Discussion Session On Coherent Pion Production

1. The Rein-Sehgal Model
   - Origins
   - Success at High Energy
   - Problems at Low Energy
2. Refinement a la Berger-Sehgal
   - Muon Mass Effects
   - π^+/π^0 Ratio
   - σ_{coh} using Pion-Carbon data
3. Dynamical Models
   - Beyond PCAC
   - Microscopic Calculations (things I don't know)

1. The Rein-Seghal Model

- Origin : First Evidence for Coherent π^0 (Aachen-Padua)

- Interpretation in terms of Adler's Theorem

$$\sigma(\nu + \mathcal{N} \to \nu + \mathcal{N} + \pi^0) \propto \beta^2 \sigma(\pi^0 \mathcal{N} \to \pi^0 \mathcal{N})$$

$Q^2 \to 0$, or $\theta \to 0$

β = isovector axial vector coupling of weak neutral current

$\beta = 1$ in SM

- Extension to non-forward scattering

$$\frac{d\sigma}{dx\,dy\,d|t|} = \frac{G^2 M E}{\pi^2} \frac{1}{2} f_\pi^2 (1-y) \frac{d\sigma}{dt}(\pi^0 \mathcal{N} \to \pi^0 \mathcal{N}) \Big|_{(Ey = E_\pi)} \cdot \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}, \quad m_A \sim 1\,\text{GeV}$$

- A simple (simplistic ?) model for $\frac{d\sigma}{dt}(\pi^0 \mathcal{N} \to \pi^0 \mathcal{N})$

Assume :

$$\frac{d\sigma(\pi^0 \mathcal{N} \to \pi^0 \mathcal{N})}{dt} = A^2 \left| F_{\mathcal{N}}(t) \right|^2 \frac{d\sigma(\pi^0 N \to \pi^0 N)}{d|t|}\Bigg|_{t=0}$$

$$F_{\mathcal{N}}(t) = e^{-bt} \cdot F_{abs}$$

$$\frac{d\sigma(\pi^0 N \to \pi^0 N)}{dt}\Bigg|_{t=0} = \frac{1}{16\pi} \left(\frac{\sigma_{tot}^{\pi^+ p} + \sigma_{tot}^{\pi^- p}}{2} \right)^2 \cdot (1 + r^2), \quad r = \frac{\mathrm{Re}\, f_{\pi N}(0)}{\mathrm{Im}\, f_{\pi N}(0)}$$

(small)

$$b = \frac{1}{3} R^2, \quad R = R_0 A^{1/3}, \quad R_0 \sim 1.06 \text{ fm}$$

$$F_{abs} = \exp\left[-\langle x \rangle / \lambda \right]$$

$\langle x \rangle$ = average path length traversed by a π^0 produced in the nucleus

$= 3/4\, R$

$$\lambda = 1 / \sigma_{inel} \cdot \rho \qquad \rho = A \left(\frac{4}{3} \pi R^3 \right)^{-1}$$

Main (Only?) Virtue of Model:

No parameters!

Specify A, model gives $\sigma_{coh}(E_\nu)$

In particular:

$$\sigma_{coh}^{\nu} = \sigma_{coh}^{\bar{\nu}}$$

$$\sigma_{coh}^{\pi^+} = \sigma_{coh}^{\pi^0}$$

(isoscalar target)

Model can be easily disproved.

- Experimental Status

Compilation of σ_{coh} by Layda (1992)

Recent Measurement: NOMAD (2009)

Conclude: Model successful at high energies ($E_\nu \gtrsim 2$ GeV)

L. M. Sehgal, Neutrino-81, Hawaii
(on behalf of Aachen-Padua)

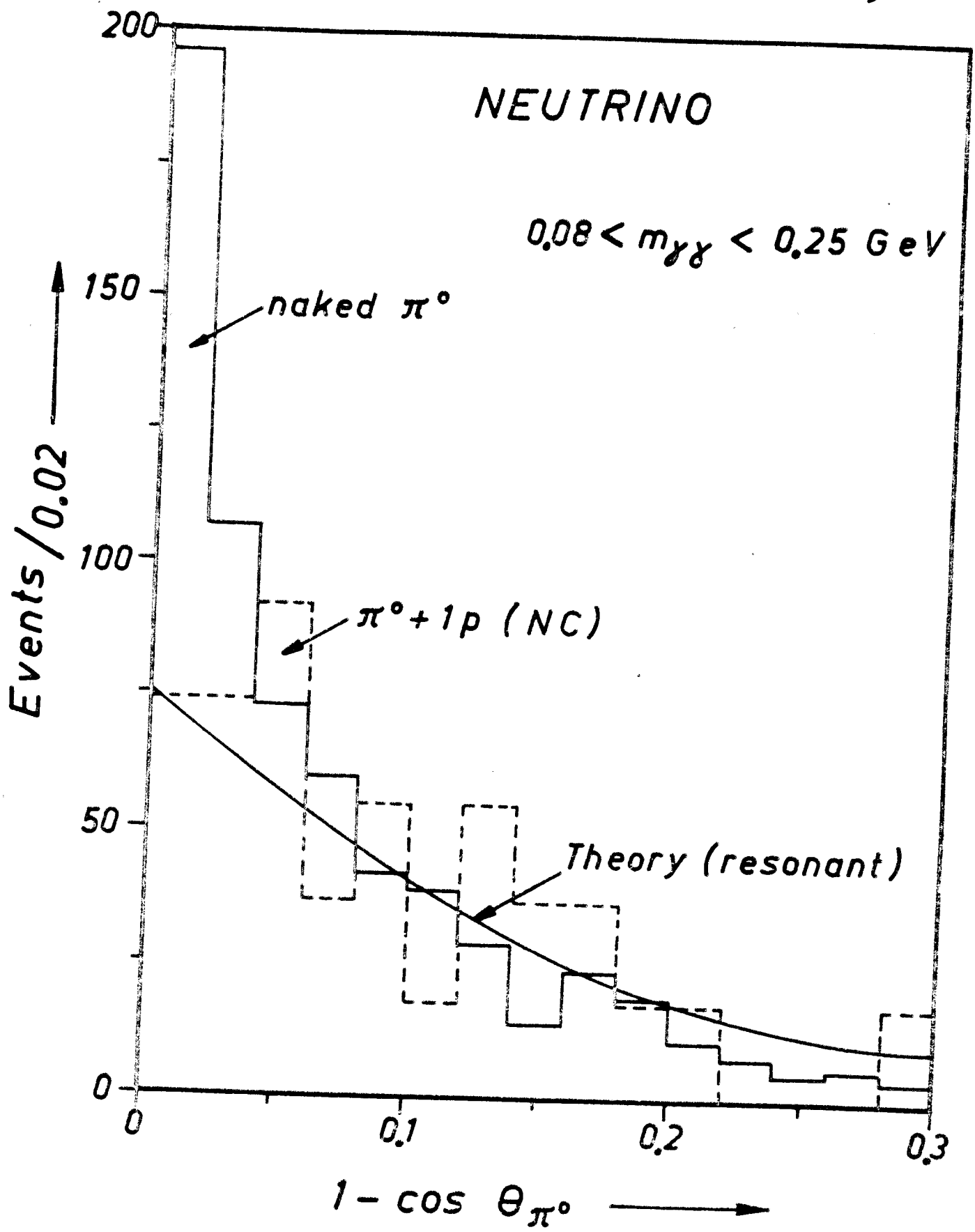


Fig. 7

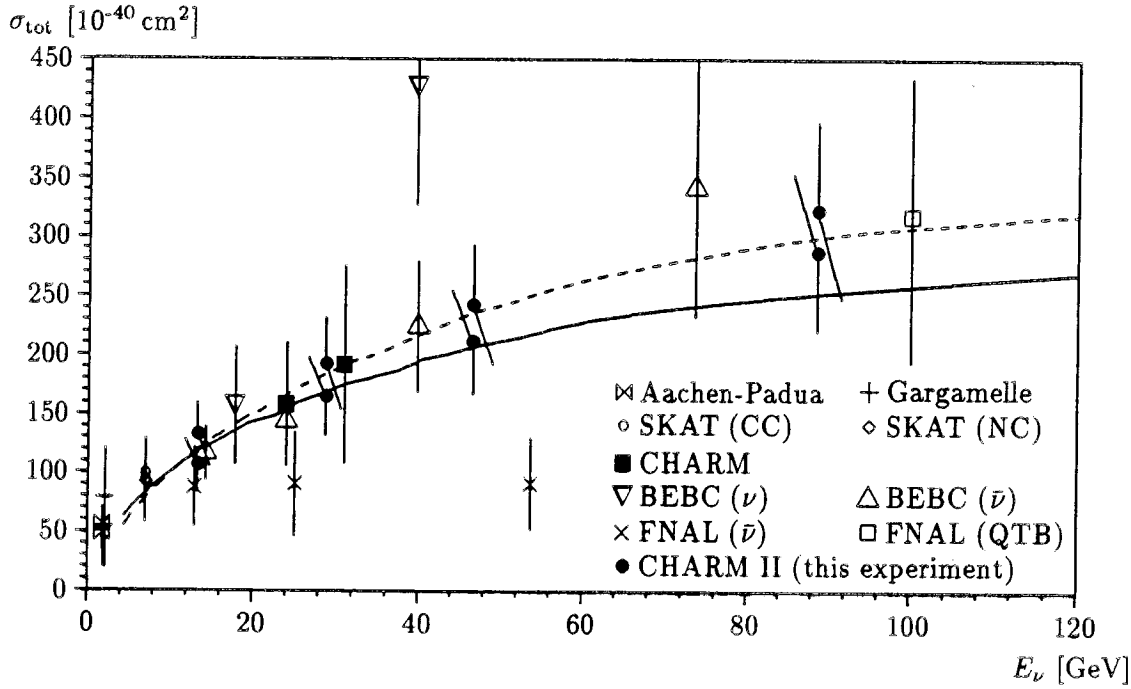


Figure 4.6: *Compilation of experiments on coherent single pion production (cf. table 4.9). For this experiment the results from a scaling according to the Rein-Sehgal model (upper points) and the Bel'kov-Kopeliovich approach (lower points) are shown, averaged over both beam types. The predictions of the Rein-Sehgal model for $m_A = 1.3\,GeV/c^2$ (full line) and the Bel'kov-Kopeliovich approach (dashed line) are indicated. Data have been duely scaled to allow comparison.*

scaling according to the models are plotted. Given the good agreement between neutrino and antineutrino data, the average of both beam types has been formed. Apart from this experiment only the WA 59 Collaboration and the FNAL group [Amm87] contribute more than one point to figure 4.6. The CHARM Collaboration extrapolated from their pion energy interval ($6\,\text{GeV} \leq E_\pi \leq 20\,\text{GeV}$) to the full range with the help of the Rein-Sehgal model.

All results are fully compatible. The values of [Amm87], obtained in an analysis in the framework of the Bel'kov-Kopeliovich scheme, seem a bit low. Overall, a consistent picture emerges. The results confirm the PCAC hypothesis, also at high energies. Both phenomenological approaches yield predictions compatible with the data. A value of m_A close to the a_1-mass of $m_{a_1} = 1.26\,\text{GeV}/c^2$ is favoured. The result $\sigma \propto E_\nu$ by Lackner [Lac79b] is ruled out.

T. Layda (CHARM II)
Ph.D. Dissertation, Hamburg
(1992)

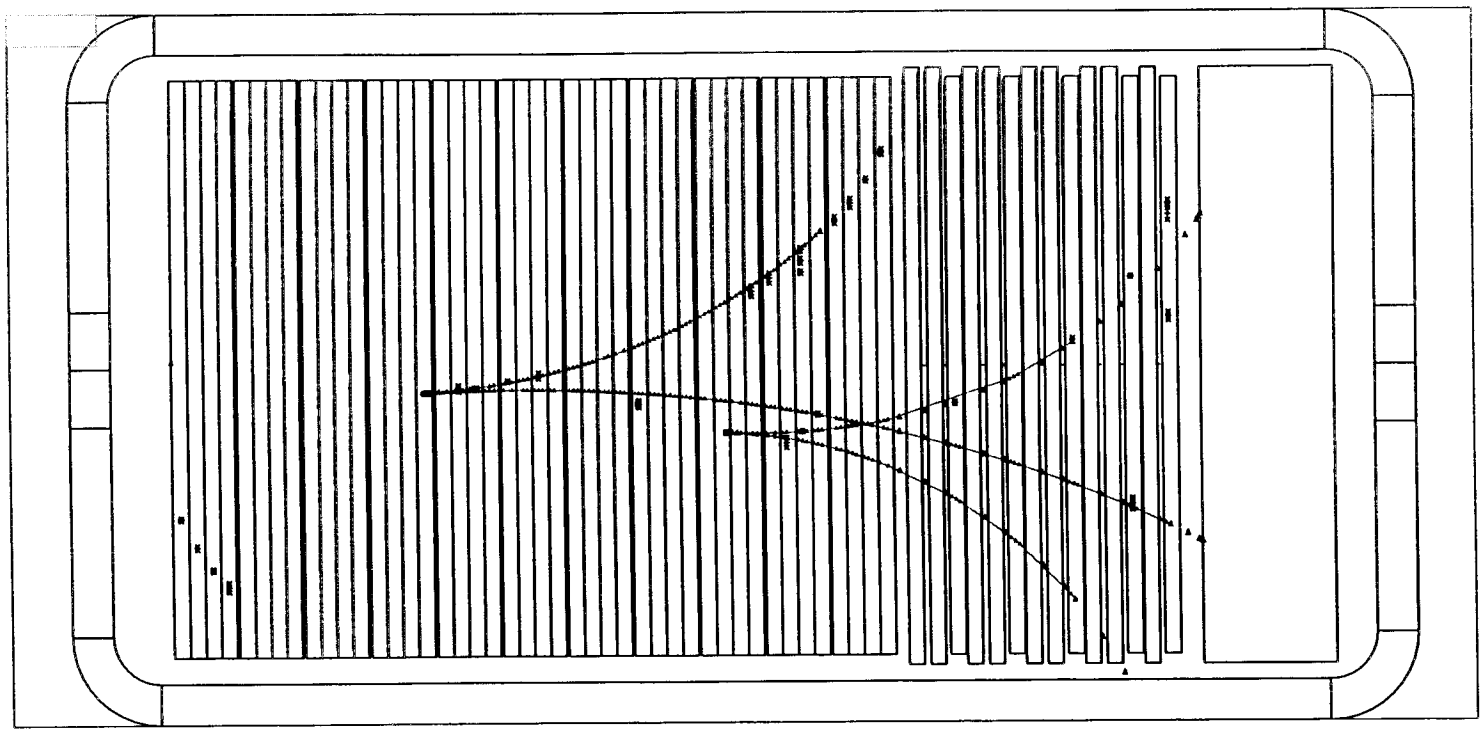


Fig. 1. Schematic of the DC tracker and a coherent π^0 event candidate in NOMAD where both photons from the π^0 decay convert in the DC's. The red crosses represent drift chamber digitizations that are used in the track-reconstruction, whereas the black ones are not. The upstream ($\gamma 1$) and downstream ($\gamma 2$) momentum vectors when extrapolated upstream intersect within the fiducial volume.

NOMAD
Phys. Lett B682, 177
(2009)

NOMAD 2009

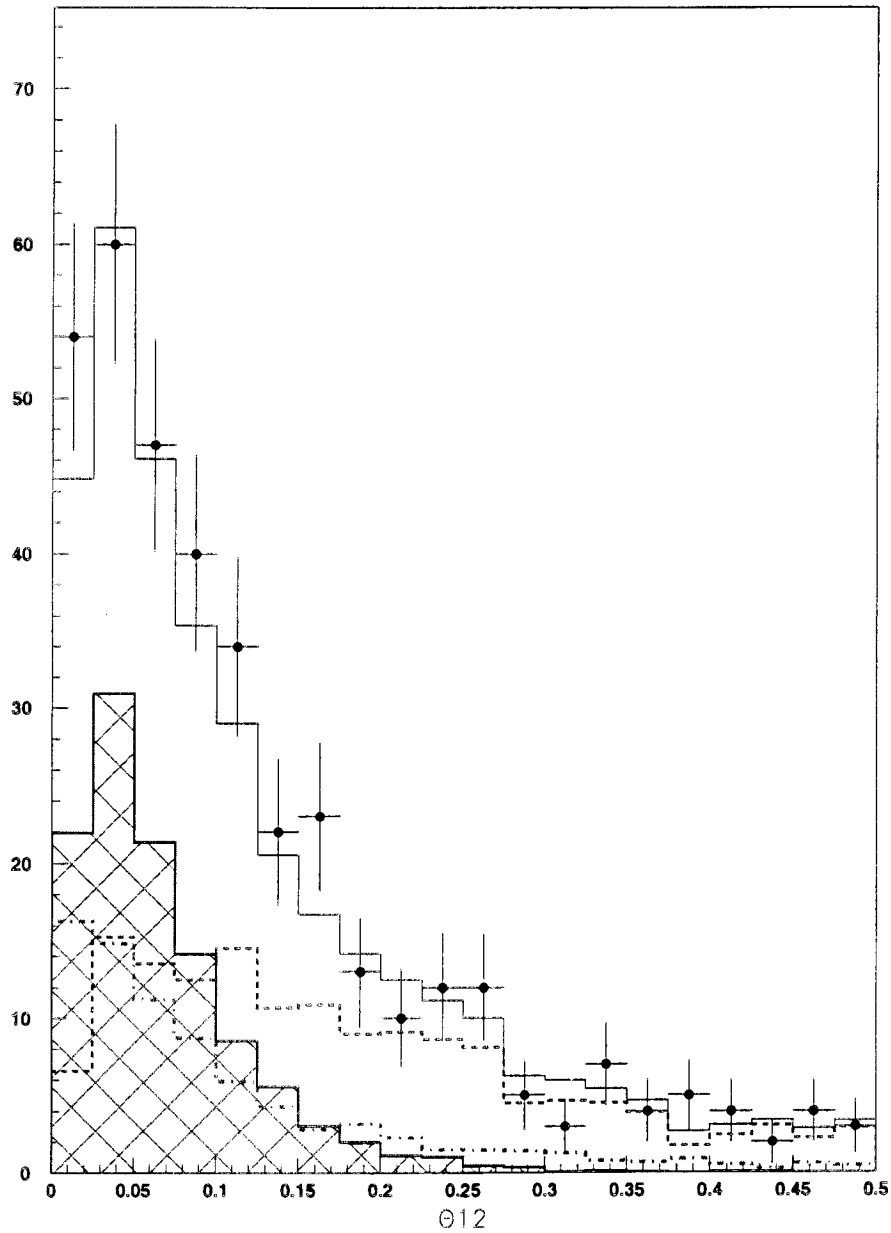


Fig. 8. Data and MC Comparison of the Θ_{12} Distribution.

Final Result: $(E_\nu = 24.3\ \mathrm{GeV}, \langle A \rangle = 12.8)$

$\sigma(\nu A \to \nu A\, \pi^0) = (72.6 \pm 8.1 \pm 6.9) \cdot 10^{-40}\ \mathrm{cm}^2$ per nucleus

R-S Prediction: $78 \times 10^{-40}\ \mathrm{cm}^2$

Problems at low energy ($E_\nu \lesssim 1\,\mathrm{GeV}$)

(1) Intrinsic limitation: Empirical data on $\pi N \to \pi N$ is not well-described by $\exp(-bt)$, with energy-independent b, when $E_\pi \ll 1\,\mathrm{GeV}$.

(2) MiniBoone measurement of coherent π^0: 35% below R-S
Not really serious.
Further: not clear if NUANCE is equivalent to R-S.

(3) K2K (2005): No evidence for coh π^+!

$$\sigma_{coh}^{\pi^+} / \sigma_{coh}^{\pi^+}(R\text{-}S) < \frac{1}{3} \quad \left(E_\nu \sim 1.3\,\mathrm{GeV}\right)$$

(4) SciBoone (2008)

$$\sigma_{coh}^{\pi^+} / \sigma(CC) < 0.67\%, \text{ at } E_\nu \sim 1.1\,\mathrm{GeV}$$
$$\left(\lesssim \tfrac{1}{3} \text{ of R-S}\right)$$
$$< 1.36\% \text{ at } E_\nu \sim 2.2\,\mathrm{GeV}$$
$$\left(\lesssim \tfrac{2}{3} \text{ of R-S}\right)$$

(5) $d\sigma/dQ^2$ for $\nu_\mu N \to \mu^- \pi^+ N$ indicates too few μ^- at small angles ($Q^2 < 0.1$ GeV^2). (MiniBoone)
$\Rightarrow$ Coherent contribution smaller than expected ?

(6) Microscopic (nuclear physics) calculations usually give crossections for $\nu_\mu N \to \nu_\mu N \pi^0$ and $\nu_\mu N \to \mu^- N \pi^+$ at low energies ($E_\nu < 1$ GeV) that are lower than R-S.
However, these models have their own uncertainties.

Hiraide (SciBoone) Nu Int 09

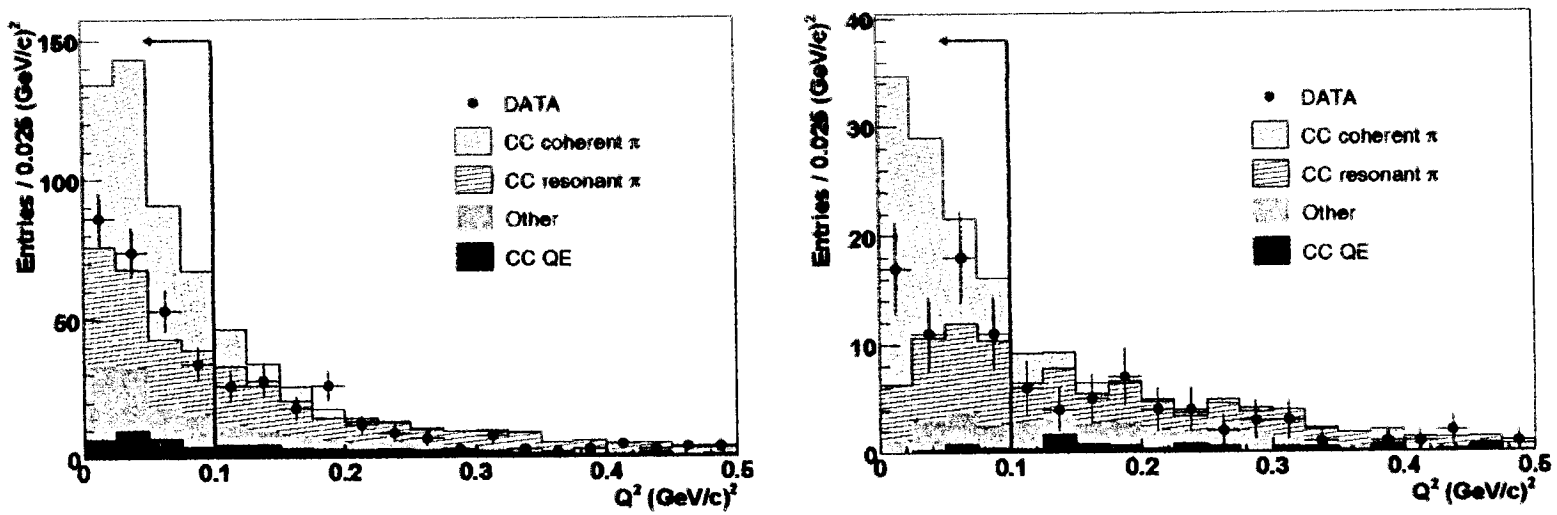


FIGURE 1. Reconstructed Q^2 for the MRD stopped charged current coherent pion sample (left), and the MRD penetrated charged current coherent pion sample (right).

where $E_\nu^{\rm rec}$ is the reconstructed neutrino energy calculated by assuming charged current quasi-elastic (CC-QE) kinematics,

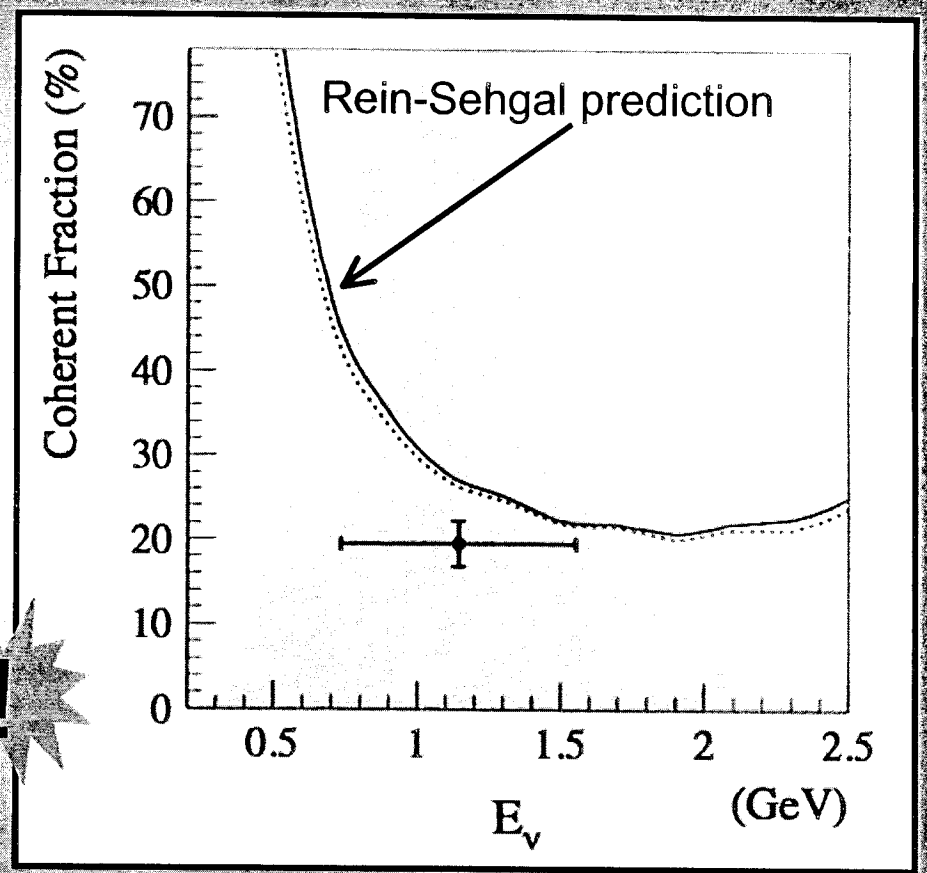
$$E_\nu^{\rm rec} = \frac{1}{2}\frac{(m_p^2 - m_\mu^2) - (m_n - V)^2 + 2E_\mu(m_n - V)}{(m_n - V) - E_\mu + p_\mu \cos\theta_\mu} \tag{2}$$

where m_p and m_n are the mass of proton and neutron, respectively, and V is the nuclear potential, which is set to 27 MeV. The fitting is described in detail in ref. [5].

Charged current coherent pion candidates are extracted from the $\mu + \pi$ events which do not have vertex activity. The sample still contains CC-QE events in which a proton is misidentified as a minimum ionizing track. We reduce this

MiniBooNE Coherent NC π^0

- 1st observation of NC coherent π^0 production at low energy (E_ν< 2 GeV)
  - 14% measurement
    (helped reduce uncertainties in MiniBooNE's $\nu_\mu \rightarrow \nu_e$ search)
  - 35% lower than most widely used model prediction
    (forced an immediate change in our π^0 background predictions)

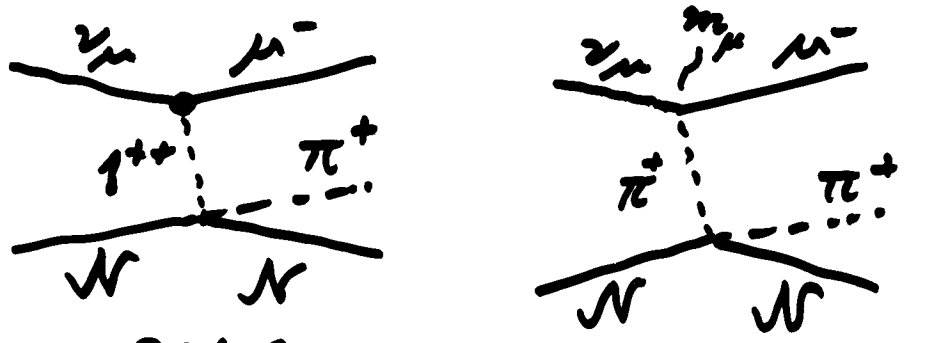


- coherent π^0 fraction = (19.5 ± 2.7)%; MC prediction = 30%
  published this month in Phys. Lett. B664, 41 (2008)

2. Berger-Sehgal Refinement of R-S Model, for $E_\nu \lesssim 1$ GeV.

(A) Include muon mass

- Phase space effect
- Important dynamical effect: Application of PCAC to π^+ coher. production: destructive interference due to $\pi^+ \to \mu^+ \nu_\mu$ coupling



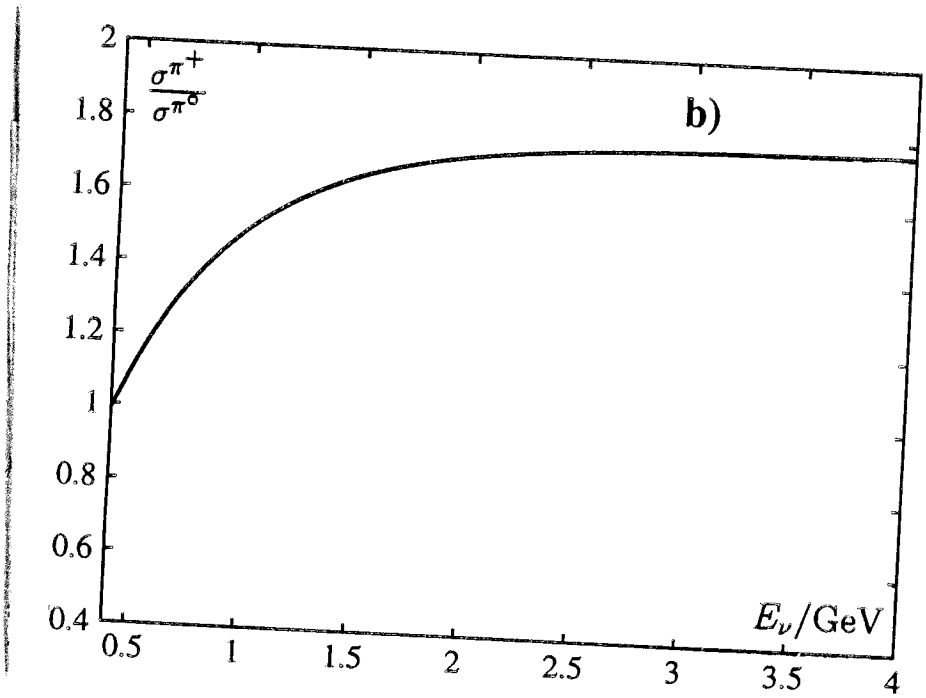
PCAC $\qquad$ π^+-exchange

$$\Rightarrow \frac{d\sigma^{cc}}{dQ^2\,dy\,dt} = (\text{Kin. Fac.})(1-y)\left\{\left[G_A - \frac{1}{2}\frac{Q^2_{min}}{Q^2+m_\pi^2}\right]^2 + \frac{1}{4}(Q^2-Q^2_{min})\frac{Q^2_{min}}{(Q^2+m_\pi^2)^2}\right\}\frac{d\sigma}{dt}(\pi^+ N)$$

Note: $Q^2_{min} = m_\ell^2\, y/(1-y)$

Thus Q^2 is not zero for forward scattering $\theta_\ell \to 0$.

$\sigma_{coh}(\pi^+)/\sigma_{coh}(\pi^0)$



Berger-Sehgal

(b) Replace R-S ansatz for $\frac{d\sigma}{dt}(\pi N \to \pi N)$ by data. For $N = C^{12}$, data available in the region $100 < T_\pi < 900$ MeV. This suffices for calculating coherent π^0 cross section up to $E_\nu \sim 1$ GeV.

Berger-Sehgal give a simple parametrization of the form

$$\frac{d\sigma}{dt}(\pi C \to \pi C) = A_1 e^{-b_1 t}$$

where A_1 and b_1 depend on E_π.

Result: σ_{coh} is <u>smaller</u> by a factor 2-3!

Consequence:

(i) Tension with K2K and SciBoone data on σ^{π^+} <u>eased</u>.

(ii) σ^{π^0} is now <u>lower</u> than MiniBoone by factor 2-3

: isospin symmetry due to lep-
in (6). The dominant cause for
ıe figure is the reduced phase
Adler screening factor reduces
ther 10% at $E = 0.6$ GeV and
ıence of the numerical value of
tio is negligible. Even setting
y less then 2%.

'ION CARBON CROSS
:TION

en presented which calculates
ing from pion nucleon scatter-

$$\left.\frac{d\sigma_{\rm el}}{dt}\right|_{t=0} e^{-bt} F_{\rm abs}. \qquad (10)$$

-hand side can be charged or
ıstic pion nucleon cross section
: right-hand side is determined
:glecting a possible real part of

$$\frac{1}{\pi}\left(\frac{\sigma_{\rm tot}^{\pi^+ p} + \sigma_{\rm tot}^{\pi^- p}}{2}\right)^2 \qquad (11)$$

ɔonential t distribution is taken
ation

$$\frac{1}{3} R_0^2 A^{2/3} \qquad (12)$$

$F_{\rm abs}$ describes the average at-
ging from a sphere of nuclear
resulting in

$$\left(-\frac{9A^{1/3}}{16\pi R_0^2}\sigma_{\rm inel}\right) \qquad (13)$$

The dotted line of Fig. 2 shows the result. The fact that this cross section is derived from a simple (classical) ansatz expressed by (10) and (13) raises doubts about its validity as a description of pion nucleus scattering in the resonance region. An alternative approach to coherent pion nucleus interaction based on the Glauber model was proposed by Bel'kov and Kopeliovich [13]. Its numerical results were similar to those in the RS model at least at high energies. The experimental groups use detailed Monte Carlo routines to simulate the scattering and absorption of the pion inside the nucleus [14,15].

Following the PCAC route we have tried to circumvent the uncertainties in modeling nuclear processes by direct appeal to data on pion nucleus elastic scattering; see also [16]. For carbon targets this can be done easily because π^+ and π^- data on differential and total cross sections exist for pion kinetic energies T_π from 30 to 870 MeV. They have been subjected to phase shift analyses yielding up to 21 complex phase shifts per energy [17]. Neglecting electromagnetic effects these phase shifts can be used to compute

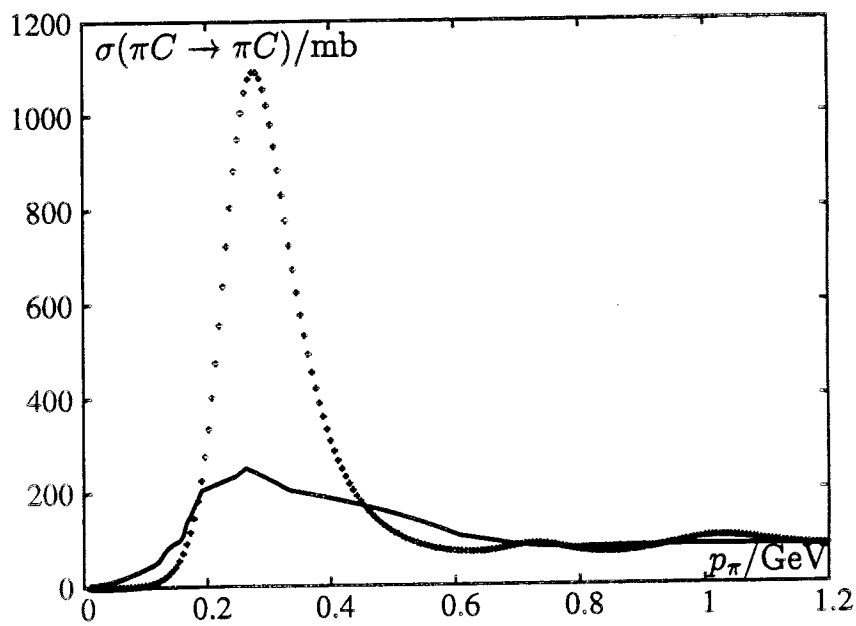


FIG. 2. Total elastic pion carbon cross section versus pion laboratory momentum. The dotted line represents the Rein-Sehgal model according to (15) and the solid line is derived from pion carbon data as explained in the text.

Berger & Sehgal

TABLE I. Coefficients A_1, b_1 of Eq. (16).

T_π (GeV)	A_1 (mb/GeV^2)	b_1 (1/GeV^2)
0.076	11 600	116.0
0.080	14 700	109.0
0.100	18 300	89.8
0.148	21 300	91.0
0.162	22 400	89.2
0.226	16 400	80.8
0.486	5730	54.6
0.584	4610	55.2
0.662	4570	58.4
0.776	4930	60.5
0.870	5140	62.2

the elastic strong interaction cross section $d\sigma_{\rm el}/dt$ for pion carbon scattering in a straightforward manner.

The phase shifts accurately reproduce even tiny effects like secondary peaks in the angular distribution. We have checked that except for the lowest two kinetic energies of 30 and 50 MeV it suffices to parametrize the cross section by the simple ansatz

$$\frac{d\sigma_{\rm el}}{dt} = A_1 e^{-b_1 t} \tag{16}$$

with energy dependent coefficients A_1, b_1 which are listed in Table I. For energies between the measured data points these coefficients are linearly interpolated which is the reason for the zigzag structure of the solid line in Fig. 2. It is obvious that $\sigma_{\rm el}$ from pion carbon data is much below the RS model in the resonance region. At the same time one observes that as $|\boldsymbol{p}_\pi|$ approaches 1 GeV, the two curves become very similar with $\sigma_{\rm el} \approx 80$ mb. This finally justifies the ansatz (9). It also suggests that the RS hadronic

model fails in the region
valid description at highe

V.

We are now ready to i
the two different models
cussed in the last section
the neutrino energy in Fi
Fig. 3(b) for π^+ productio
the empirical pion carbon
assuming the coefficients
valid up to $T_\pi = 1.7$ Ge
sumption results in a cro
2 GeV.

The curve using carbon
the curve obtained by app
Cross sections for NC and
duction on carbon have
ansatz based mainly on t
$\mu^-\Delta^{++}$ and its modificati
[18–21]. (For an early refe
Remarkably our calculatio
sponding results given in [2
approach to coherent neut
cross sections of [16] depen
ξ. Referring to footnote 41
are close to the ones obtaine
cross sections $d\sigma/dQ^2$ or d
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prediction for $d\sigma/dQ^2$ at a
the CC reaction a pronounc
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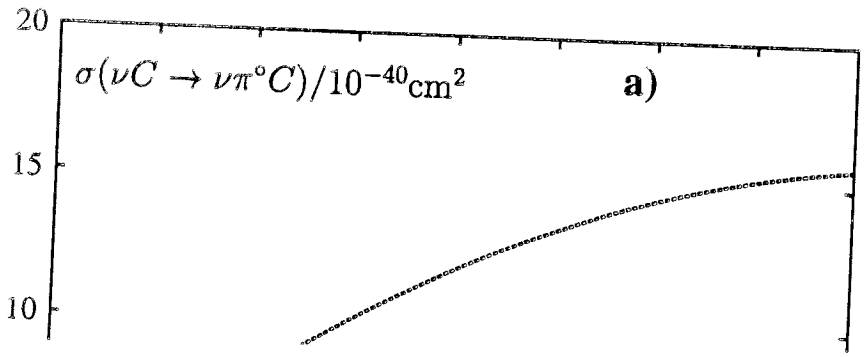
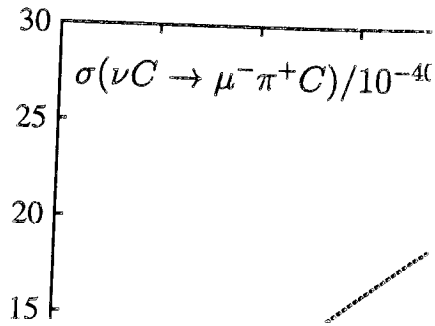




3. Dynamical Models

At low energies, PCAC is not the whole story.

All inelastic cross sections are writable as

$$\frac{d\sigma^{cc}}{dQ^2\,dy} = \frac{G_F^2 \cos^2\theta_c}{4\pi^2} K E_\nu \frac{Q^2}{|q|^2} \cdot$$

$$\left[u^2 \sigma_L + v^2 \sigma_R + 2uv\,\sigma_S\right]$$

Only σ_S part survives at $Q^2 \to 0$ (this is determined by PCAC) but σ_L, σ_R contribute in non-forward directions.

Thus a first-principle calculation desirable / essential.

Examples (from Nu Int 09):

Amaro et al

Nakamura et al

Leitner et al

+ several others

One illustrative result:

Nakamura et al, $E_\nu = 1\,\mathrm{GeV}$, C^{12}

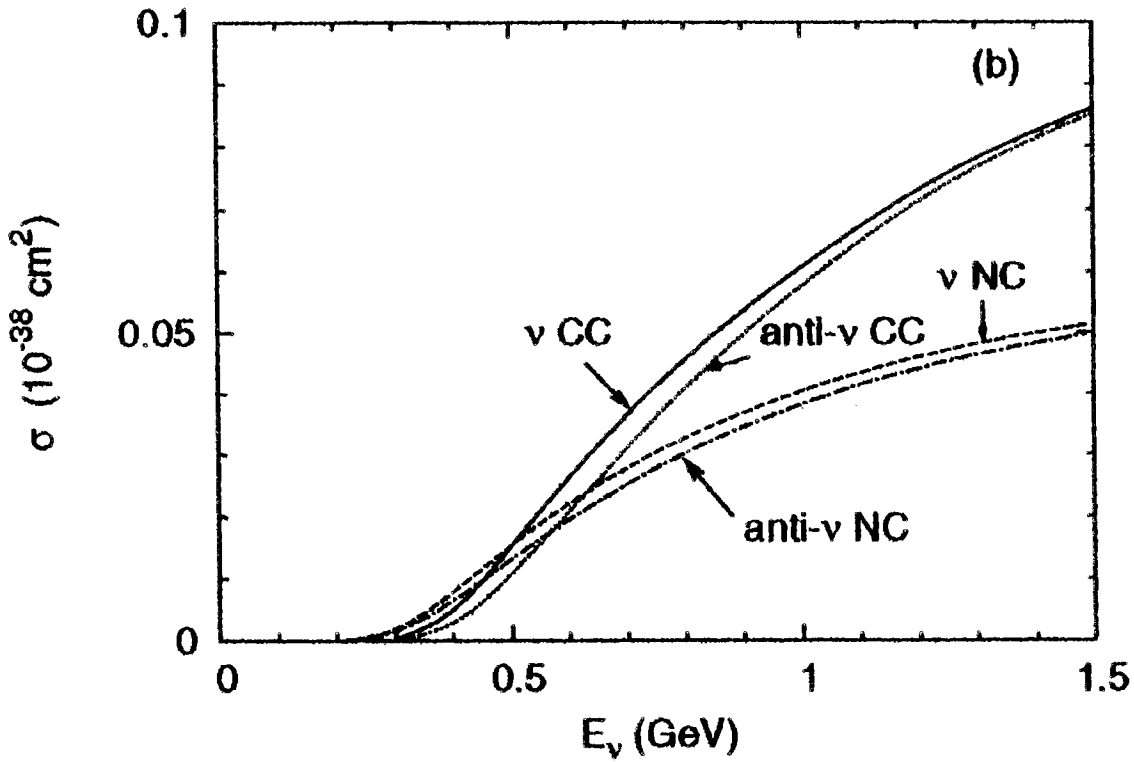
(i) $\frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} \approx 0.95$ (B-S : 1)

$\sigma_{coh}^{\pi^0} = 4 \times 10^{-40}\,\mathrm{cm}^2$ (B-S : 3.5×10^{-40})

$\sigma_{coh}^{\pi^+} = 6 \times 10^{-40}\,\mathrm{cm}^2$ (B-S : 6×10^{-40})

(ii) Differences between different dynamical calculations suggest that the uncertainty may be 20-25% (personal guess).

Nakamura, Sato, Lee, Szczerbinska, Kubodera
(NuInt 09)



$^{0} + {}^{12}C_{g.s.}$. The solid line represents the result of the full calculation.
e medium effects on the Δ-propagation, while the short-dash line is
h-dotted line corresponds to a case in which the pion production
Ref. [10]. (b) The E_ν-dependence of the total cross section for
$\nu + \pi^0 + {}^{12}C$ (dashed line) $\bar{\nu} + {}^{12}C \to \mu^+ + \pi^- + {}^{12}C$