# Nuclear effects in QE neutrino CC scattering on carbon

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- Motivations.
- Nuclear model.
- Results.
- Summary.

For more details of the present approaches see:
A.Butkevich, S.Kulagin,Phys.Rev.C76:045502,2007;
A.Butkevich,Phys.Rev.C78:015501,2008;
A.Butkevich,arXiv:0904.1472[nucl-th];

# Motivations

- NuInt09, *Sam Zeller:* Topic 1: QE scattering

($\star$) What about handling of low $Q^2$ region, are there better alternatives to the $\kappa$ parameter?

($\star$) Is the axial form factor dipole?

($\star$) Do we understand the difference in $M_A$ values from different experiments?

($\star$) Do we need to start thinking beyond $M_A$? Is $M_A$ measuring nuclear effects or axial form factor? Is there are more reliable way to make comparison?

- How $NN$-correlation effects can change the total CC QE $\nu A$- cross section?

# Formalism of the quasi-elastic scattering

We consider lepton charged-current (CC) QE exclusive

$$l(k_i) + A(p_A) \to l'(k_f) + N(p_x) + B(p_B),$$

and inclusive

$$l(k_i) + A(p_A) \to l'(k_f) + X$$

scattering off nuclei, where

- ⋆ $l$ is incident lepton $(e/\nu_\mu)$, $l'$ is scattered lepton $(e/\mu)$, $k_i = (\varepsilon_i, \boldsymbol{k}_i)$ and $k_f = (\varepsilon_f, \boldsymbol{k}_f)$ are initial and final lepton momenta
- ⋆ $p_A = (\varepsilon_A, \boldsymbol{p}_A)$, and $p_B = (\varepsilon_B, \boldsymbol{p}_B)$ are the initial and final target momenta, $p_x = (\varepsilon_x, \boldsymbol{p}_x)$ is ejectile nucleon momentum
- ⋆ $q = (\omega, \boldsymbol{q})$ is momentum transfer carried by the virtual photon (W-boson), and $Q^2 = -q^2 = \boldsymbol{q}^2 - \omega^2$ is photon (W-boson) virtuality
- ⋆ $m$, $m_A$ and $m_B$ are masses of nucleon, target and recoil nucleus, respectively. The missing momentum and energy are defined by $\boldsymbol{p}_m = \boldsymbol{p}_x - \boldsymbol{q}$, $\quad \varepsilon_m = m + m_B - m_A$

## QE lepton-nucleus cross section

In the lab frame the differential cross section for exclusive (anti)neutrino CC scattering can be written as

$$\frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} = R\frac{|\boldsymbol{p}_x|\varepsilon_x}{(2\pi)^5}\frac{|\boldsymbol{k}_f|}{\varepsilon_i}\frac{G^2\cos^2\theta_c}{2}L_{\mu\nu}W^{\mu\nu},$$

where $\Omega_f$ is the solid angle for the lepton momentum, $\Omega_x$ is the solid angle for the ejectile nucleon momentum, $G \simeq 1.16639 \times 10^{-11}$ MeV$^{-2}$ is the Fermi constant, $\theta_C$ is the Cabbibo angle ($\cos\theta_C \approx 0.9749$). The recoil factor $R$ is given by

$$R = \left|1 - \frac{\varepsilon_x}{\varepsilon_B}\frac{\boldsymbol{p}_x\cdot\boldsymbol{p}_B}{\boldsymbol{p}_x\cdot\boldsymbol{p}_x}\right|^{-1},$$

and $\varepsilon_x$ is solution to equation

$$\varepsilon_x + \varepsilon_B - m_A - \omega = 0,$$

where $\varepsilon_B = \sqrt{m_B^2 + \boldsymbol{p}_B^2}$, $\boldsymbol{p}_B = \boldsymbol{q} - \boldsymbol{p}_x = -\boldsymbol{p}_m$, $\boldsymbol{p}_x = \sqrt{\varepsilon_x^2 - m^2}$, and $m_A$, $m_B$, and $m$ are masses of the target, recoil nucleus, and nucleon respectively.

The lepton tensor can be written as the sum of symmetric $L_S^{\mu\nu}$ and antisymmetric $L_A^{\mu\nu}$ tensors

$$L^{\mu\nu} = L_S^{\mu\nu} + L_A^{\mu\nu}$$
$$L_S^{\mu\nu} = 2\left(k_i^\mu k_f^\nu + k_i^\nu k_f^\mu - g^{\mu\nu} k_i k_f\right)$$
$$L_A^{\mu\nu} = h 2 i \epsilon^{\mu\nu\alpha\beta} (k_i)_\alpha (k_f)_\beta,$$

where $h$ is $+1$ for positive lepton helicity and $-1$ for negative lepton helicity, $\epsilon^{\mu\nu\alpha\beta}$ is the antisymmetric tensor

The weak CC hadronic tensor $W_{\mu\nu}$ is given by bilinear product of the transition matrix elements of the nuclear CC operator $J_\mu$ between the initial nucleus state $|A\rangle$ and the final state $|B_f, \boldsymbol{p}_x\rangle$ as

$$W_{\mu\nu} = \sum_f \langle B_f, p_x | J_\mu | A \rangle \langle A | J_\nu | B_f, p_x \rangle$$

where the sum is taken over undetected states.

It is also useful to define a reduced cross section

$$\sigma_{red} = \frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} / K\sigma_{lN},$$

where $K = Rp_x\varepsilon_x/(2\pi)^5$ is phase-space factor for neutrino scattering, and $\sigma_{lN}$ is corresponding elementary cross section for the neutrino scattering from moving free nucleon. It should be similar for electron and neutrino scattering (apart from small differences due to FSI effects for electron and neutrino induced reactions).

In the inclusive reactions only the outgoing lepton is detected and the differential cross sections can be written as

$$\frac{d^3\sigma}{d\varepsilon_f d\Omega_f} = \frac{1}{(2\pi)^2}\frac{|\boldsymbol{k}_f|}{\varepsilon_i}\frac{G^2\cos^2\theta_c}{2}L_{\mu\nu}\mathcal{W}^{\mu\nu},$$

where $\mathcal{W}^{\mu\nu}$ is inclusive hadronic tensor.

- We describe the lepton-nucleon scattering in the Impulse Approximation (IA), in which only one nucleon of the target is involved in reaction and the nuclear current is written as the sum of single-nucleon currents. Then, the nuclear matrix element takes the form

$$\langle p, B|J^\mu|A\rangle \quad = \quad \int d^3r \ \exp(i\boldsymbol{t}\cdot\boldsymbol{r})\overline{\Psi}^{(-)}(\boldsymbol{p},\boldsymbol{r})\Gamma^\mu\Phi(\boldsymbol{r}),$$

  where $\Gamma^\mu$ is the vertex function, $\boldsymbol{t} = \varepsilon_B\boldsymbol{q}/W$ is the recoil-corrected momentum transfer, $W = \sqrt{(m_A+\omega)^2 - \boldsymbol{q}^2}$ is the invariant mass, $\Phi$ and $\Psi^{(-)}$ are relativistic bound-state and outgoing wave functions.

- The single-nucleon charged current has $V-A$ structure $J^\mu = J^\mu_V + J^\mu_A$. For a free nucleon vertex function $\Gamma^\mu = \Gamma^\mu_V + \Gamma^\mu_A$ we use CC2 vector current vertex function

$$\Gamma^\mu_V = F_V(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_M(Q^2)$$

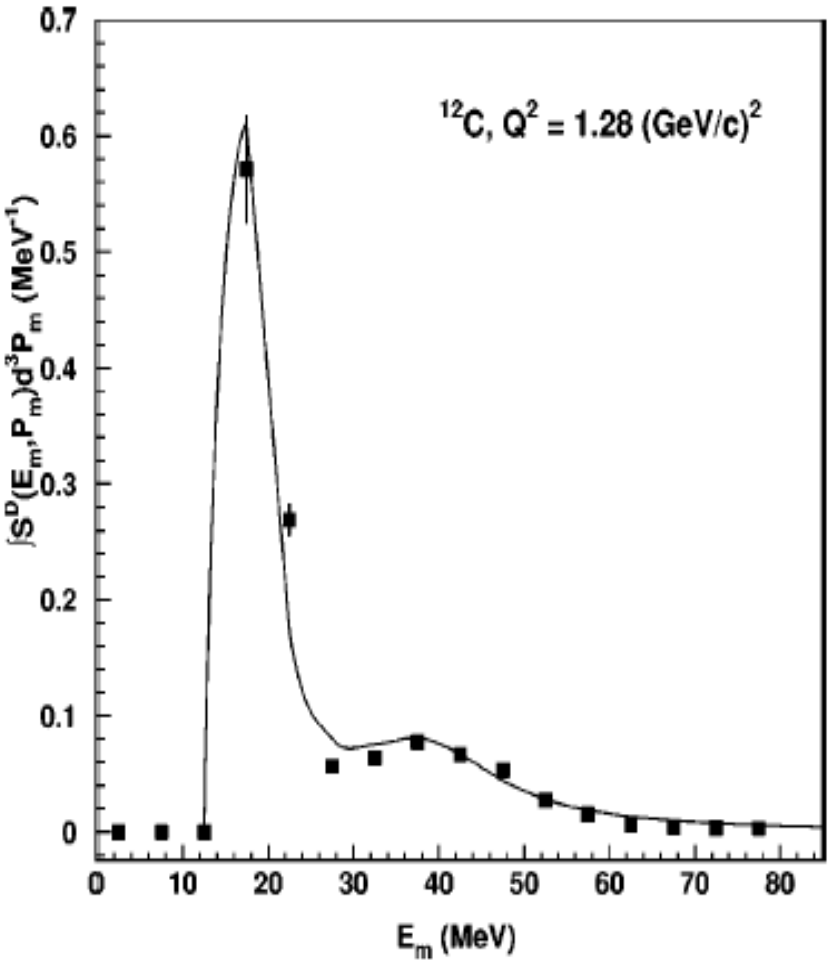
  and the axial current vertex function

$$\Gamma^\mu_A = F_A(Q^2)\gamma^\mu\gamma_5 + F_P(Q^2)q^\mu\gamma_5.$$

  We use the MMD approximation [P.Mergell et al (1996)] of the nucleon form factors.

- The axial $F_A$ and pseudoscalar $F_P$ form factors in the dipole approximation are parameterized as $F_A(Q^2) = F_A(0)/(1+Q^2/M_A^2)^2$, $F_P(Q^2) = 2mF_A(Q^2)/m_\pi^2 + Q^2$, where $F_A(0) = 1.267$, $m_\pi$ is the pion mass, and $M_A$ is the axial mass.

# Model




Shell occupancy for Carbon:

$S(P_{3/2})$=0.84

$S(S_{1/2})$=1 is regarded as discrete state

Average occupancy of nuclear shells $\overline{S}$=0.89

[supported by JLab measurement D. Dutta et al. (2003)] and [J.J.Kelly (2004)]

Missing Energy

Neutron: $E_m(1p_{3/2})$=17.9 MeV, $E_m(1s_{1/2})$=39.8 MeV

Proton: $E_m(1p_{3/2})$=16 MeV, $E_m(1s_{1/2})$=37.9 MeV

Missing strength (11%) can be attributed to the short-range $NN$-correlations in ground state. It is in agreement with direct measurement of the spectral function [JLab measurement D.Rohe at al. (2006)] observed approximately 0.6 proton in a region with $p_m$ >240 MeV and $E_m$ >50 MeV

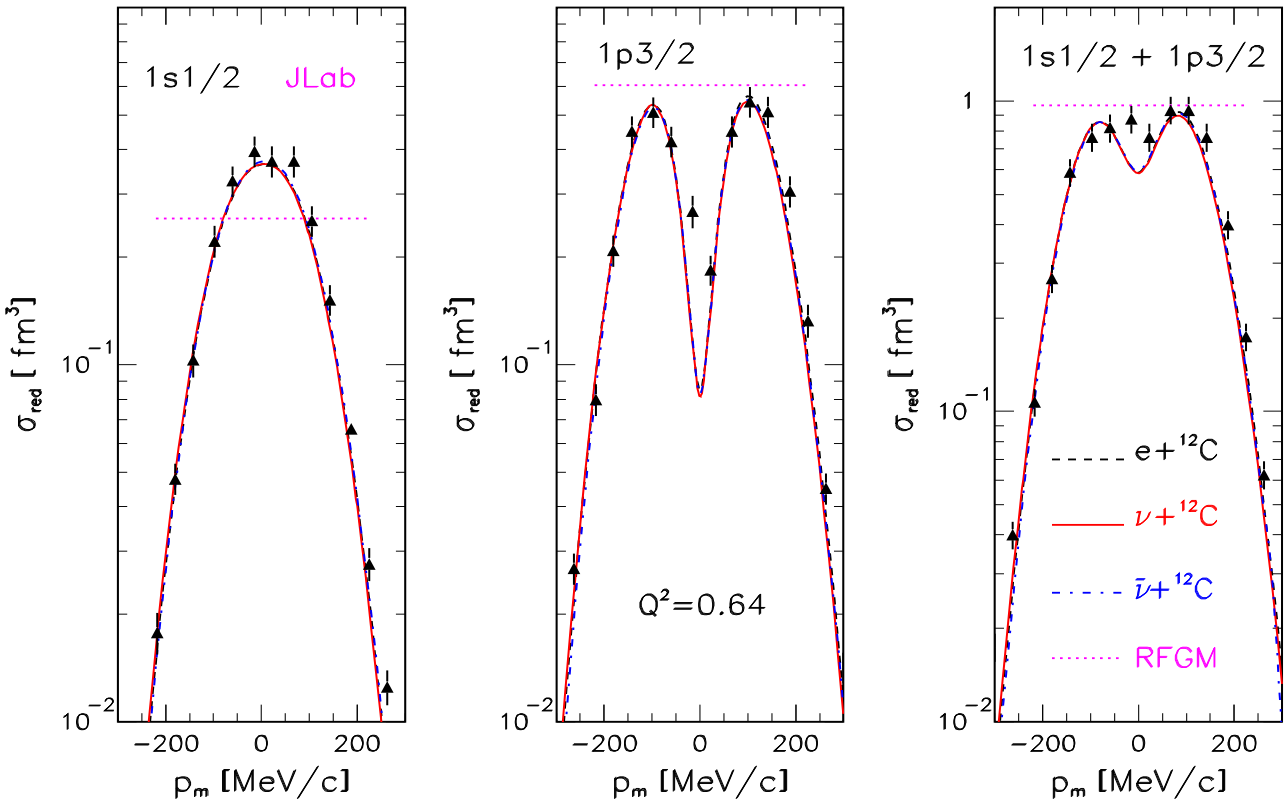
- In independ particle shell model the relativistic bound-state functions $\Phi$ are obtained within the Hartree–Bogolioubov approximation in the $\sigma - \omega$ model [B.Serot et al. 1986].
- In the RDWIA the final state interaction between the outgoing nucleon and the residual nucleus is taking into account and the ejectile wave function $\Psi$ is solution of a Schrödinger equation containing equivalent central and spin-orbit potentials, which are functions of the scalar and vector optical potentials $S$ and $V$, and are energy dependent.
- We use the LEA program [J.J. Kelly (1995)] for numerical calculation of the distorted wave functions with EDAD1 SV relativistic optical potential [E. Cooper (1993)].

- In the PWIA the final state interaction between the outgoing nucleon and the residual nucleus is neglected and the ejectile wave function $\Psi$ is plane wave.

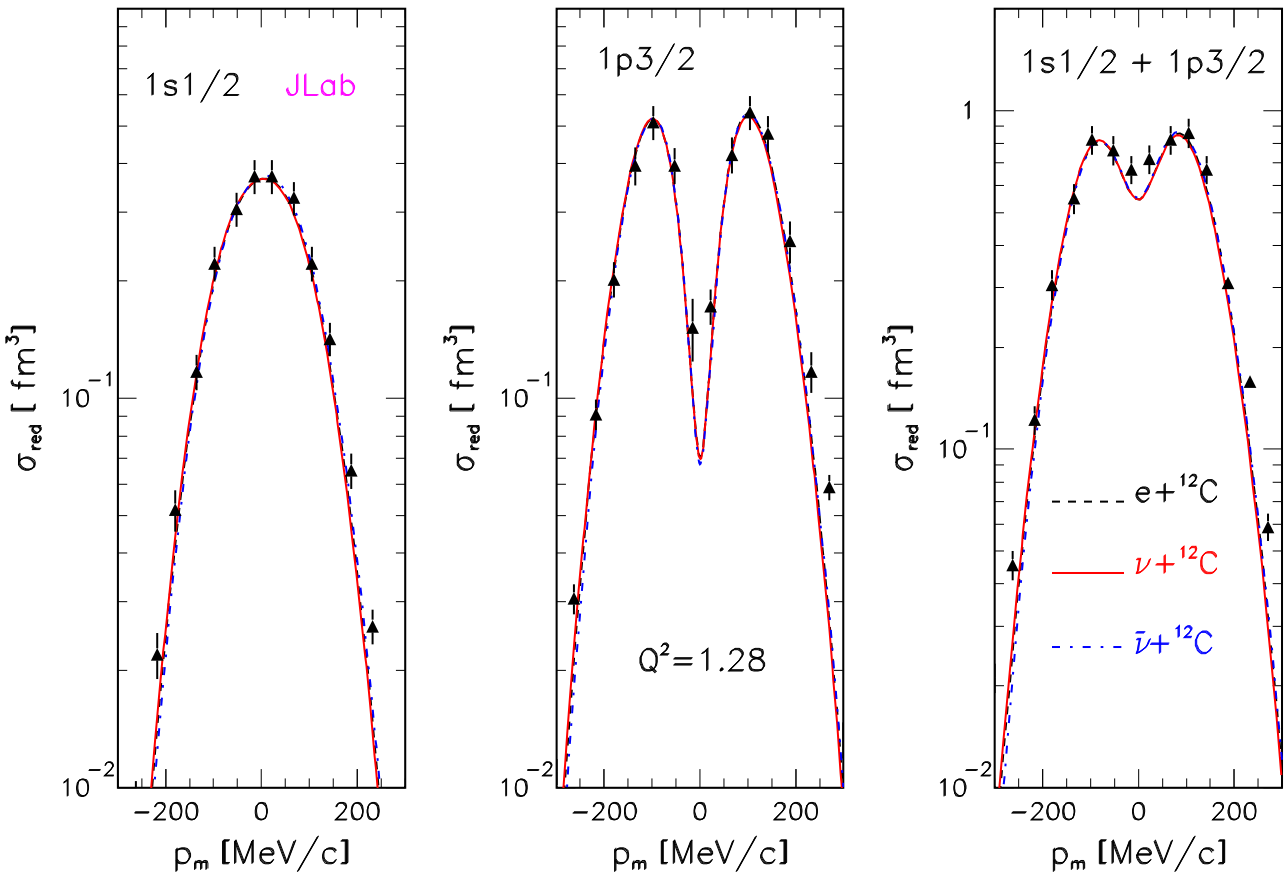
- In the PWIA and RDWIA approaches effects of the $NN$-correlation on the inclusive and total cross section are taking into account. In the RDWIA calculation the inclusive cross section with FSI effects in the presence of short-range $NN$-correlations was calculated using the approach which was proposed [ A. Butkevich et al. (2007)].

- In Fermi Gas model for carbon we use $p_F$=221 MeV/c and $\varepsilon$=25 MeV. The RFGM does not account nuclear shell structure, FSI effect, and the presence of NN-correlations.

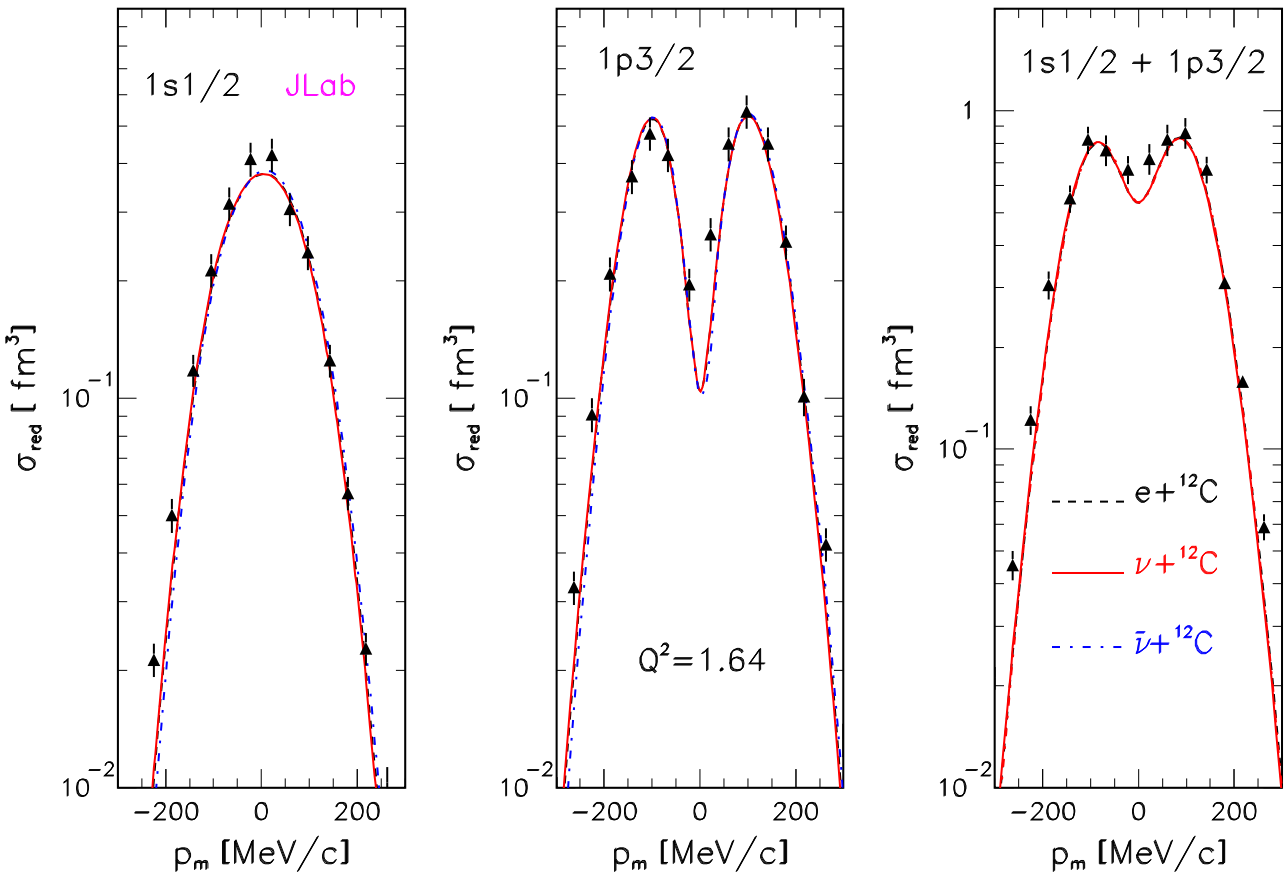
# Results



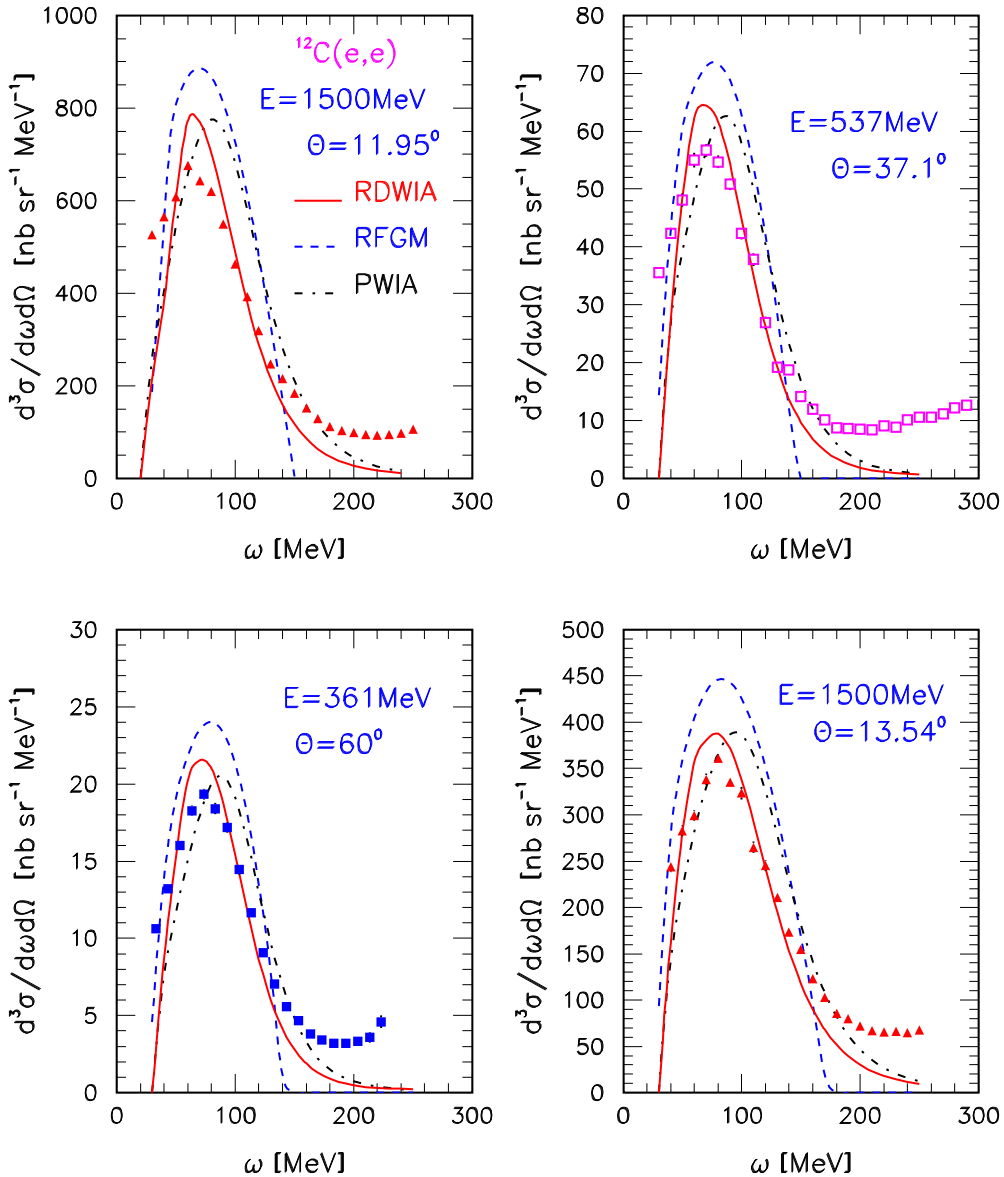

Comparison of the RDWIA and RFGM calculations for electron, neutrino, and antineutrino reduced cross sections for the removal of nucleon from $1p$ and $1s$ shells of $^{12}C$ as a function of missing momentum. The cross sections were calculated for JLab [D.Dutta et al. (2003)] kinematics: $\varepsilon_i = 2.445 GeV, Q^2 = 0.64 (GeV/c)^2$ and $T_x = 350 MeV$.
The RFGM predictions are completely off of the exclusive data (CCQE two-track events).



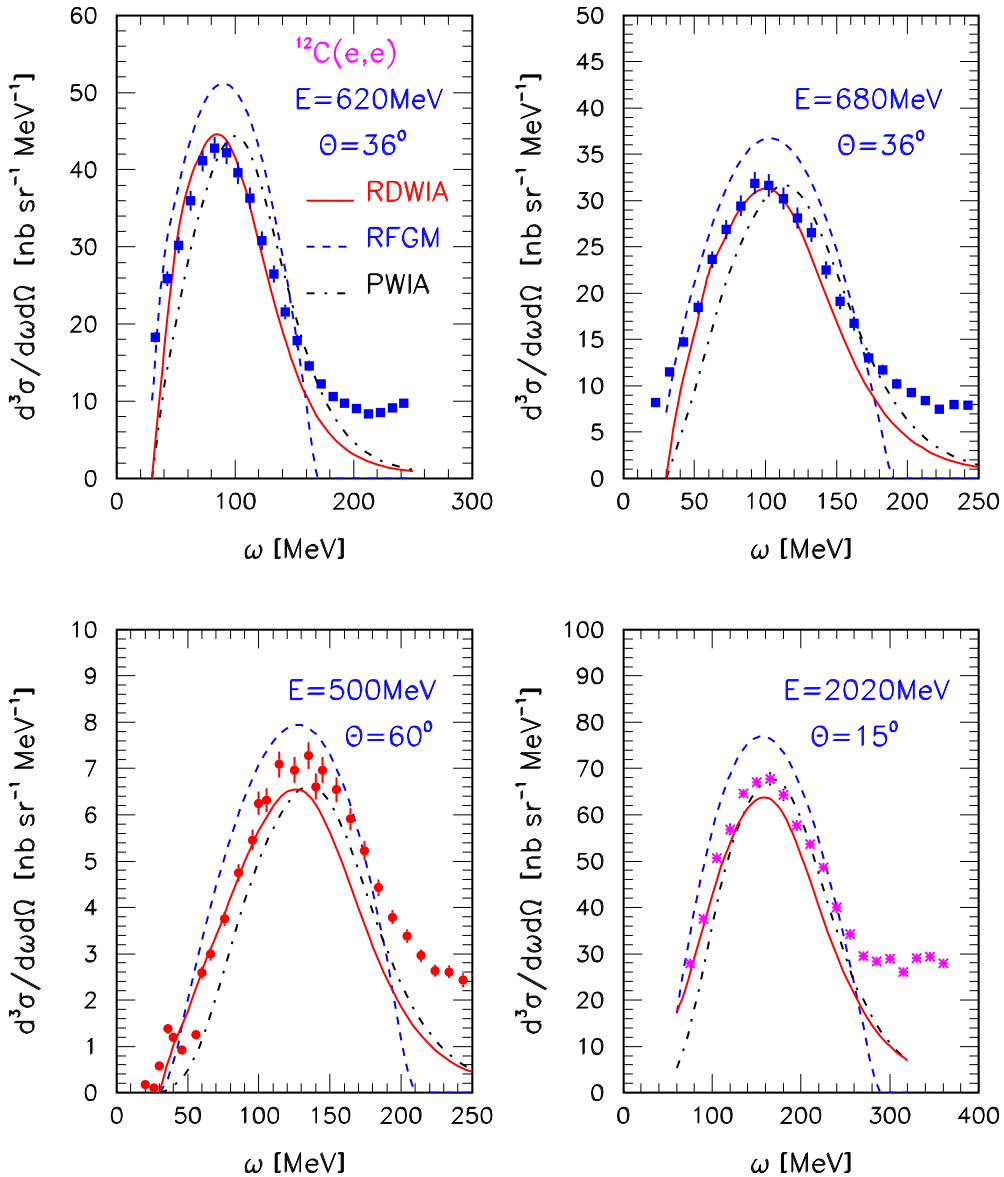

Comparison of the RDWIA calculations for electron, neutrino, and antineutrino reduced cross sections for the removal of nucleon from $1p$ and $1s$ shells of $^{12}C$ as a function of missing momentum. The cross sections were calculated for JLab [D.Dutta et al. (2003)] kinematics: $\varepsilon_i = 2.445 GeV, Q^2 = 1.28 (GeV/c)^2$ and $T_x = 700 MeV$.



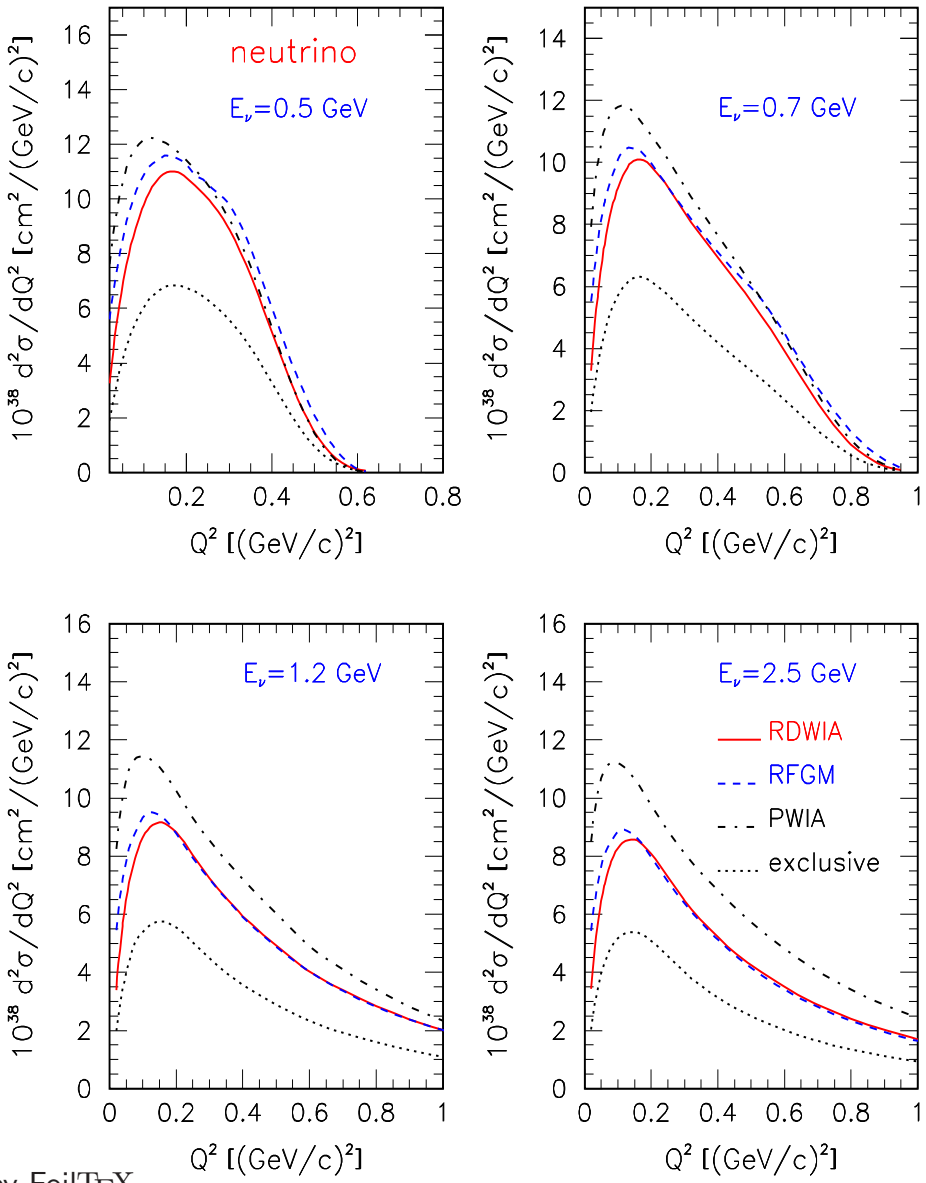

Comparison of the RDWIA calculations for electron, neutrino, and antineutrino reduced cross sections for the removal of nucleon from $1p$ and $1s$ shells of $^{12}C$ as a function of missing momentum. The cross sections were calculated for JLab [D.Dutta eta al. (2003)] kinematics: $\varepsilon_i = 2.445 GeV, Q^2 = 1.84 (GeV/c)^2$ and $T_x = 970 MeV$.




Inclusive cross section versus the energy transfer $\omega$ for electron scattering on $^{12}$C. The data are from SLAC (filled triangles), Bates (open squares) and Saclay (filled squares). SLAC data are for electron beam energy $E_e$=1500 MeV and scattering angle $\theta$=11.95° and $\theta$=13.54°. Bates data are for $E_e$=537 MeV and $\theta$=37.1°. Saclay data are for $E_e$=361 MeV and $\theta$=32°. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.

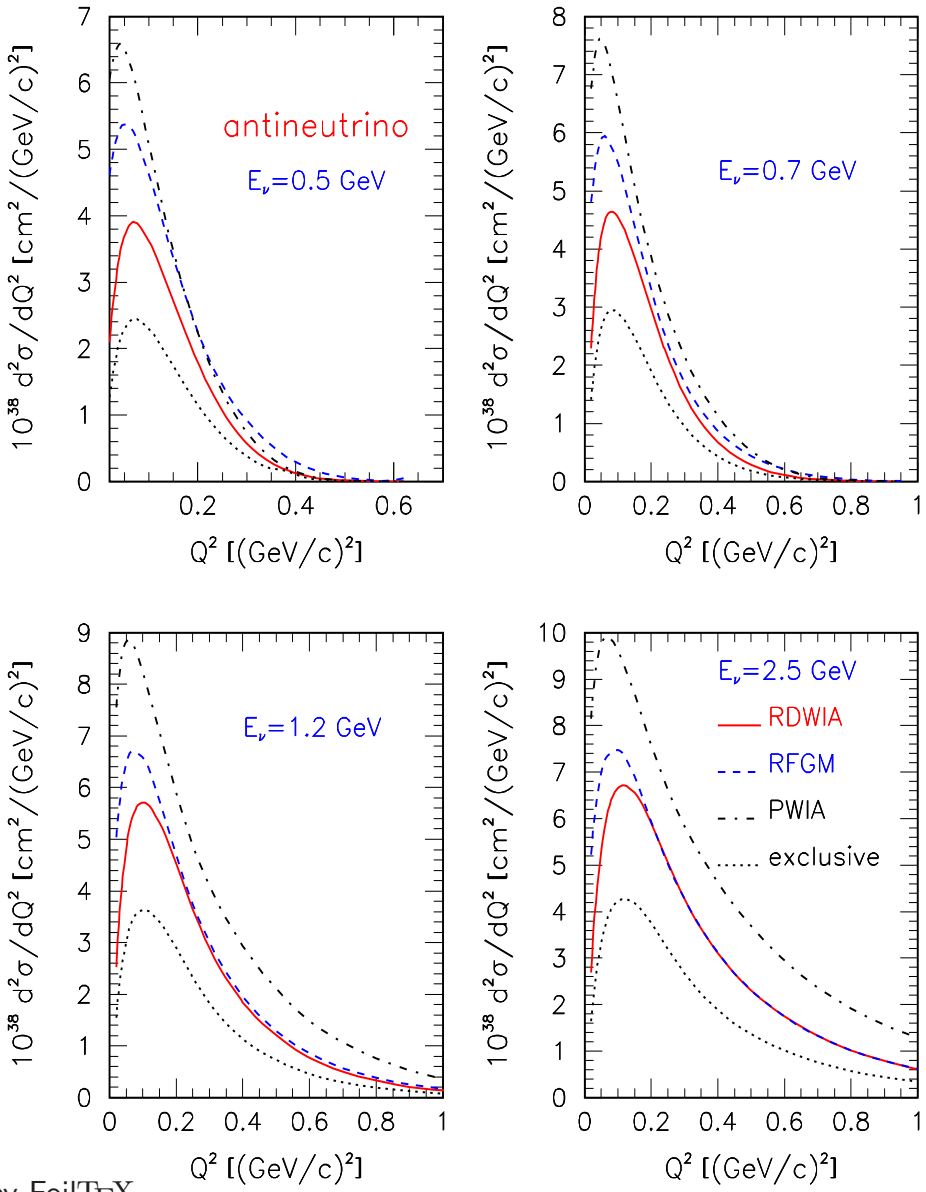



Inclusive cross section versus the energy transfer $\omega$ for electron scattering on $^{12}$C. The data are from SLAC (filled circles), JLab (stars) and Saclay (filled squares). SLAC data are for electron beam energy $E_e$=500 MeV and scattering angle $\theta$=60°. JLab data are for $E_e$=2020 MeV and $\theta$=15°. Saclay data are for $E_e$=620, 680 MeV and $\theta$=36°. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.



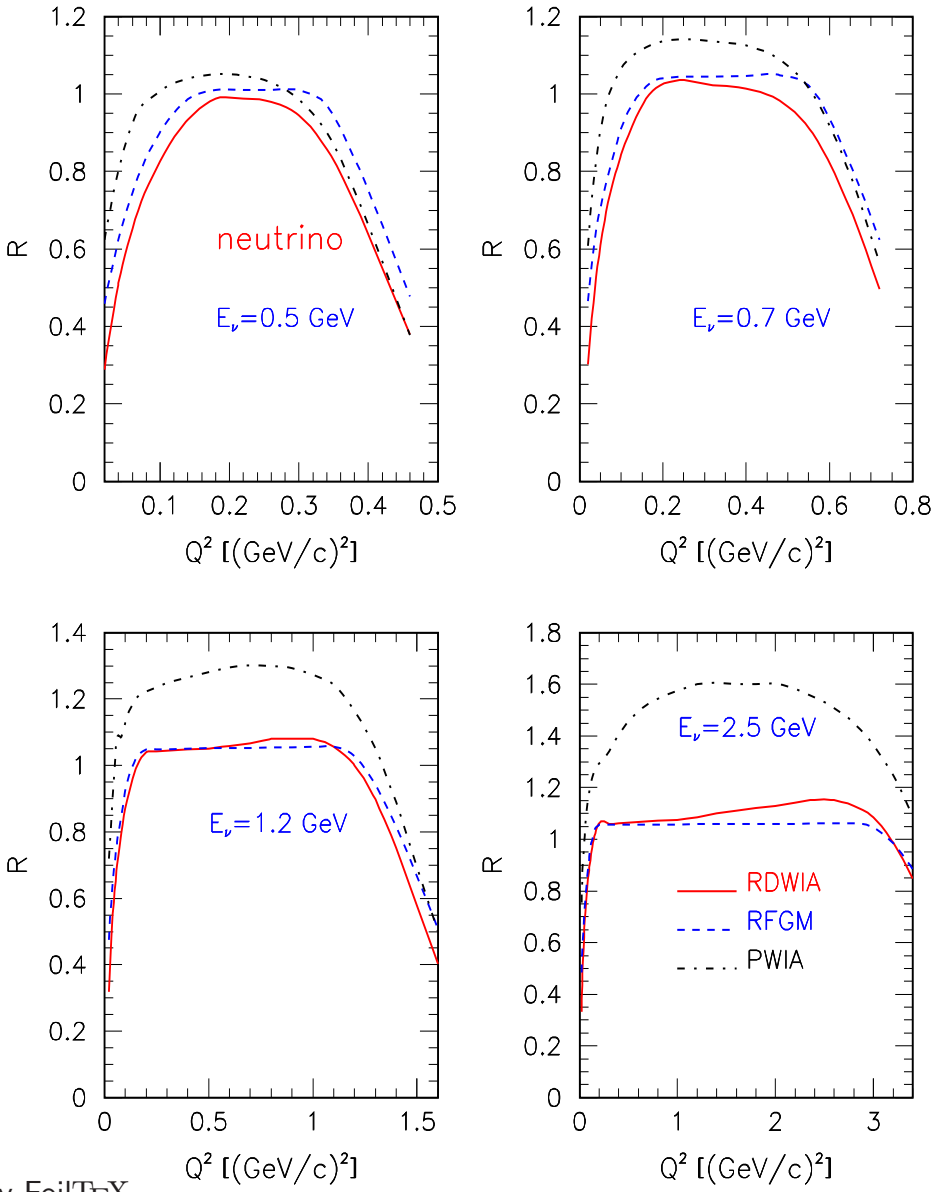

Inclusive cross section versus $Q^2$ for neutrino scattering on $^{12}$C and for the four values of incoming neutrino energy: $\varepsilon_\nu$=0.5, 0.7, 1.2 and 2.5 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the RDWIA result for the exclusive reaction (two-track events).

In the region $< 0.2$ (GeV/c)$^2$ the RFGM results are higher than those obtained in the RDWIA and at $Q^2 = 0.1$ (GeV/c)$^2$ the the difference is about $\sim 12\%$ for $\varepsilon_\nu = 0.5$ GeV and $\sim 7\%$ for $\varepsilon_\nu = 2.5$ GeV.



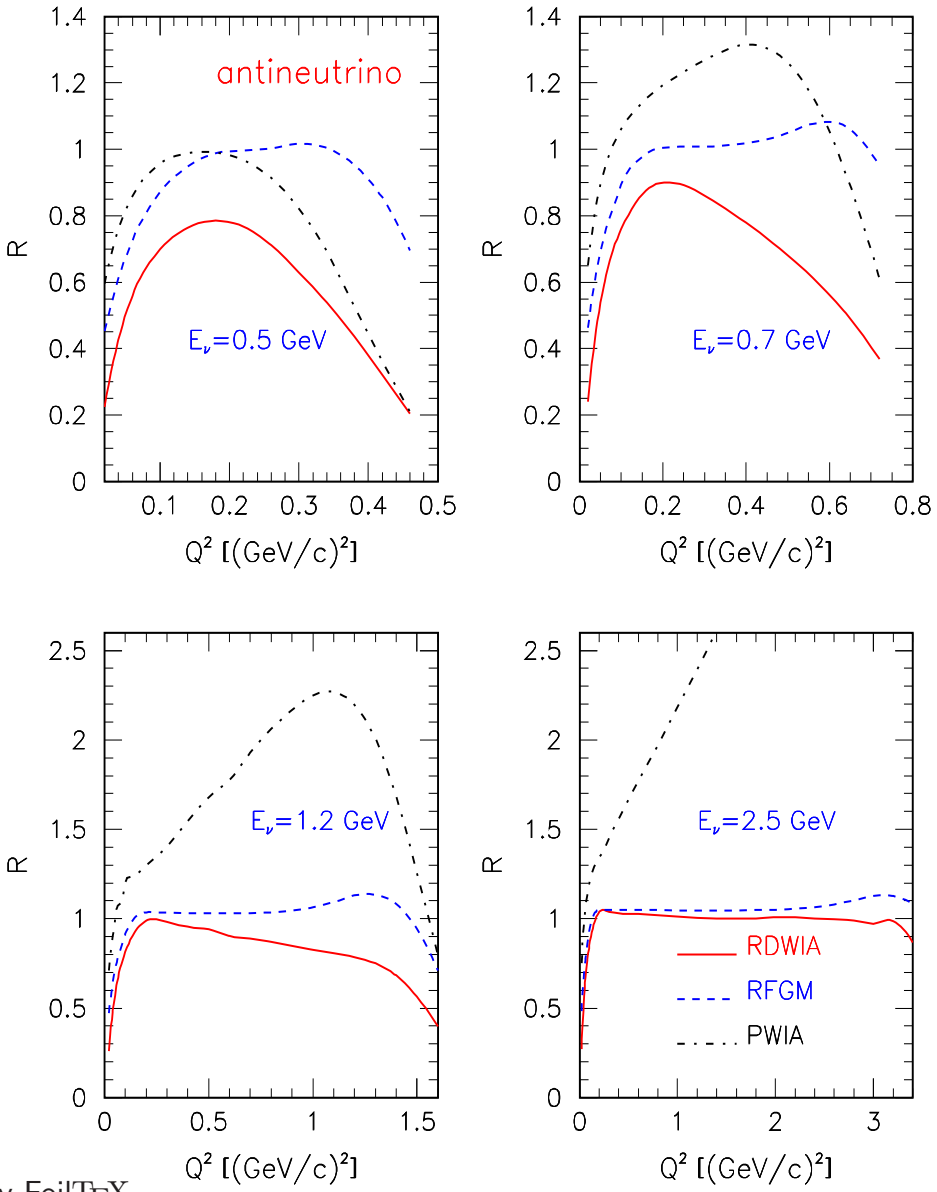

Inclusive cross section versus $Q^2$ for antineutrino scattering on $^{12}$C and for the four values of incoming neutrino energy: $\varepsilon_\nu$=0.5, 0.7, 1.2 and 2.5 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the RDWIA result for the exclusive reaction (two-track events).

In the region $< 0.2$ (GeV/c)$^2$ the RFGM results are higher than those obtained in the RDWIA and at $Q^2 = 0.1$ (GeV/c)$^2$ the the difference is about $\sim 28\%$ for $\varepsilon_\nu = 0.5$ GeV and $\sim 12\%$ for $\varepsilon_\nu = 2.5$ GeV.

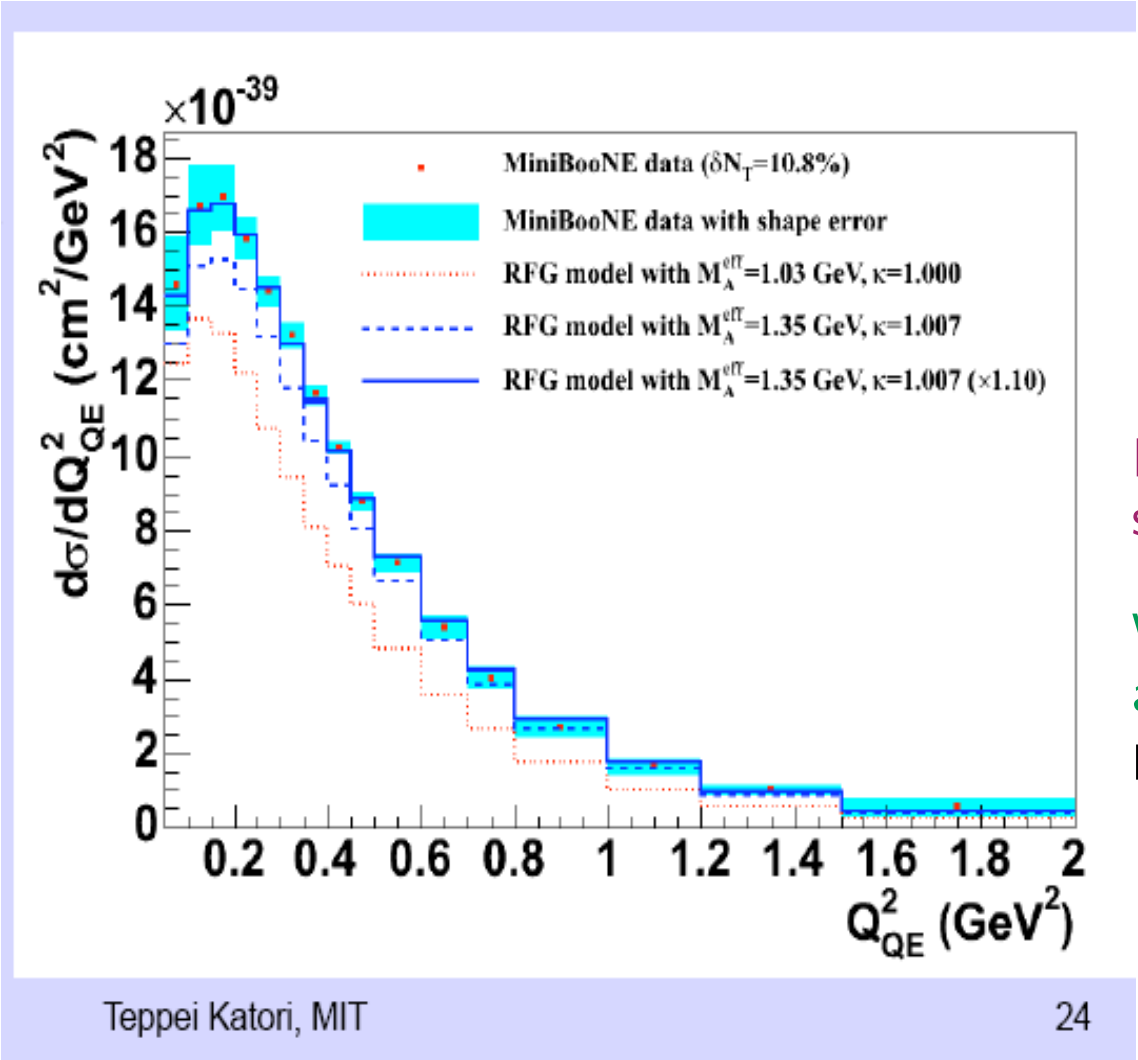



$R = (d\sigma/dQ^2)_{nuc}/(d\sigma/dQ^2)_{freenuc}$ versus $Q^2$ for neutrino scattering on $^{12}$C and for the four values of incoming neutrino energy: $\varepsilon_\nu$=0.5, 0.7, 1.2 and 2.5 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.

Range of $Q^2$ where $R \approx const$, (i.e. nuclear effects cannot modify the value of $M_A$) increases with neutrino energy




$R = (d\sigma/dQ^2)_{nuc}/(d\sigma/dQ^2)_{freenuc}$ versus $Q^2$ for antineutrino scattering on $^{12}$C and for the four values of incoming neutrino energy: $\varepsilon_\nu$=0.5, 0.7, 1.2 and 2.5 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.




NuInt09: MiniBooNE data

CC QE absolute cross section

Flux averaged single differential cross section.

What about handling of low $Q^2$ region, are there better alternatives to the $\kappa$ parameter ?

$d\sigma/dQ^2$ cross sections averaged over (anti)neutrino flux

# $\nu$-mode

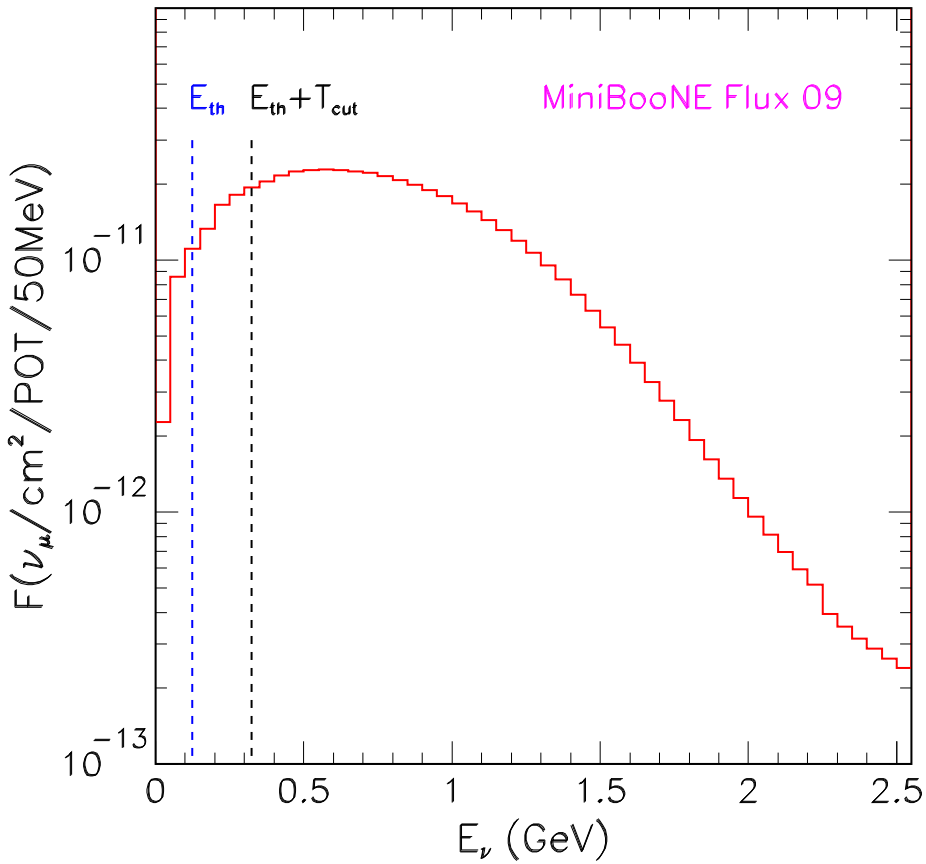
- Normalization of the flux: $N^{\nu}_{tot}(T_{cut}) = \int\limits_{\varepsilon_{min}}^{\varepsilon_{max}} [I^{\nu}_{\nu}(\varepsilon) + I^{\nu}_{\bar{\nu}}(\varepsilon)]d\varepsilon$

$$\varepsilon_{min} = E_{th} + T_{cut}$$

- Weights: $W^{\nu}_{\nu}(\varepsilon, T_{cut}) = I^{\nu}_{\nu}(\varepsilon)/N^{\nu}_{tot}(T_{cut})$ and $W^{\nu}_{\bar{\nu}}(\varepsilon, T_{cut}) = I^{\nu}_{\bar{\nu}}(\varepsilon)/N^{\nu}_{tot}(T_{cut})$
- $\left(d\sigma^{\nu(\bar{\nu})}/dQ^2\right)(Q^2, T_{cut}, \varepsilon) = \int\limits_{\omega_{min}(Q^2)}^{\omega_{cut}} [\left(d\sigma^{\nu(\bar{\nu})}/dQ^2 d\omega\right)(Q^2, \omega, \varepsilon)d\omega]$

$$\omega_{cut} = min\{\omega_{max}(Q^2), E_{\nu} - T_{cut} - m_{\mu}\}$$

- $\langle d\sigma/dQ^2\rangle^{\nu}(T_{cut}) = \int\limits_{\varepsilon_{min}(Q^2)}^{\varepsilon_{max}} [W^{\nu}_{\nu}(\varepsilon, T_{cut})\left(d\sigma^{\nu}/dQ^2\right)(T_{cut}, \varepsilon) + W^{\nu}_{\bar{\nu}}(\varepsilon, T_{cut})\left(d\sigma^{\bar{\nu}}/dQ^2\right)(T_{cut}, \varepsilon)]d\varepsilon$




MiniBooNE $\nu$-flux09 [Aguilar-Arevalo et al., PRD79,2009]

$$E_{th} = 125 \text{ MeV}$$

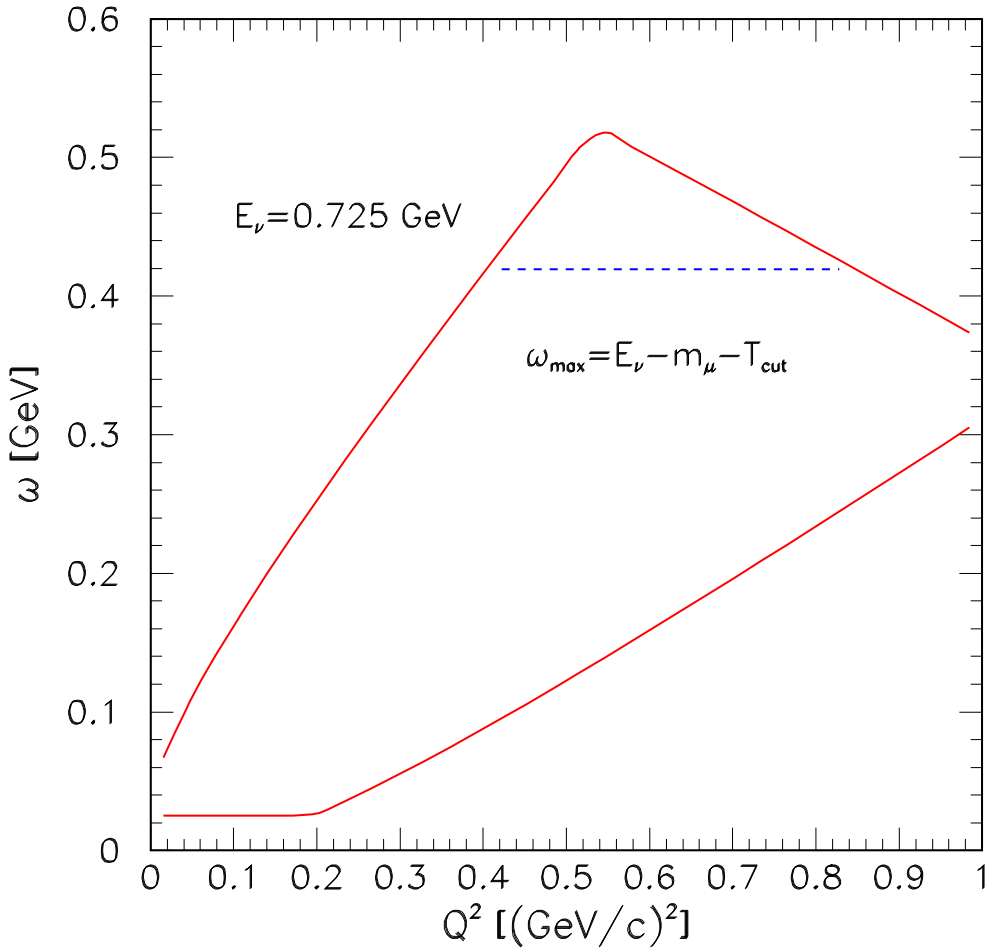
CUT:$\mu$-track length $> 1m$, $T_{cut} = 200$ MeV

$$N_{tot}(T_{cut} = 200)/N_{tot}(T_{cut} = 0) = 0.873$$

$$W(T_{cut} = 200) > W(T_{cut} = 0)$$

At $T_{cut} = 200$ the contribution of the high energy bins to $d\sigma/dQ^2$ is higher.

The effect of $T_{cut}$ on the normalization of the $\nu$-flux depends on $\nu$-spectrum (peak position).




## Kinematic region

$E_{th} = 125$ MeV

CUT:$\mu$-track length $> 1m$, $T_{cut} = 200$ MeV

$\omega_{cut} = min\{\omega_{max}(Q^2), E_\nu - T_{cut} - m_\mu\}$

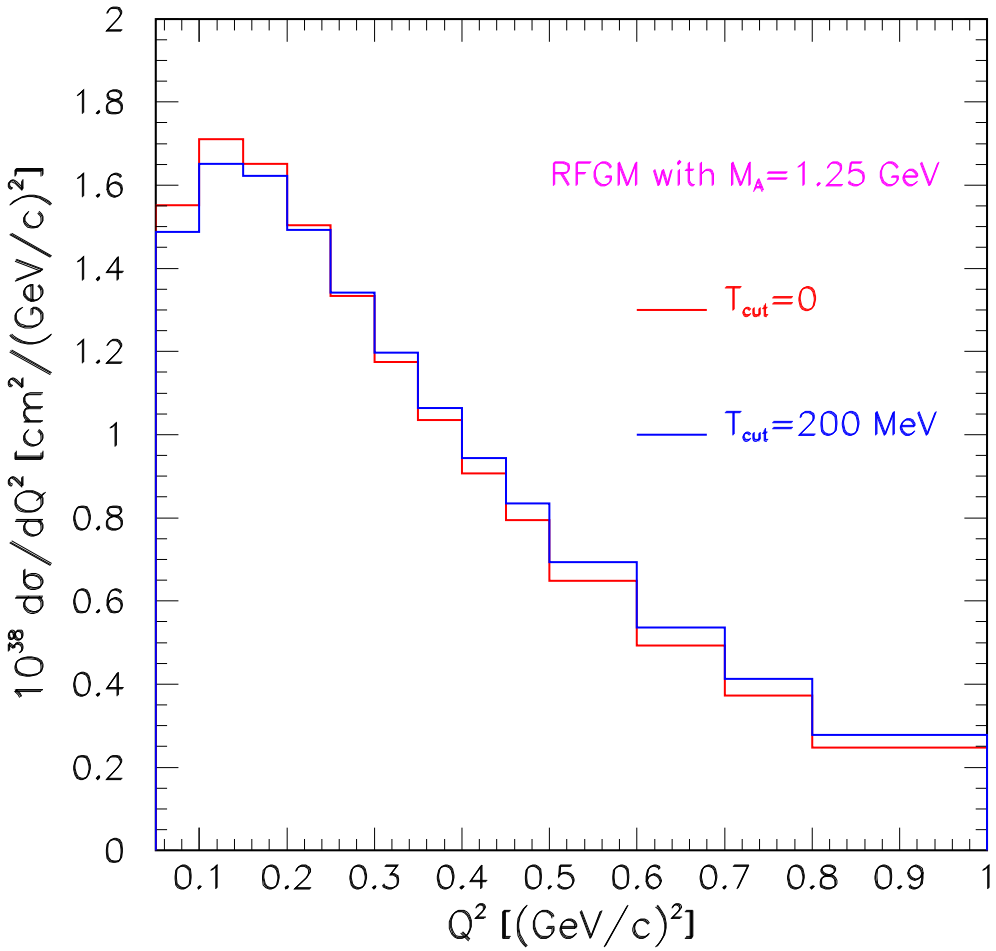
Method 1

$(d\sigma/dQ^2)_{data}$ for $T_{cut} = 200$ MeV

Method 2

$(d\sigma/dQ^2)_{data} + (d\sigma/dQ^2)_{extrapol}$ for all allowed kinematical range

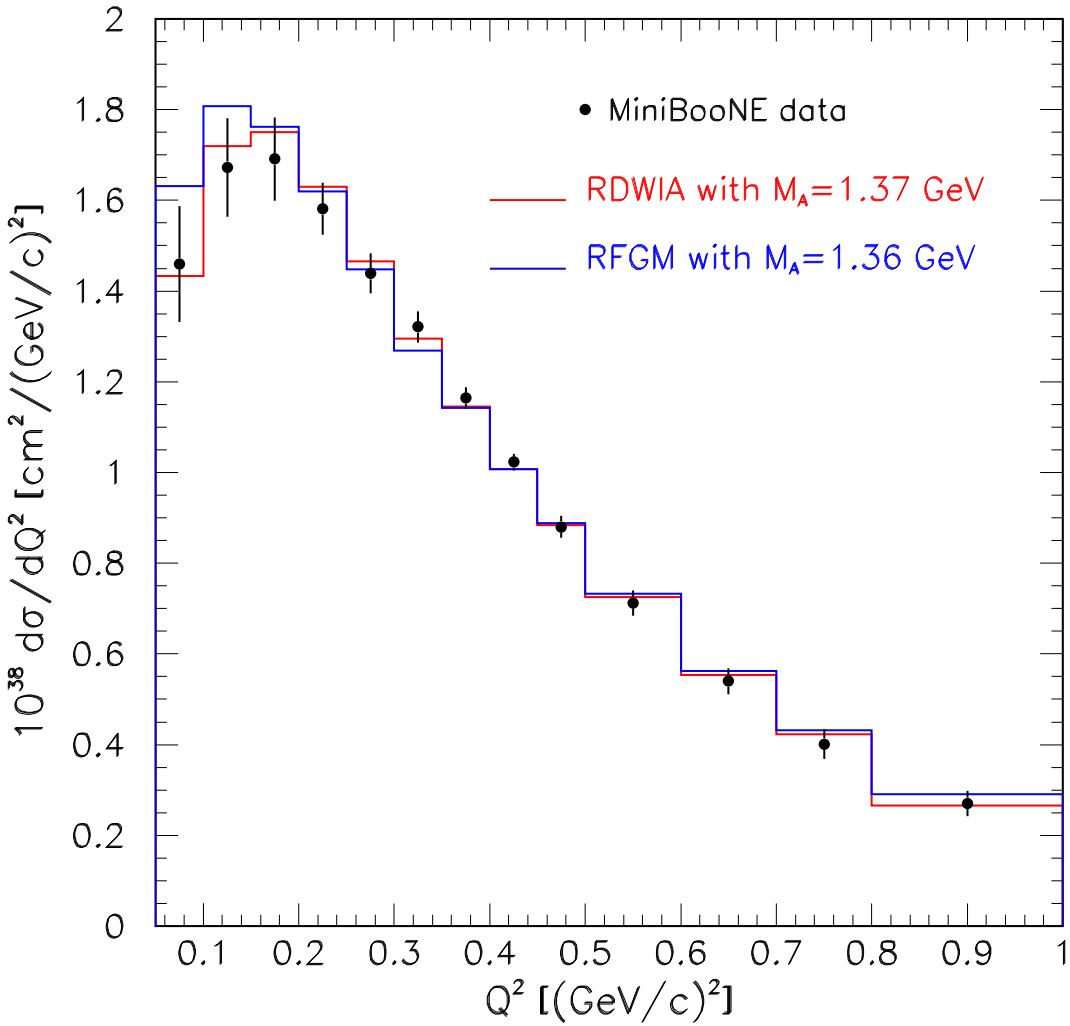
$(d\sigma/dQ^2)_{extrapol}$ is obtained using a Model with the values of parameters $(M_A, \kappa, ...)$ which were extracted from $(d\sigma/dQ^2)_{data}$ or the events distribution.




## Effect of the flux averaging of $d\sigma/dQ^2$ cross section

Method 1 or Method 2

$M_A$ is fixed but slopes of the $Q^2$-distributions are different.




## NuInt09: MiniBooNE data

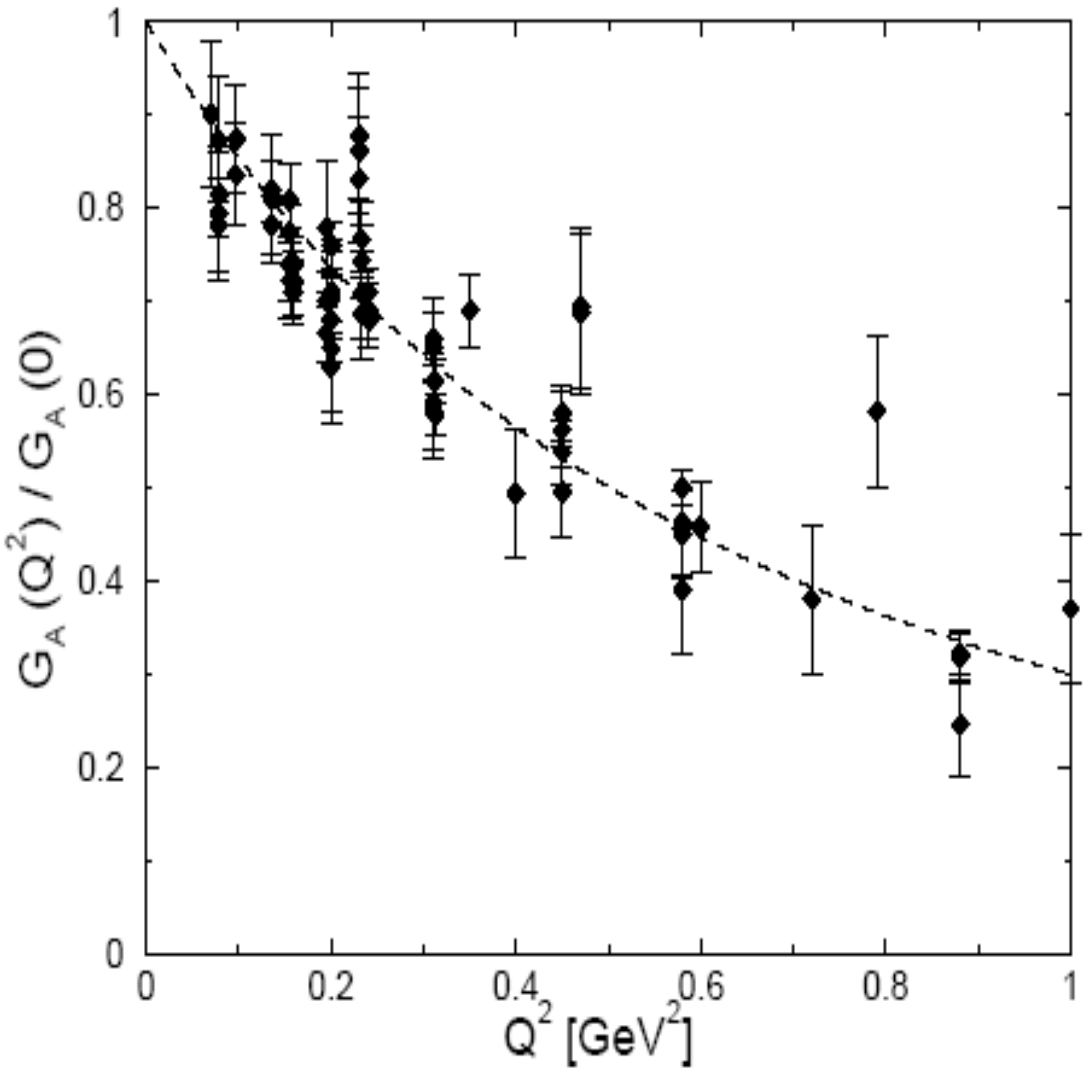
Flux averaged single differential cross section. Comparison with the RDWIA and RFGM results.

Do we understand the difference in $M_A$ values from different experiments?

MiniBooNE with $\kappa = 1.007$:
$M_A = 1.35 \pm 0.17$(stat+syst)
RDWIA: $M_A = 1.37 \pm 0.05$
RFGM: $M_A = 1.36 \pm 0.05$
(with shape error only)

MINOS: RFGM $(Q^2 > 0.3GeV^2)$

$$M_A = 1.26^{+0.09(fit)+0.12(sys)}_{-0.1(fit)-0.14(sys)}$$




## Is the axial form factor dipole ?

Experimental data for normalized axial form factor extracted from pion electroproduction experiments.
For orientation, the dashed line shows a dipole fit with an axial mass $M_A$ =1.1 GeV
[V.Bernard et al., 2001]

# Axial form factor extraction from neutrino experiments.

## The difference

$$\frac{d\sigma^{\nu}}{dQ^2} - \frac{d\sigma^{\overline{\nu}}}{dQ^2} = \frac{G^2}{\pi}\cos^2\theta_c \frac{Q^2}{m\varepsilon_i}\left(1 - \frac{Q^2}{4m\varepsilon_i}\right)(F_V + F_M)F_A$$

is proportional to $F_A$ and decreases with neutrino energy. In the range of $\varepsilon_i \sim 0.5 \div 1$ GeV it can be used for measuring of the axial form factor $F_A$ at $Q^2 < 1$ GeV.

# Do we need to start thinking beyond $M_A$ ?

- The axial form factor admits the following expansion for small momentum transfer $Q^2 < 1\ GeV^2$

$$G_A(Q^2) = g_A\Big(1 - \frac{1}{6} < r_A^2 > Q^2\Big),$$

where $g_A$ =1.267 and $< r_A^2 >^{1/2}$ is the nucleon axial radius.

- Radius of the nucleon axial charge distribution

$$< r_A^2 >= \frac{6}{g_A}\frac{dG_A(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

- For dipole fit on can express the axial radius in term of the axial mass

$$< r_A^2 >= 12/M_A^2$$

The nucleon size as deduced from electron scattering experiment is $r \approx 0.85 fm$. The axial charge radius $0.51 < r_A < 0.66 fm$ for $1.032 < M_A < 1.35 GeV$

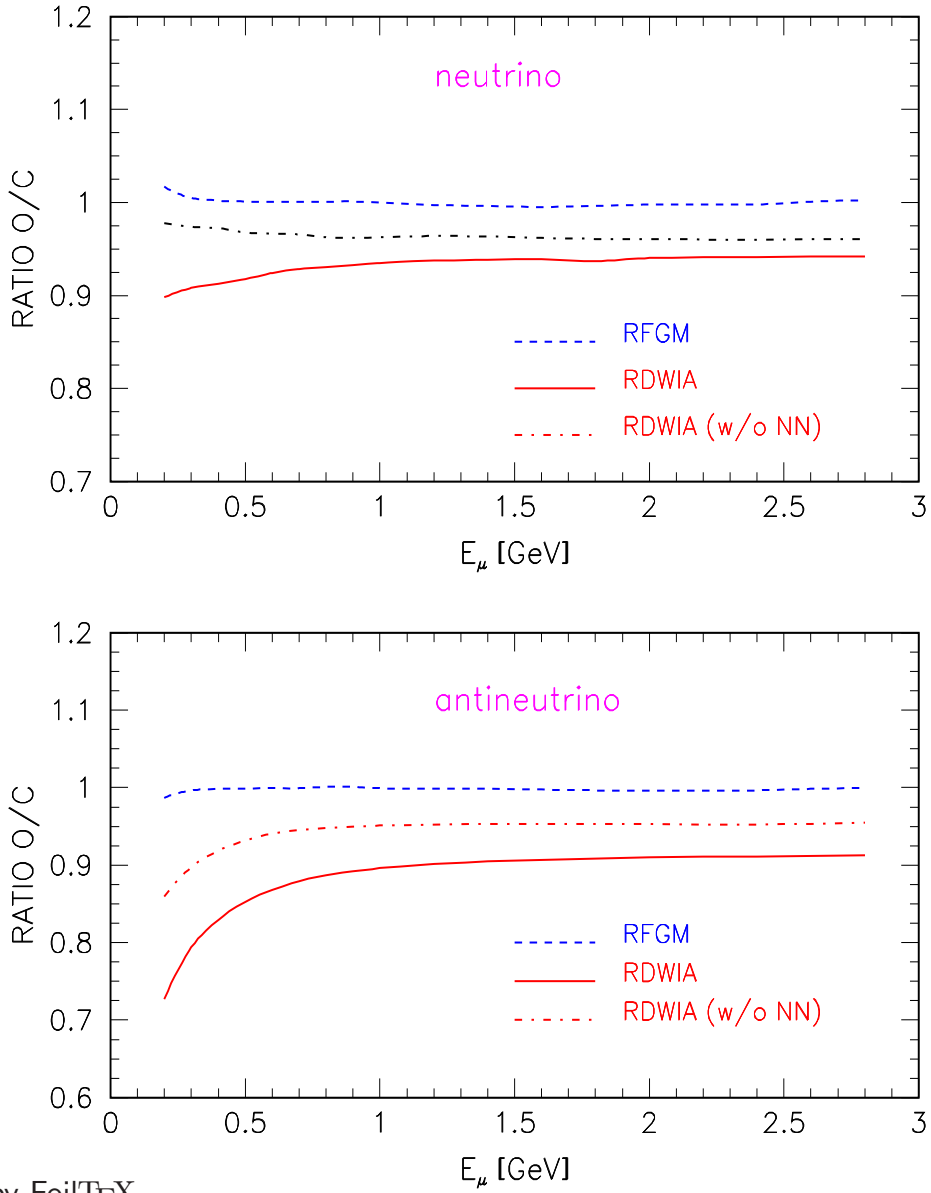
## How $NN$-correlation effects can change the total CC QE $\nu A$- cross section?

Average occupancy of nuclear shells:

Oxygen: $\overline{S}_O$ =75% and missing strength is 25%

Carbon: $\overline{S}_C$ =89% and missing strength is 11%

Missing strength is attributed to the short-range NN-correlations in ground state.




Ratio of the total cross section per neutron R=O/C versus neutrino energy for neutrino (top panel) and antineutrino (bottom panel) scattering off $^{16}$O and $^{12}$C. The RFGM predicts almost identical values cross section for $^{16}$O and $^{12}$C. In the RDWIA approach the total cross section calculated for oxygen is lower than that for carbon. This ration is $\sim 0.9$. The ratio without NN-correlation contributions, i.e. $\overline{S}$=1 for $^{16}$O and $^{12}$C is about 0.95 at $\varepsilon_\nu$ >1 GeV.

# Summary

QE CC $\nu(\bar{\nu})^{12}$C cross sections were studied in different approaches.

- The reduced and inclusive cross sections for neutrino and electron scattering off $^{12}$C were tested against $^{12}C(e,e'p)$ and $^{12}C(e,e')$ data.
  - ⋆ In RDWIA the cross sections for $\nu(\bar{\nu})$ scattering are similar to those of electron scattering and in a good agreement with data.
  - ⋆ The RFGM fails completely when compared to exclusive cross section data.
- $d\sigma/dQ^2$ cross sections were calculated for different neutrino energies and the nuclear effects on the shape of the $Q^2$ distribution were estimated.
- The flux averaged $d\sigma/dQ^2$ cross section was calculated within the RDWIA and RFGM models and compared with MiniBooNE data.
  - ⋆ At low $Q^2 < 0.2GeV^2$ the RDWIA result agree well with data, whereas the Fermi gas model overestimates the inclusive cross section.
  - ⋆ At higher $Q^2$ a good math between the RDWIA (RFGM) calculation with $M_A$ =1.36(1.37) GeV and measured $Q^2$ distribution is observed at $Q^2 < 1GeV^2$.
  - ⋆ The extracted values of $M_A$ are in agreement within errors.

- The CC QE total cross sections (per nucleon in target) for (anti)neutrino scattering on the oxygen and carbon targets were compared. It was found that the cross section calculated within the RDWIA for oxygen are lower than for carbon and effects of the NN-correlations increases the difference.