\mu = \mathbf{k}_\mu - \mathbf{k}'_\mu$$

$$\mathbf{Q}^2 \equiv -(\mathbf{q}_\mu)^2$$

The vector current contribution vanishes at Q=0 due to CVC

$$\mathbf{q}_\mu\,\mathbf{V}_\mu = 0$$

$$\frac{d\sigma_{\mathbf{V}}(\nu \mathbf{p} \to \mathbf{l}\mathbf{X})}{d\mathbf{Q}^2 d\nu} = \frac{\mathbf{G}^2}{4\pi^2}\,\frac{|\mathbf{q}|}{\mathbf{E}^2}\,\frac{\mathbf{Q}^2}{1-\epsilon}\,\left[\sigma_{\mathbf{V}}^{\mathbf{T}}(\mathbf{Q}^2) + \epsilon\,\sigma_{\mathbf{V}}^{\mathbf{L}}(\mathbf{Q}^2)\right]$$

$$\epsilon = \frac{4\mathbf{E}(\mathbf{E}-\nu) - \mathbf{Q}^2}{4\mathbf{E}(\mathbf{E}-\nu) + \mathbf{Q}^2 + 2\nu^2}$$

Besides the overall factor $\mathbf{Q}^2$, the longitudinal cross section vanishes at $\mathbf{Q}^2 \to 0$: $\sigma_{\mathbf{V}}^{\mathbf{L}}(\mathbf{Q}^2)/\sigma_{\mathbf{V}}^{\mathbf{T}}(\mathbf{Q}^2) \propto \mathbf{Q}^2$

The axial longitudinal cross section behaves very differently.

Nontrivial conservation of axial current

In the chiral limit of massless quarks both the vector and axial current are conserved: $\mathbf{q}_\mu \left[\bar{q}(k')\,\gamma_\mu\,q(k)\right] = 0;\ \mathbf{q}_\mu \left[\bar{q}(k')\,\gamma_\mu\gamma_5\,q(k)\right] = 0$

In the regime of chiral symmetry hadrons acquire large masses via the mechanism of spontaneous symmetry breaking. Therefore the hadronic currents should be still conserved. For the vector current this is obvious:

$$\mathbf{q}_\mu\,\mathbf{j}^{\mathbf{V}}_\mu = \mathbf{q}_\mu\,\bar{\mathbf{p}}(\mathbf{k}')\,\gamma_\mu\,\mathbf{n}(\mathbf{k}) = (\mathbf{m_n} - \mathbf{m_p})\bar{\mathbf{p}}\,\mathbf{n} = \mathbf{0}$$ **(up to QED corrections)**

However $$\mathbf{q}_\mu\,\mathbf{j}^{\mathbf{A}}_\mu = \mathbf{q}_\mu\,\bar{\mathbf{p}}(\mathbf{k}')\,\gamma_\mu\gamma_\mathbf{5}\,\mathbf{n}(\mathbf{k}) = (\mathbf{m_n} + \mathbf{m_p})\bar{\mathbf{p}}\,\gamma_\mathbf{5}\,\mathbf{n} \neq \mathbf{0}$$

Nevertheless, the axial current can be conserved. In the general form

$$\mathbf{j}^{\mathbf{A}}_\mu = \bar{\mathbf{p}}(\mathbf{k}')\left[\gamma_\mu\gamma_\mathbf{5} - \mathbf{q}_\mu\,\mathbf{g_p}\right]\mathbf{n}(\mathbf{k})$$

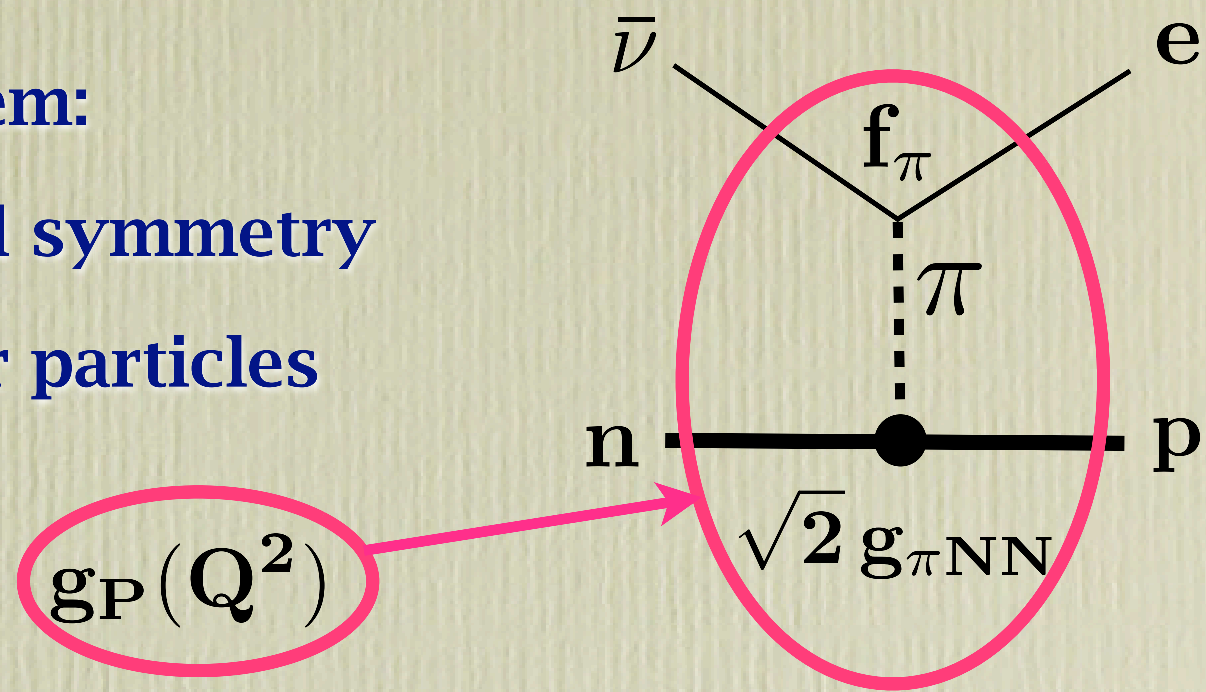
the current can be conserved if

$$\mathbf{g_P}(\mathbf{Q^2}) = \frac{\mathbf{g_A}(\mathbf{Q^2})}{\mathbf{2m_N}\,\mathbf{Q^2}}$$

This pole behavior shows presence of massless Golstone particles

Nontrivial conservation of axial current

This proves the Goldstone theorem: spontaneous breakdown of chiral symmetry generates massless pseudo-scalar particles identified with pions.



Assuming the current to be conserved at the pion pole and pion dominance in the dispersion relation one arrives at the Goldberger-Treiman relation

$$(\mathbf{m_p} + \mathbf{m_n})\mathbf{g_A}(\mathbf{0}) = \sqrt{\mathbf{2}}\,\mathbf{f}_\pi\,\mathbf{g}_{\pi\mathbf{NN}}$$

which well agrees with data on β-decay and muon capture.

PCAC: $$\partial_\mu\,\mathbf{j}^{\mathbf{A}}_\mu = \mathbf{m}^{\mathbf{2}}_\pi\,\phi_\pi$$

Adler relation

At $Q^2 \to 0$ the vector current contribution and the transverse part of the axial term vanish, only σ_L^A survives.

In this limit the lepton factor $L_{\mu\nu} \propto q_\mu q_\nu$, and according to PCAC

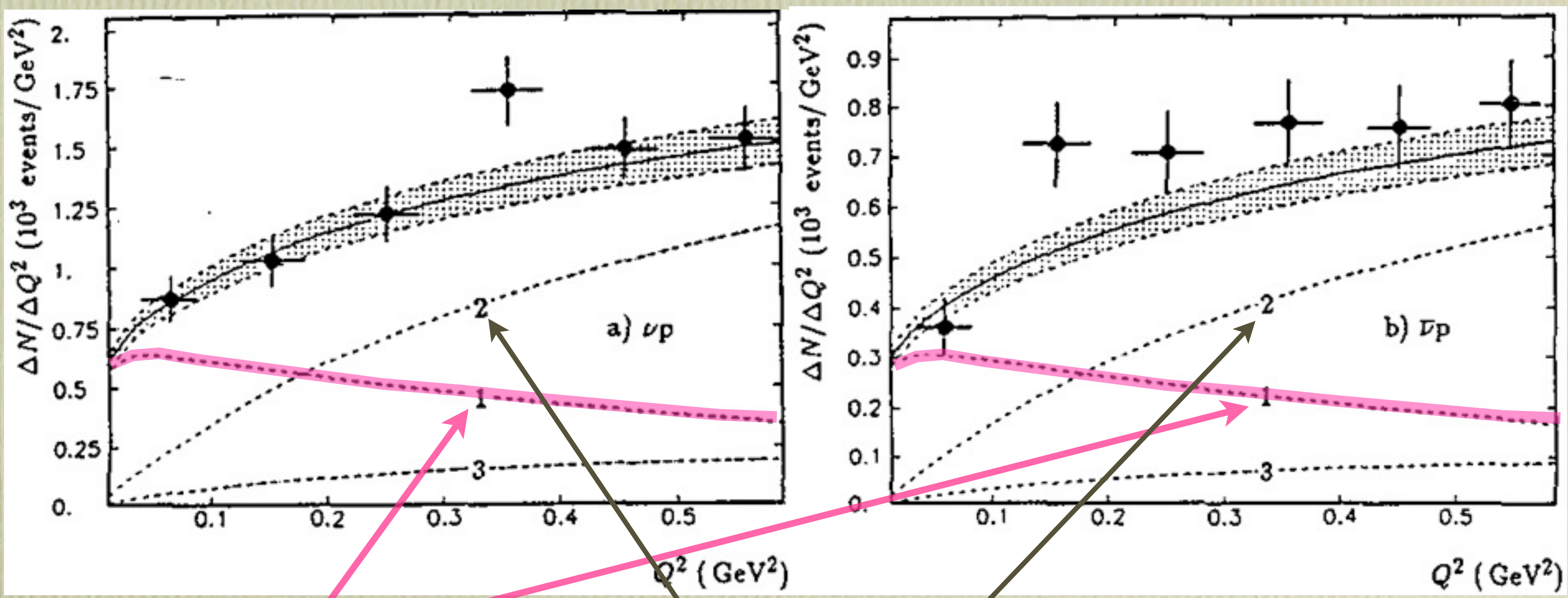
$$q_\mu \, A_{\mu\nu} \, q_\nu = \frac{1}{\nu} \, f_\pi \, \sigma(\pi p \to X)$$

what leads to the Adler relation (AR):

$$\left. \frac{d^2\sigma(\nu p \to l \, X)}{dQ^2 \, d\nu} \right|_{Q^2=0} = \frac{G^2}{2\pi^2} \, f_\pi^2 \, \frac{E-\nu}{E\nu} \, \sigma(\pi p \to X)$$

Adler relation

Comparison with WA21 data for (anti)neutrino-proton total cross section at $\nu > 2$ GeV



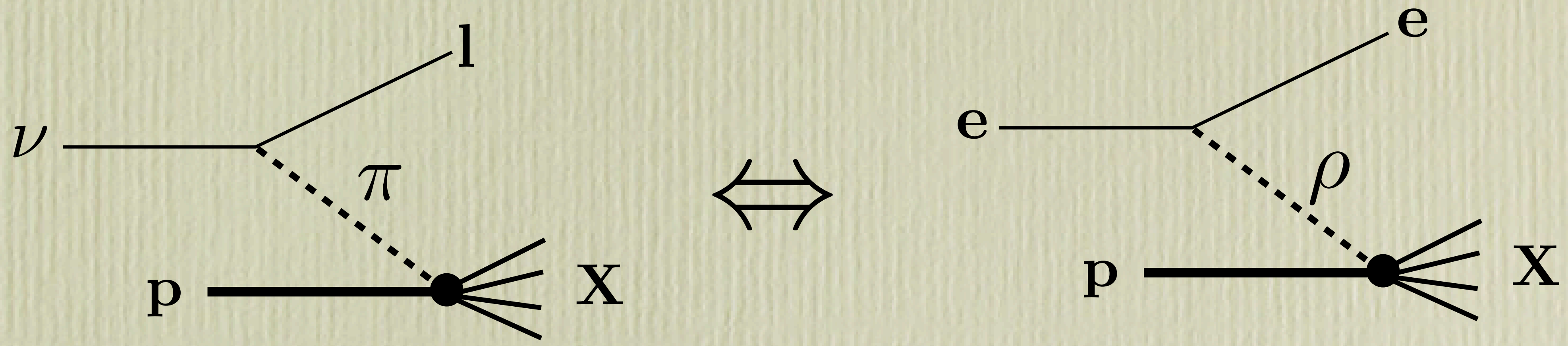
1. Adler relation for the longitudinal cross section extrapolated to $Q^2 \neq 0$

2. Vector + axial transverse cross sections

3. The upper bound for the axial-vector interference

Pion dominance?

It is tempting to interpret the AR as pion dominance, in analogy to the ϱ-dominance of the vector current.



However, neutrinos do not fluctuate to pions because of conservation of the lepton current $\mathbf{q}_\mu\,\mathbf{l}_\mu = \mathbf{0}$

$$\mathbf{A}_\mu = \mathbf{q}_\mu\,\frac{\mathbf{f}_\pi}{\mathbf{Q^2} + \mathbf{m}_\pi^2}\,\mathbf{T}(\pi\mathbf{p} \to \mathbf{X}) + \frac{\mathbf{f}_{\mathbf{a_1}}}{\mathbf{Q^2} + \mathbf{m}_{\mathbf{a_1}}^2}\,\mathbf{M}_\mu(\mathbf{a_1 p} \to \mathbf{X}) + \ldots$$

The pion pole does not contribute

The contribution of heavy states

Piketty-Stodolsky paradox

Miracle of PCAC

The Goldberger-Treiman relation for β-decay is based on the pion dominance. However, in high-energy neutrino interactions PCAC does not mean that the pion pole dominates, moreover it is forbidden. PCAC only relates the combined contribution of all heavy axial-vector states to the pion cross section.

Such a fine tuning look miraculous, and the PCAC hypothesis for neutrino interactions should be tested thoroughly

The Adler relation was challenged by Piketty & Stodolsky, who assumed that among heavy states the $\mathbf{a_1}$ pole dominates. Then PCAC leads to the relation $\sigma_{\mathbf{diff}}(\pi \mathbf{p} \to \mathbf{a_1 p}) \approx \sigma_{\mathbf{el}}(\pi \mathbf{p} \to \pi \mathbf{p})$ which contradicts data by factor ~20 (!)

Piketty-Stodolsky paradox

The problem is relaxed by inclusion of the ϱπ cut and other diffractive excitations. Indeed, the relation $\sigma_{\mathbf{diff}}(\pi\mathbf{p} \to \mathbf{Xp}) \approx \sigma_{\mathbf{el}}(\pi\mathbf{p} \to \pi\mathbf{p})$ does not contradict data.

However, such a similarity of diffractive and elastic cross section is accidental and holds only for a proton target. On heavy nuclei this relation is severely broken: diffraction is vanishing, while the elastic cross section saturates at the maximal value imposed by the unitarity bound.

$$\frac{\sigma_{\mathbf{diff}}(\pi\mathbf{A} \to \mathbf{XA})}{\sigma_{\mathbf{tot}}(\pi\mathbf{A} \to \pi\mathbf{A})} \propto \mathbf{A}^{-1/3} \qquad\qquad \frac{\sigma_{\mathbf{el}}(\pi\mathbf{A} \to \mathbf{XA})}{\sigma_{\mathbf{tot}}(\pi\mathbf{A} \to \pi\mathbf{A})} \to \frac{1}{2}$$

Thus, the Adler relation is incurable, the P&S puzzle kills it.

One clear, unresolvable contradiction would be enough to destroy the trustworthiness of the whole.

Formal proof

Hadrons can be expanded over the basis of eigen states of interaction.

$$|\mathbf{h}\rangle = \sum_\alpha \mathbf{C}^{\mathbf{h}}_\alpha |\alpha\rangle$$

$$\hat{\mathbf{f}}\,|\alpha\rangle = \mathbf{f}_\alpha\,|\alpha\rangle$$

Orthogonality:

$$\langle \mathbf{h}'|\mathbf{h}\rangle = \sum_\alpha \left(\mathbf{C}^{\mathbf{h}'}_\alpha\right)^* \mathbf{C}^{\mathbf{h}}_\alpha = \delta_{\mathbf{hh}'}$$

$$\mathbf{f}_{\mathbf{hh}'} \equiv \langle \mathbf{h}'|\hat{\mathbf{f}}|\mathbf{h}\rangle = \sum_\alpha \mathbf{f}_\alpha \left(\mathbf{C}^{\mathbf{h}'}_\alpha\right)^* \mathbf{C}^{\mathbf{h}}_\alpha$$

$$\mathbf{f}_{\mathbf{hh}} = \sum_\alpha \mathbf{f}_\alpha \left|\mathbf{C}^{\mathbf{h}}_\alpha\right|^2$$

In the black disc limit (heavy nucleus) all elastic eigen-amplitudes saturate unitarity $\mathbf{f}_\alpha = \mathbf{1}$, then the diffractive amplitude vanishes

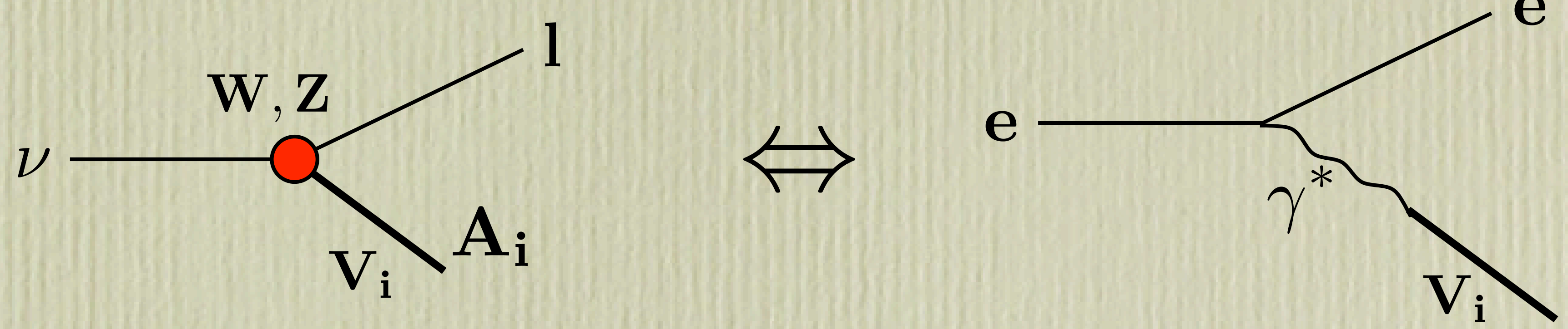
$$\mathbf{f}_{\mathbf{hh}'} = \sum_\alpha \left(\mathbf{C}^{\mathbf{h}'}_\alpha\right)^* \mathbf{C}^{\mathbf{h}}_\alpha = \mathbf{0}$$

but

$$\mathbf{f}_{\mathbf{hh}} = \sum_\alpha \left|\mathbf{C}^{\mathbf{h}}_\alpha\right|^2 = \mathbf{1}$$

Strongly interacting neutrino

Neutrino interacts strongly via hadronic fluctuations, like photons

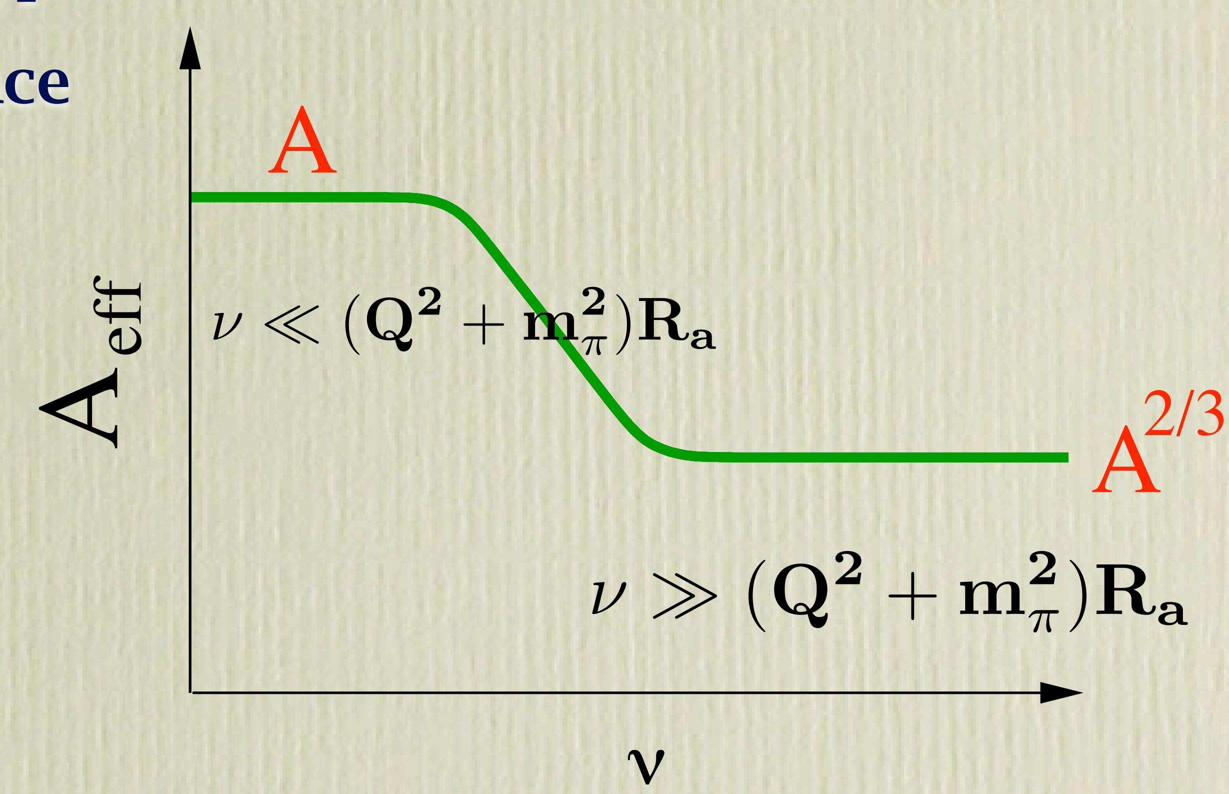


The lifetime of a hadronic fluctuation h of mass $\mathrm{m_h}$ is $\mathrm{t^h_{fluct}} = \dfrac{2\,\nu}{\mathrm{m_h^2 + Q^2}}$

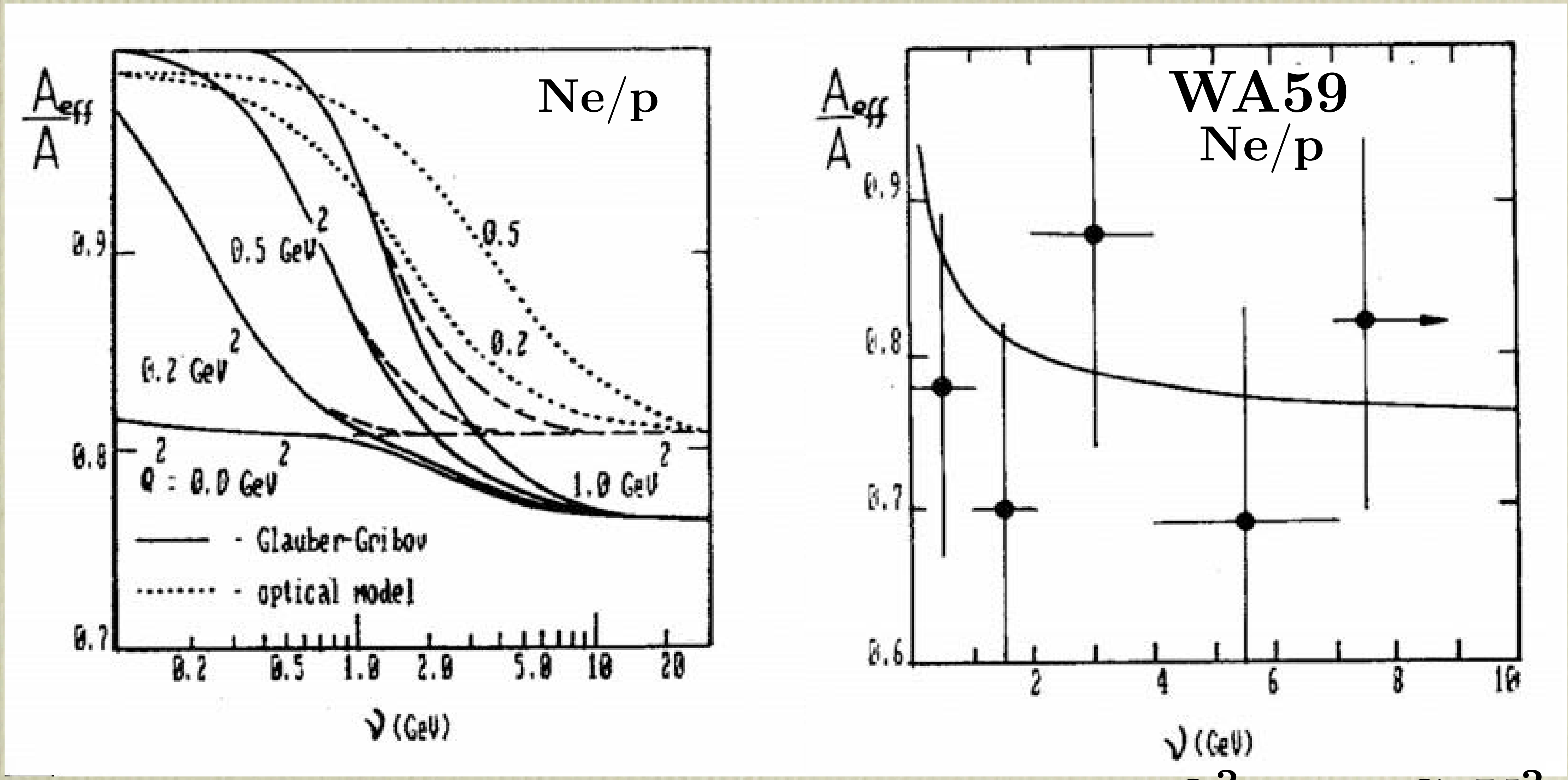
This time scale for heavy fluctuations V or A at small Q is much shorter than the coherence time for pion production

$$\mathrm{t^{A,V}_{fluct}} \ll \mathrm{t^{\pi}_{c}} = \frac{2\,\nu}{\mathrm{m_\pi^2 + Q^2}}$$

Therefore, the onset of nuclear shadowing in $\sigma^{\nu\mathrm{A}}_{\mathrm{tot}}$ is controlled by $\mathrm{t^{\pi}_{c}}$



Onset of nuclear shadowing



$x < 0.2; \quad Q^2 < 0.2\ \mathrm{GeV}^2$

Nuclear shadowing for axial current onsets at small Q

at much lower energies than for vector current,

Diffractive neutrino-production of pions

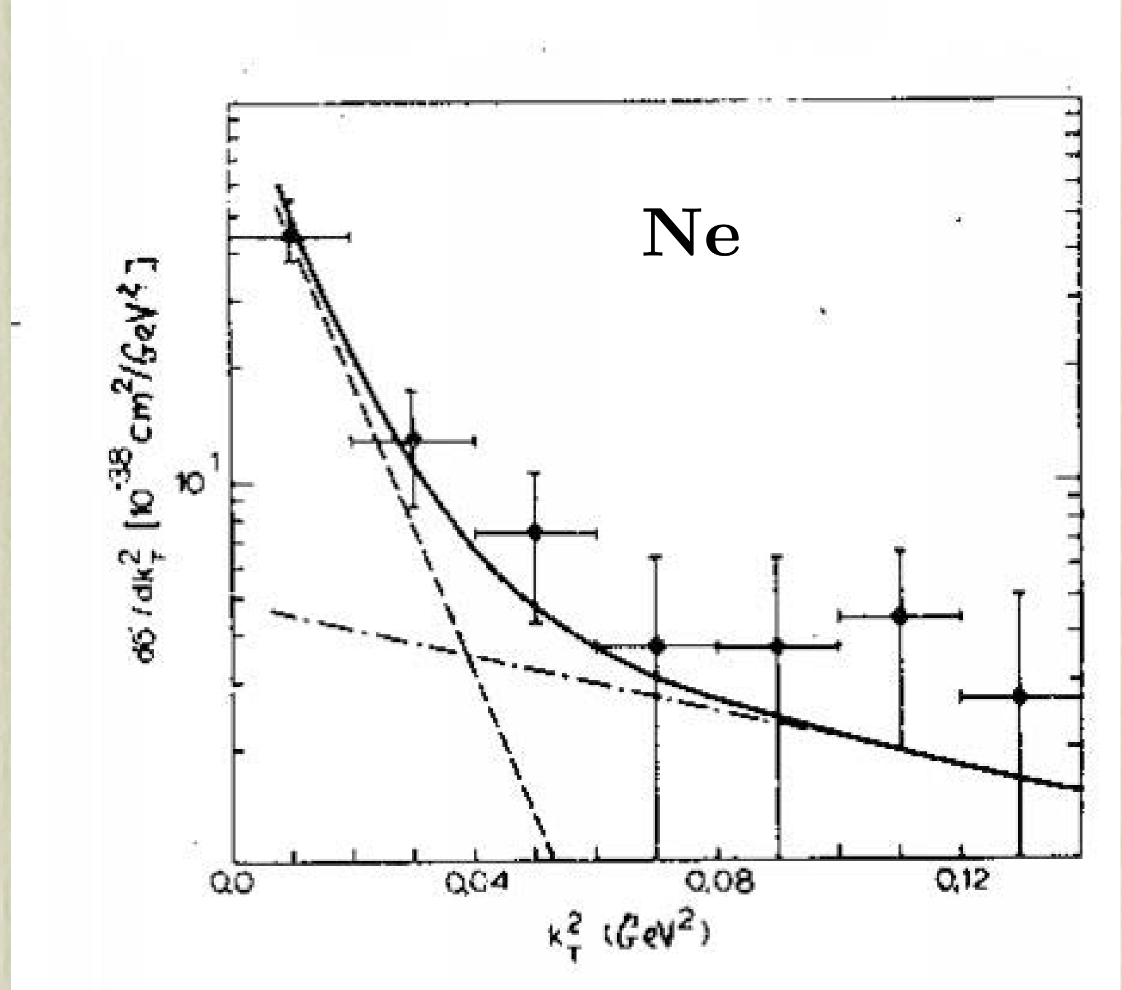
Diffractive pion production on a nucleus may be coherent

$\nu + \mathbf{A} \rightarrow \mathbf{l} + \pi + \mathbf{A}$ (the nucleus remains intact)

or incoherent

$\nu + \mathbf{A} \rightarrow \mathbf{l} + \pi + \mathbf{A}^*$ (the nucleus decays to fragments without particle production)

The two processes have very different p_T distributions, which help to separate them (statistically)



They also have different energy and Q dependences. Much can be learned from our experience with nuclear effects for vector current.

Characteristic time scales

$$\mathbf{t_c^{\pi} = \frac{2\,\nu}{m_\pi^2 + Q^2}} \gg \mathbf{t_c^{A} = \frac{2\nu}{Q^2 + m_A^2}}$$

Correspondingly, there are 3 energy regimes

III

$$\nu > (\mathbf{Q^2 + m_A^2)R_A}$$

$$\nu > \mathbf{40GeV}$$

maximal shadowing

II

$$(\mathbf{Q^2 + m_\pi^2)R_A} < \nu < (\mathbf{Q^2 + m_A^2)R_A}$$

$$\mathbf{0.5} < \nu < \mathbf{40GeV}$$

some shadowing

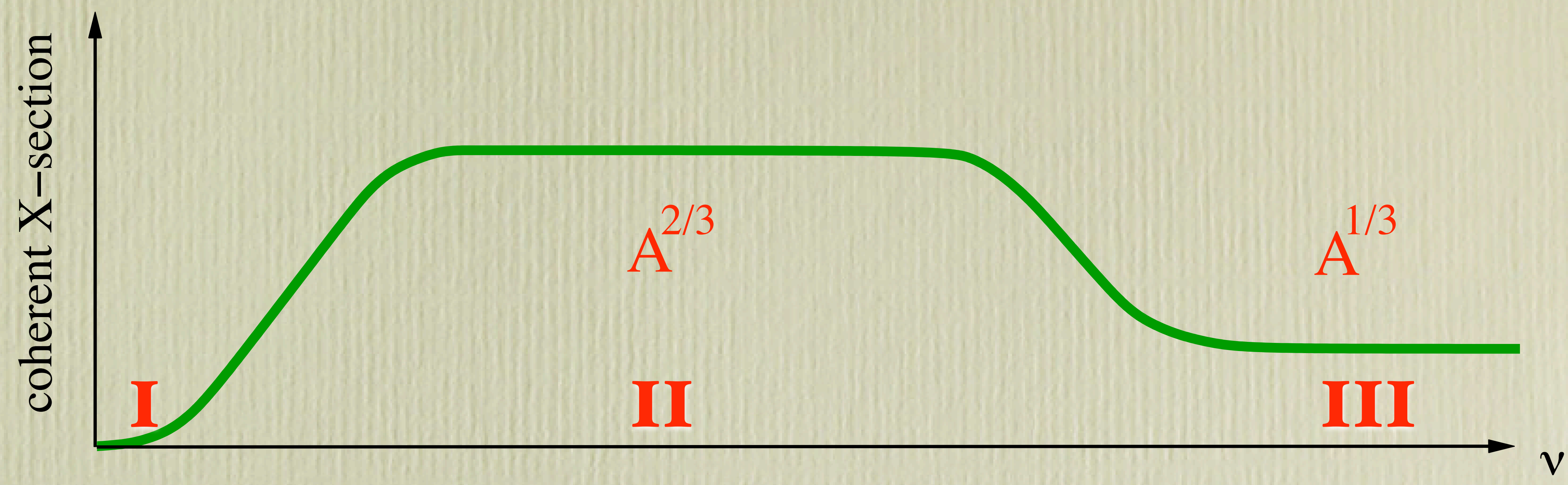
I

$$\nu < (\mathbf{Q^2 + m_\pi^2)R_A}$$

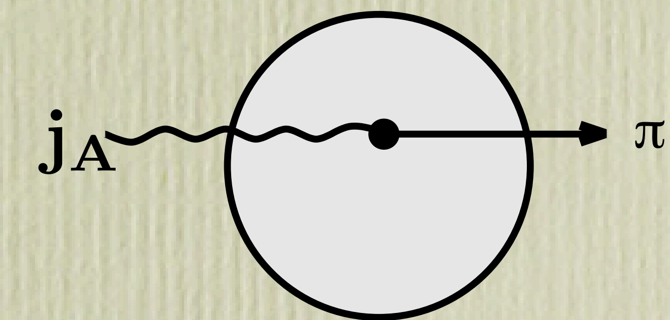
$$\nu < \mathbf{500MeV}$$

no shadowing

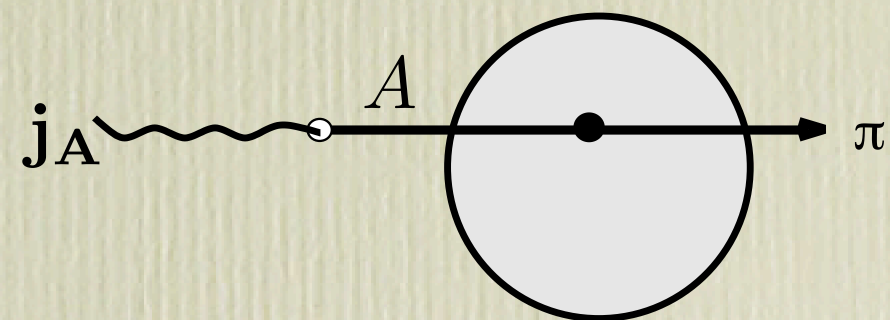
Coherent production of pions



In the regime II
the AR is as good,
as for a proton target.



$$\sigma_{\mathbf{coh}}^{\nu\mathbf{A}\to\pi\mathbf{A}} = \frac{\sigma^{\nu\mathbf{N}\to\pi\mathbf{N}}}{\sigma_{\mathbf{el}}^{\pi\mathbf{N}}}\,\sigma_{\mathbf{el}}^{\pi\mathbf{A}}$$

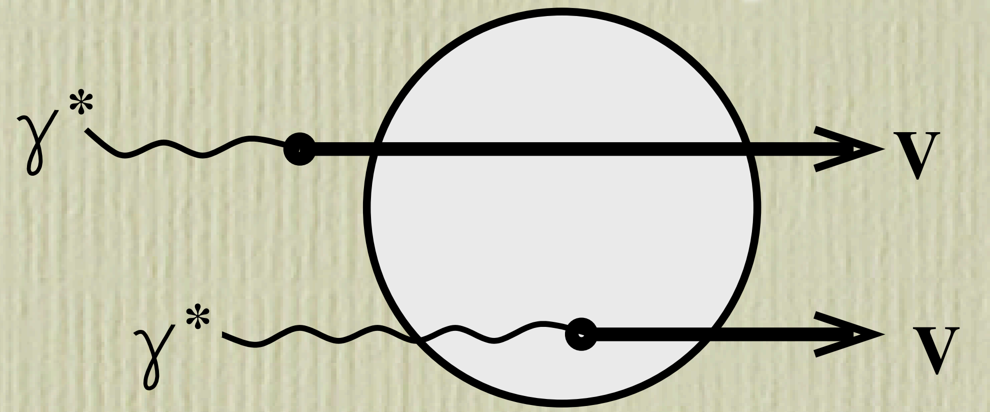


In the regime III
the AR is badly broken

$$\sigma_{\mathbf{coh}}^{\nu\mathbf{A}\to\pi\mathbf{A}} = \frac{16\pi\mathbf{B}_{\mathbf{el}}^{\pi\mathbf{N}}\,\sigma^{\nu\mathbf{N}\to\pi\mathbf{N}}}{\left(\sigma_{\mathbf{tot}}^{\mathbf{AN}} - \sigma_{\mathbf{tot}}^{\pi\mathbf{N}}\right)^{2}} \int \mathbf{d^2b}\left|\mathbf{e}^{-\frac{1}{2}\sigma_{\mathbf{tot}}^{\pi\mathbf{N}}\mathbf{T_A(b)}} - \mathbf{e}^{-\frac{1}{2}\sigma_{\mathbf{tot}}^{\mathbf{AN}}\mathbf{T_A(b)}}\right|^{2}$$

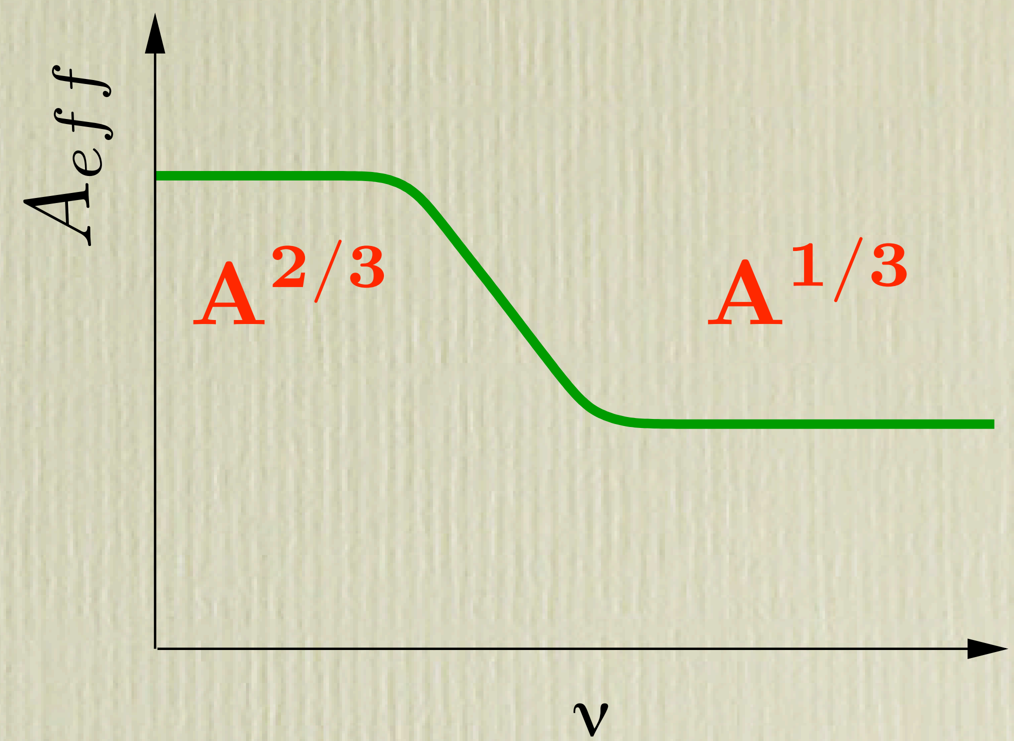
Lessons from diffractive photoproduction

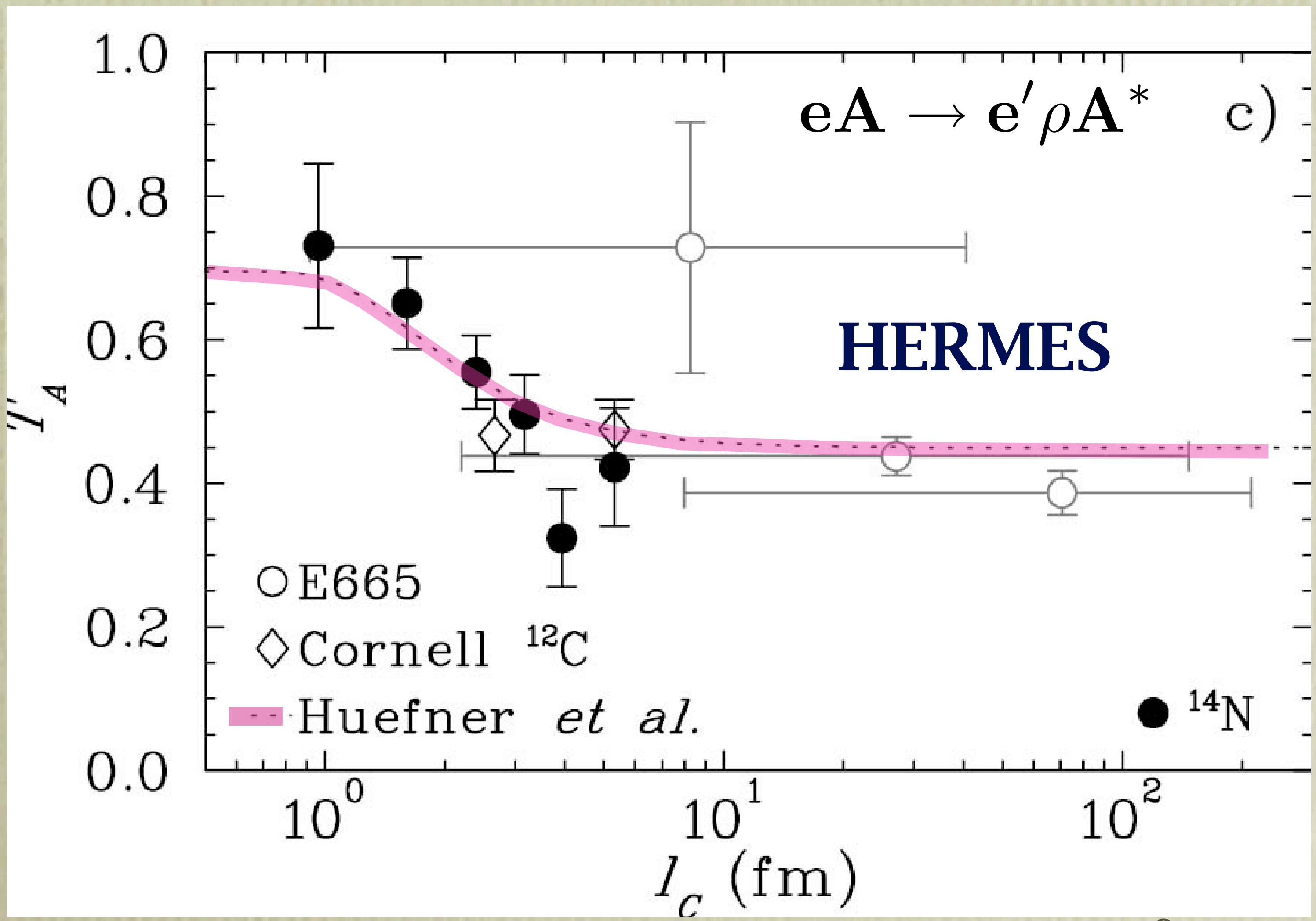
In this case the important time scale is $\mathbf{t_c^\rho} = \dfrac{\mathbf{2\,\nu}}{\mathbf{m_\rho^2 + Q^2}}$



$t_c >> R_A$ $A^{1/3}$

$t_c << R_A$ $A^{2/3}$





Current evaluations

- **D. Rein & L. Sehgal, 1983**

Based on the AR. The pion-nucleon elastic cross section is incorrect.

$\sigma_{\mathrm{el}}^{\mathrm{hA}} \propto \mathrm{A}^{2/3}$ **(any textbook on quantum mechanics)**

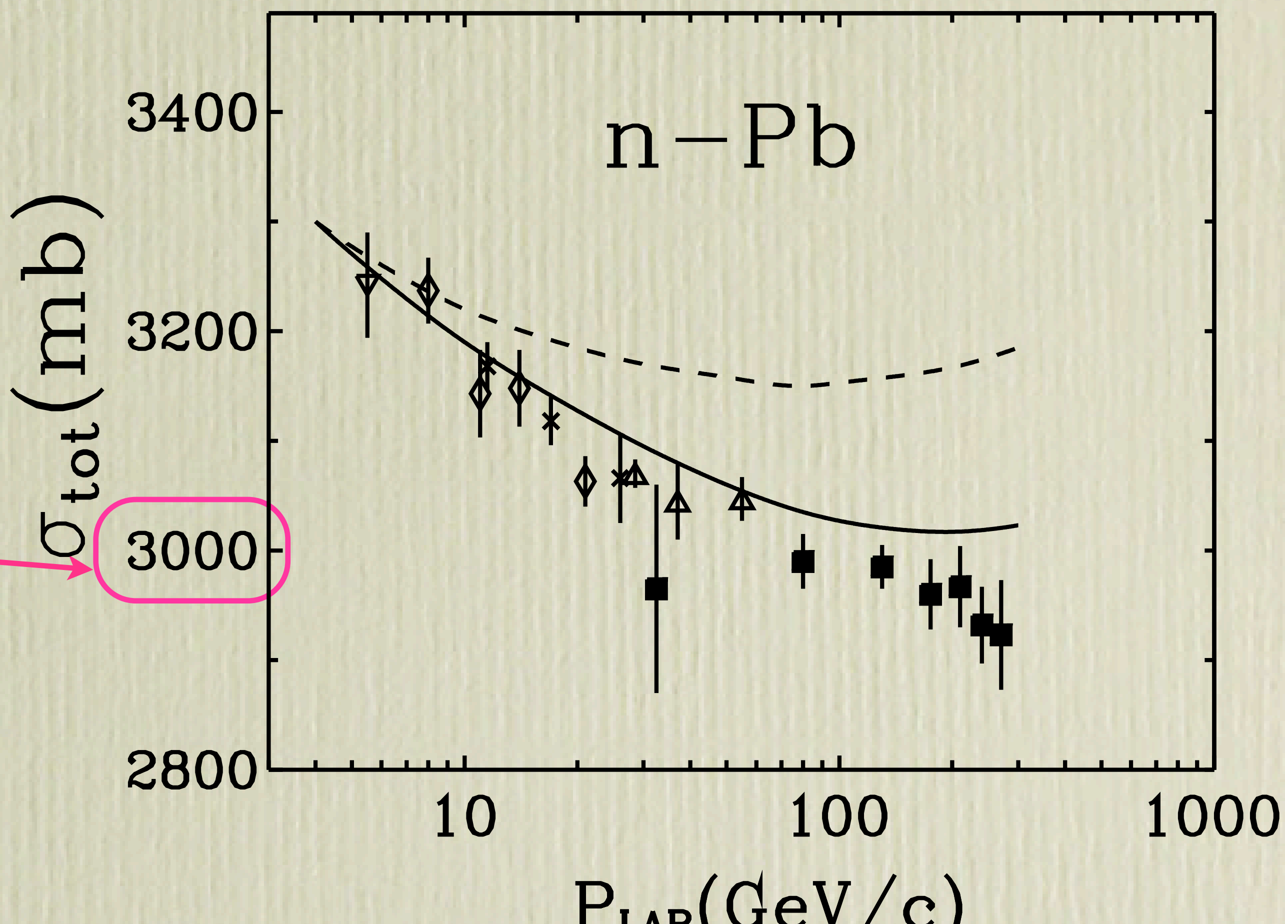
R&S: $\sigma_{\mathrm{el}}^{\mathrm{hA}} \rightarrow 0$

Numerical test

The R&S shadowing/absorption

factor $\mathrm{F_{abs}} = \exp\left(-\frac{9\mathrm{A}}{16\pi}\frac{\sigma_{\mathrm{in}}^{\mathrm{hN}}}{\mathrm{R_A^2}}\right)$

R&S: $\boxed{\sigma_{\mathrm{tot}}^{\mathrm{nPb}} = 580\mathrm{mb}}$



The elastic cross section is factor 25 below the data.

Summary

- The (partial) nontrivial conservation of axial current leads to the Goldberger-Treiman relation for β-decay and muon capture. The pseudo-Goldstone pion pole dominates.

- The application of PCAC to high-energy diffractive neutrino interactions is quite peculiar: pion pole is forbidden by the conserved lepton current.

- Absence of the pion pole leads to the Piketty-Stodolsky puzzle, which cannot be cured for nuclear targets.

- The Adler relation is severely broken for pion production on nuclear targets at high energies. Nevertheless it may be restored at medium-high energies.