

# QE neutrino CC scattering cross section on carbon

Anatoli Butkevich (INR, Moscow)

Neutrino Department Seminar, Fermilab, USA, October 29, 2008

- Motivations.
- Nuclear model.
- Results.
- Summary.

## Motivations

- Carbon (liquid scintillator, plastic) and Oxygen (water) are widely used targets in fully active, segmented, and fine grained neutrino detector.
- There are two strategies for neutrino oscillation experiment.
  - (★) One option for a near detector design is to make the near detector as similar as possible to the far detector to reduce systematical uncertainties (MINOS experiment).
  - (★) A different option for a near detector is one that is much more segmented and fine-grained to measure the flux and cross sections as independently as possible and then that information is correctly extrapolated to the far detector. Because the near and far detectors are not necessarily the same target material, the nuclear effects on the cross sections must be taken into account (T2K experiment and O/C ratio).
- How nuclear effects can change the slope in the  $Q^2$  distribution of the CCQE events and impact on the axial mass ( $M_A$ ) analysis.

## Formalism of the quasi-elastic scattering and nuclear model

Within the Relativistic Distorted-Wave Impulse Approximation (RDWIA) with LEA code [J.J. Kelly 1995], Plane-Wave Impulse Approximation (PWIA), and Relativistic Fermi Gas Model (RFGM) we study (anti)neutrino scattering off Carbon.

For more details of the present approaches see:

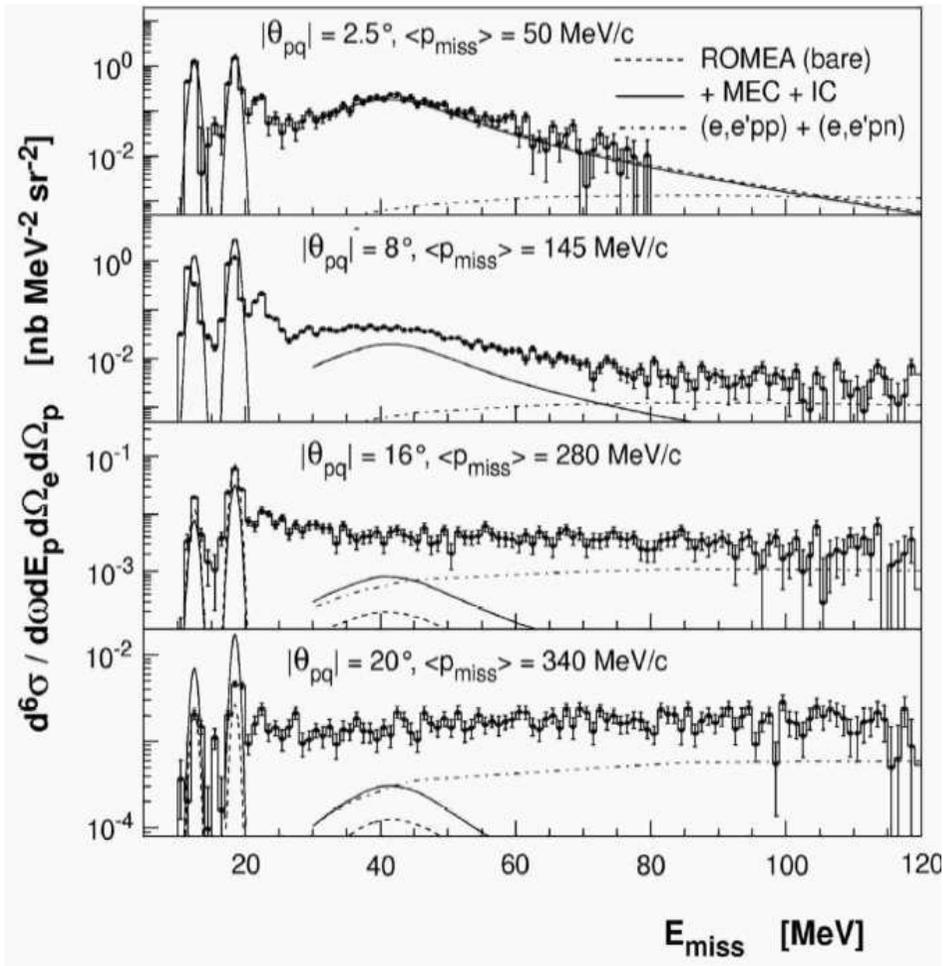
A.Butkevich, S.Kulagin, Phys.Rev.C76:045502,2007;

A.Butkevich, Phys.Rev.C78:015501,2008

In our calculations we employ:

- ★  $\Gamma_2$  off shell single nucleon vertex function in Coulomb gauge
- ★ MMD form-factors [P. Mergell et al. 1996]
- ★ SH bound nucleon wave function for carbon [C. Horowitz 1991]
- ★ EDAD1 relativistic optical potential [E.Cooper et al. 1993]
- ★ Dipole approximation of axial the form factor with  $M_A = 1.032$  GeV

## Model



Shell occupancy for Oxygen:

$$S(P_{1/2})=0.7$$

$$S(P_{3/2})=0.66$$

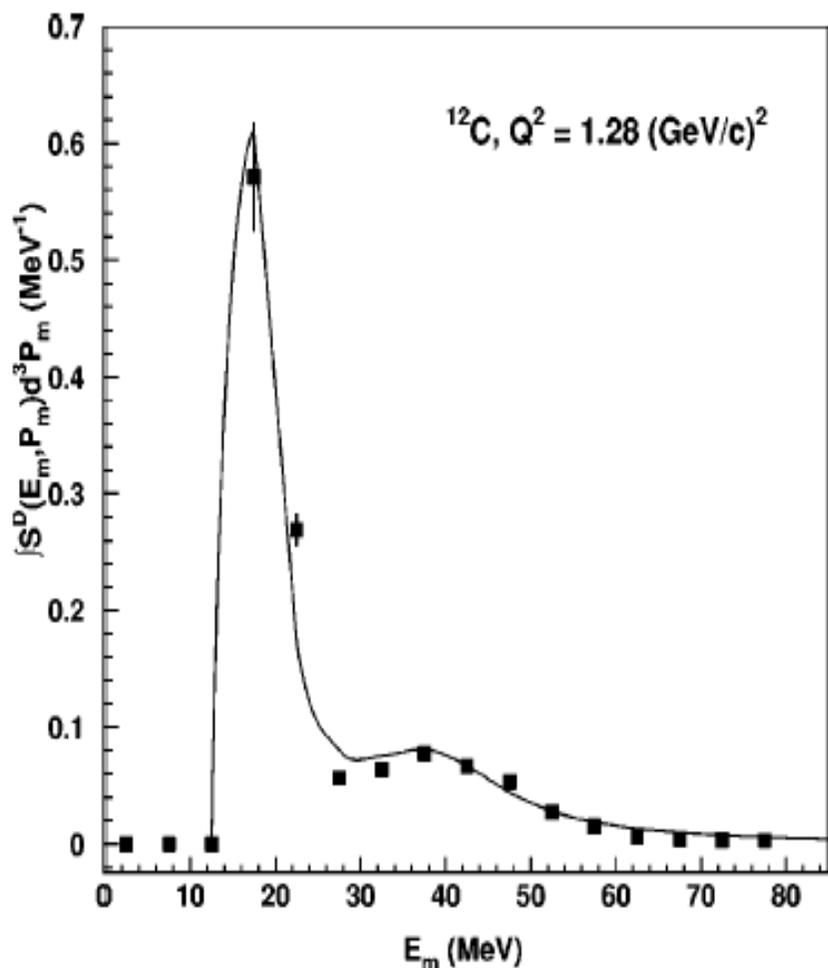
$$S(S_{1/2})=1$$

Average occupancy of nuclear shells  $\bar{S}=0.75$   
 [supported by JLab measurement K. G. Fissum et al. (2005)]

Missing Energy

Neutron: 15.7 MeV ( $1p_{1/2}$ ), 21.2 MeV ( $1p_{3/2}$ ),  
 42.9 MeV ( $1s_{1/2}$ )

Proton:  $E_m(1p_{1/2})=12.1$  MeV,  $E_m(1p_{3/2})=18.4$  MeV,  
 $E_m(1s_{1/2})=40.1$  MeV



Shell occupancy for Carbon:

$$S(P_{3/2})=0.84$$

$$S(S_{1/2})=1$$

Average occupancy of nuclear shells  $\overline{S}=0.89$

[supported by JLab measurement D. Dutta et al. (2003)  
and [J.J.Kelly (2004)]

Missing Energy

Neutron: 17.9 MeV ( $1p_{3/2}$ ), 39.8 MeV ( $1s_{1/2}$ )

Proton:  $E_m(1p_{3/2})=16$  MeV,  $E_m(1s_{1/2})=37.9$  MeV

Missing strength can be attributed to the short-range NN-correlations in ground state.

## Results

Reduced  $^{12}\text{C}(e, e'p)$  cross sections as test of neutrino-nuclear exclusive cross sections

Lepton CC QE exclusive reaction

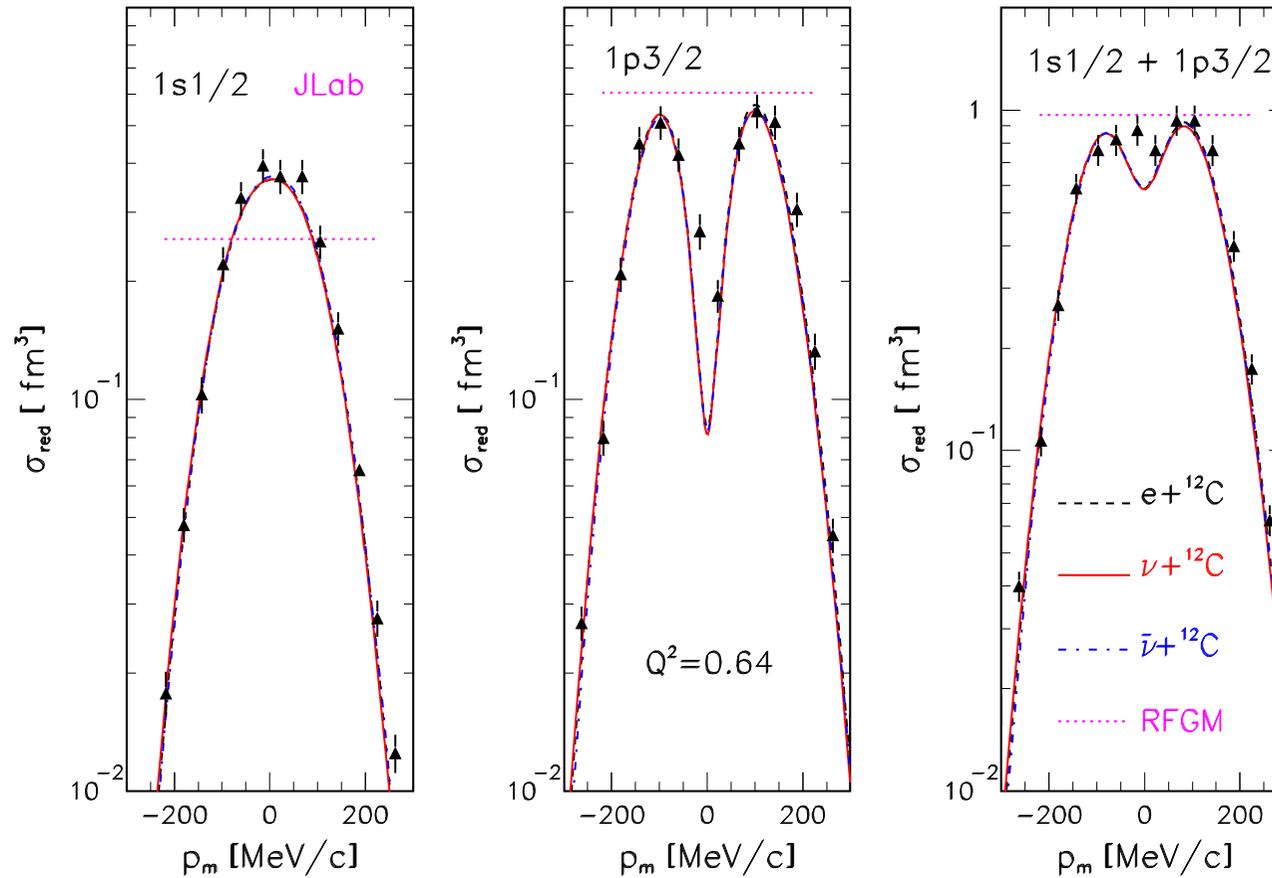
$$l(k_i) + A(p_A) \rightarrow l'(k_f) + N(p_x) + B(p_B),$$

Reduced cross section (measure the strength of nuclear effects)

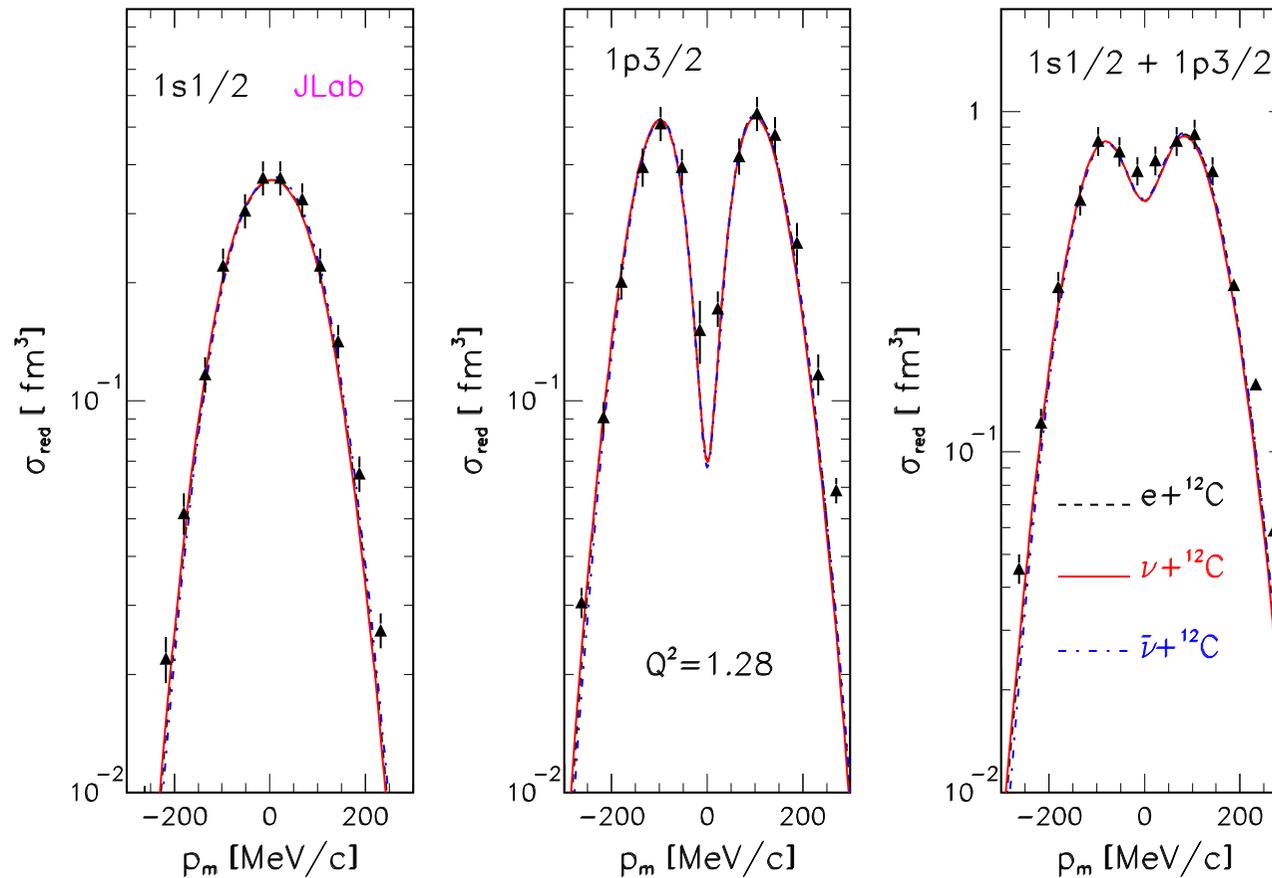
$$\sigma_{red} = \frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} / K \sigma_{lN},$$

where  $d^5\sigma/d\varepsilon_f d\Omega_f d\Omega_x$  is differential exclusive cross section,  $\varepsilon_f, \Omega_f$  are energy and solid angle of the scattered lepton,  $\Omega_x$  is solid angle for ejectile nucleon momentum,  $K$  is kinematical phase space factors and  $\sigma_{lN}$  is elementary cross section for the lepton scattering from moving free nucleon.

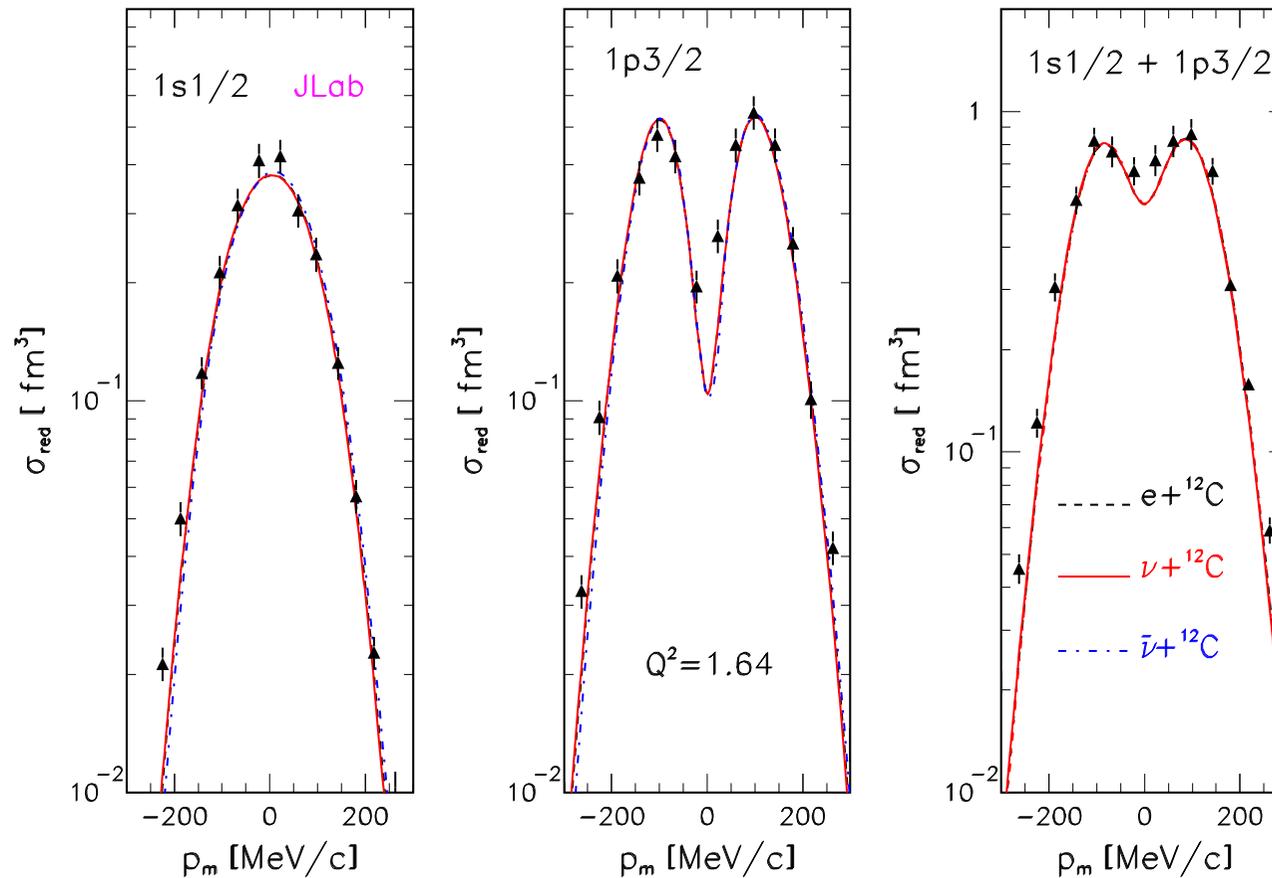
$\sigma_{red}$  and should be similar for electron and neutrino scattering (apart from small differences due to FSI effects for electron and neutrino induced reactions).



Comparison of the RDWIA and RFGM calculations for electron, neutrino, and antineutrino reduced cross sections for the removal of nucleon from  $1p$  and  $1s$  shells of  $^{12}\text{C}$  as a function of missing momentum. The cross sections were calculated for JLab [D.Dutta et al. (2003)] kinematics:  $\varepsilon_i = 2.445\text{GeV}$ ,  $Q^2 = 0.64(\text{GeV}/c)^2$  and  $T_x = 350\text{MeV}$ . The RFGM predictions are completely off of the exclusive data (CCQE two-track events).

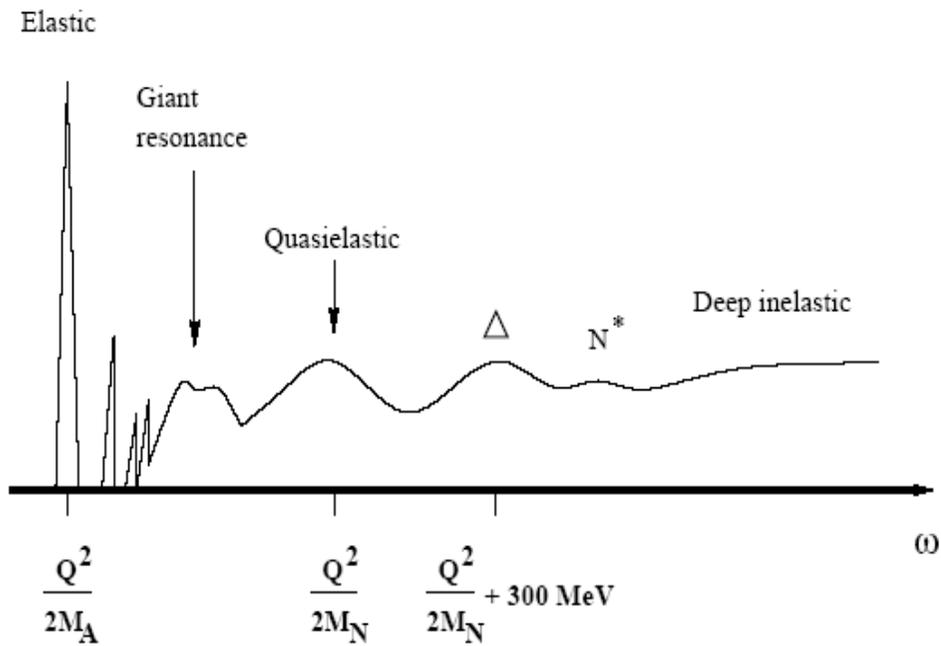


Comparison of the **RDWIA** calculations for electron, neutrino, and antineutrino reduced cross sections for the removal of nucleon from  $1p$  and  $1s$  shells of  $^{12}\text{C}$  as a function of missing momentum. The cross sections were calculated for **JLab** [D.Dutta et al. (2003)] kinematics:  $\varepsilon_i = 2.445\text{GeV}$ ,  $Q^2 = 1.28(\text{GeV}/c)^2$  and  $T_x = 700\text{MeV}$ .

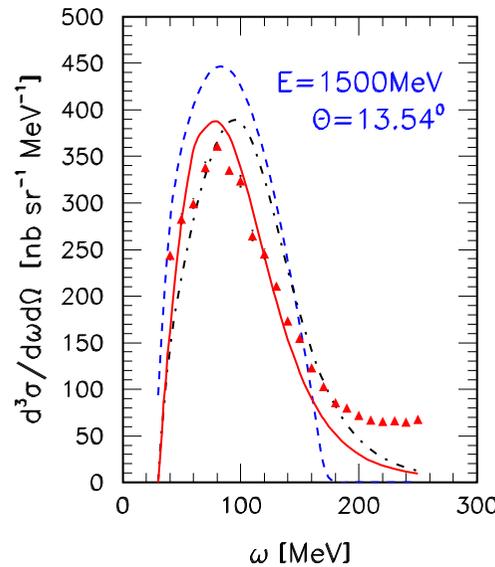
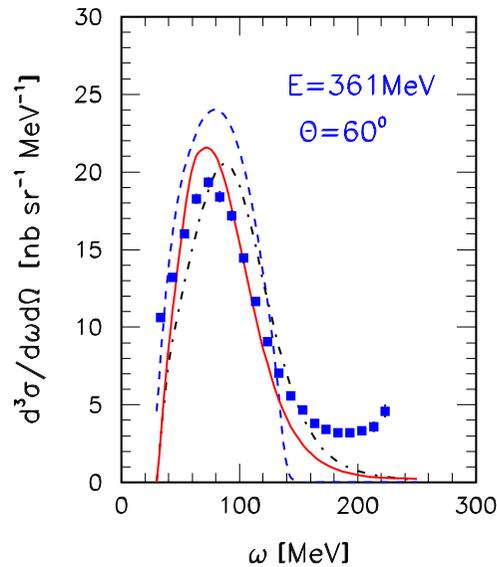
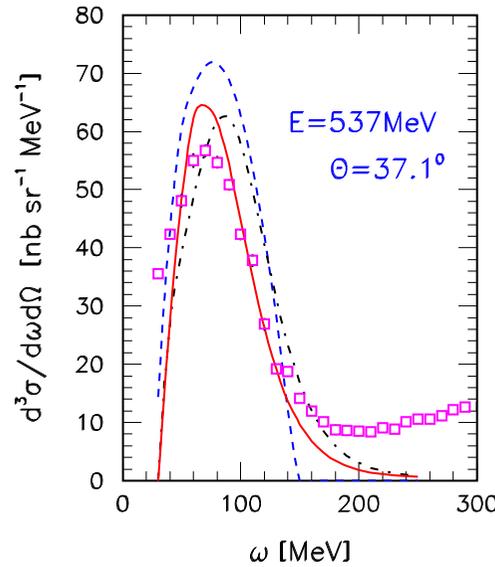
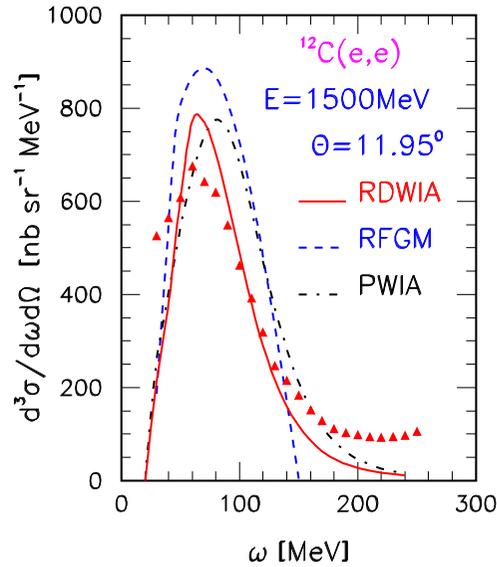


Comparison of the **RDWIA** calculations for electron, neutrino, and antineutrino reduced cross sections for the removal of nucleon from  $1p$  and  $1s$  shells of  $^{12}\text{C}$  as a function of missing momentum. The cross sections were calculated for JLab [D.Dutta et al. (2003)] kinematics:  $\varepsilon_i = 2.445\text{GeV}$ ,  $Q^2 = 1.84(\text{GeV}/c)^2$  and  $T_x = 970\text{MeV}$ .

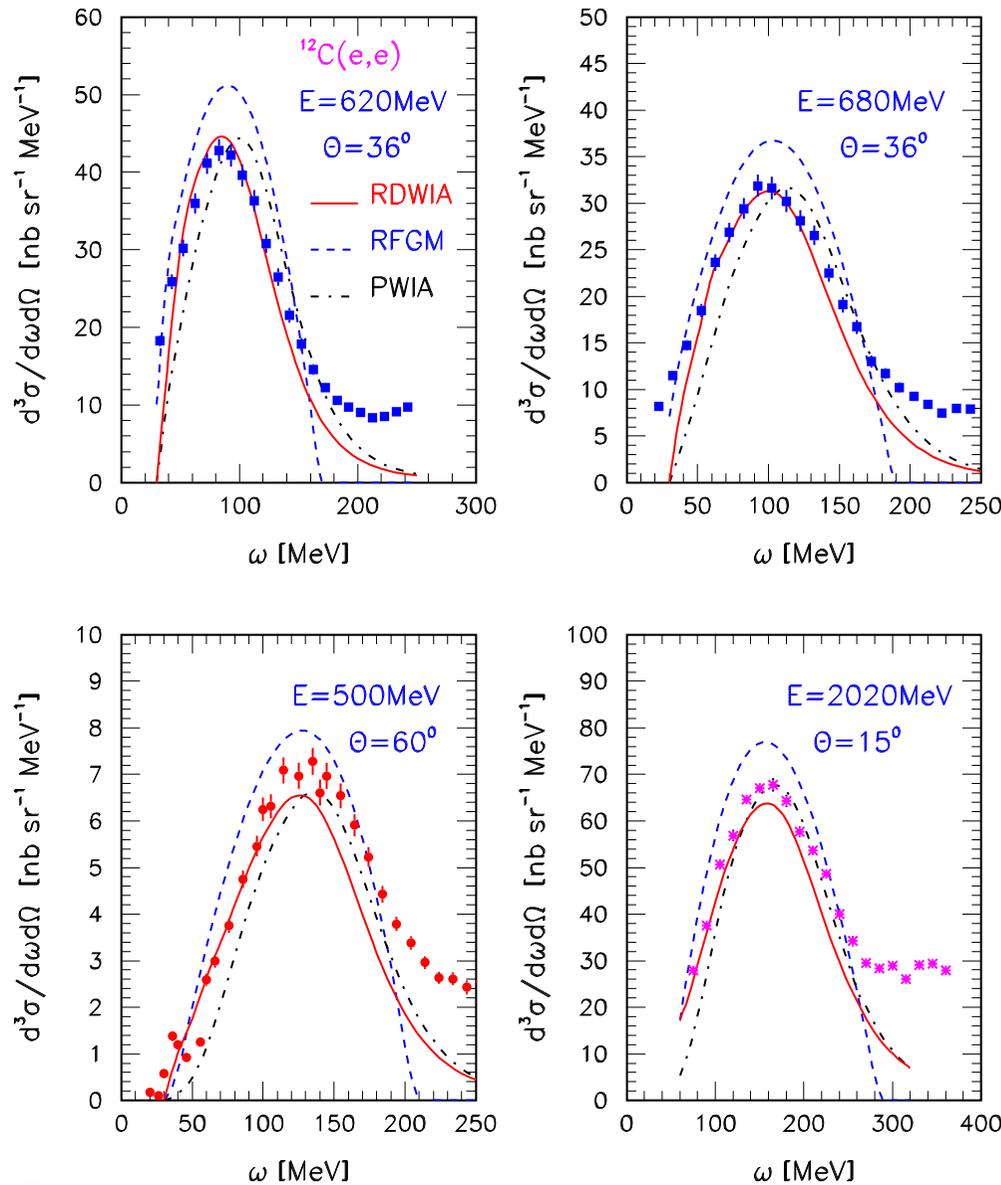
## Inclusive $^{12}\text{C}(e, e')$ cross sections



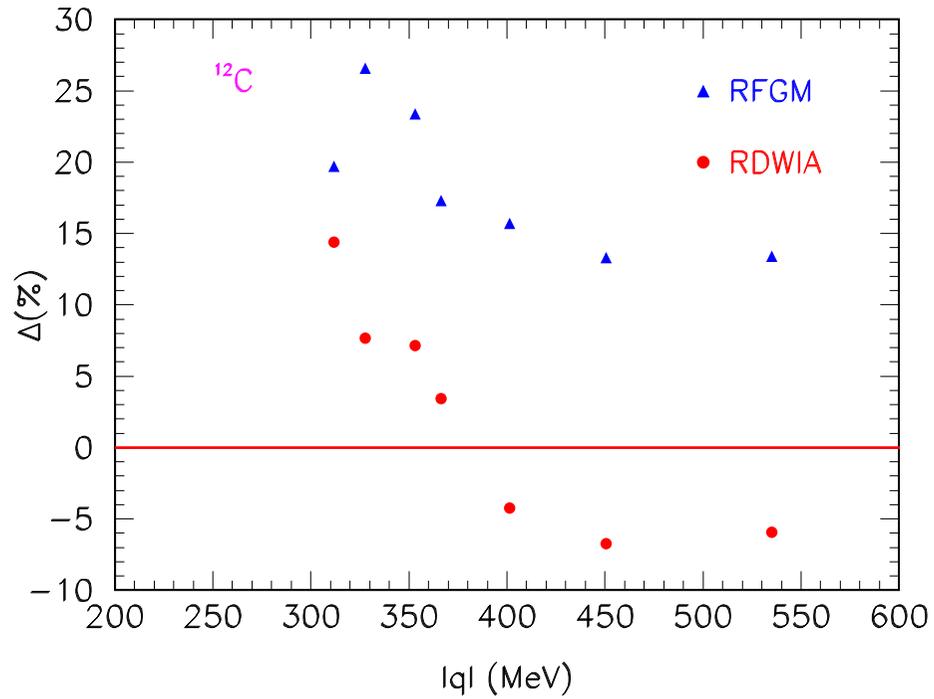
General inclusive spectrum at fixed  $Q^2$ .



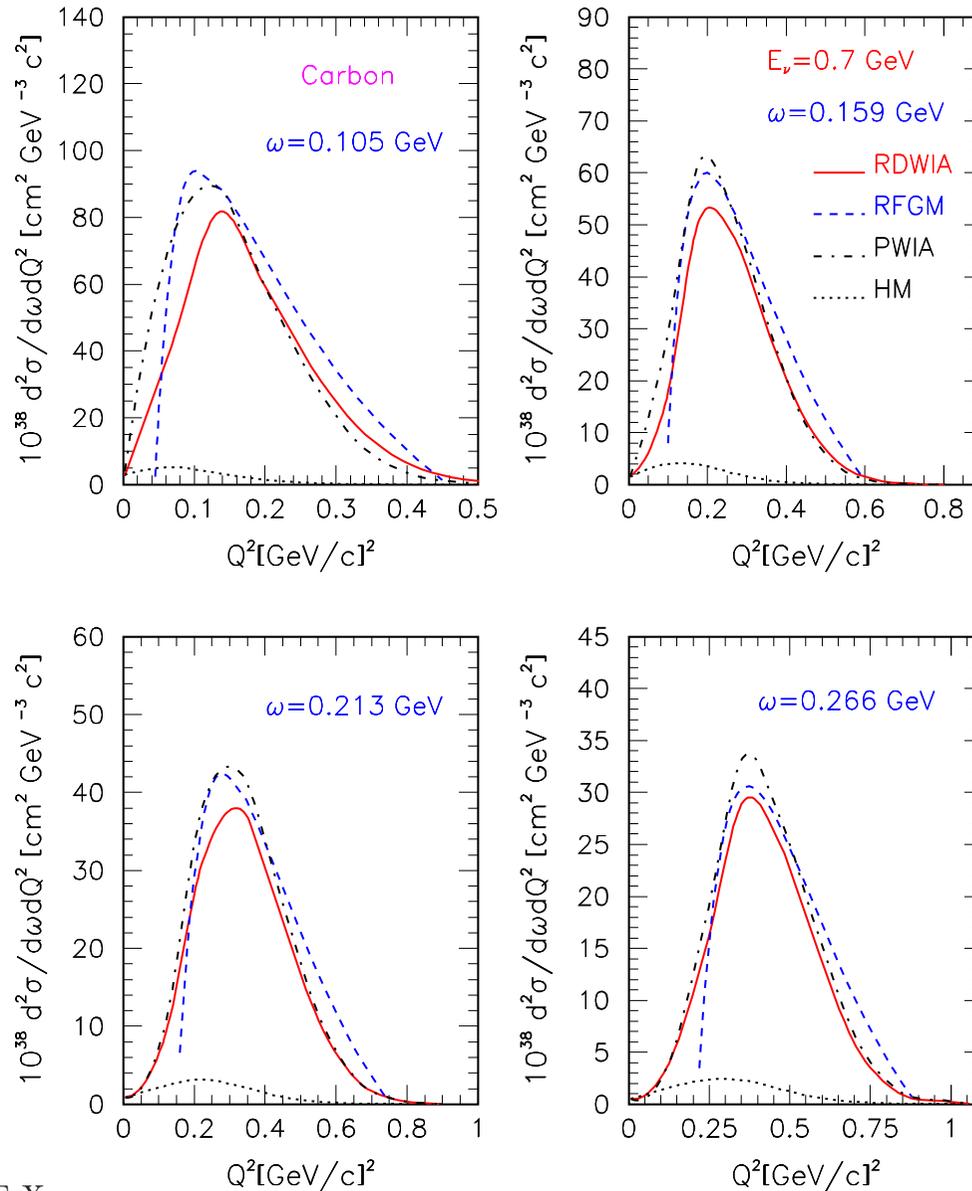
Inclusive cross section versus the energy transfer  $\omega$  for electron scattering on  $^{12}\text{C}$ . The data are from SLAC (filled triangles), Bates (open squares) and Saclay (filled squares). SLAC data are for electron beam energy  $E_e=1500$  MeV and scattering angle  $\theta=11.95^\circ$  and  $\theta=13.54^\circ$ . Bates data are for  $E_e=537$  MeV and  $\theta=37.1^\circ$ . Saclay data are for  $E_e=361$  MeV and  $\theta=32^\circ$ . The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.



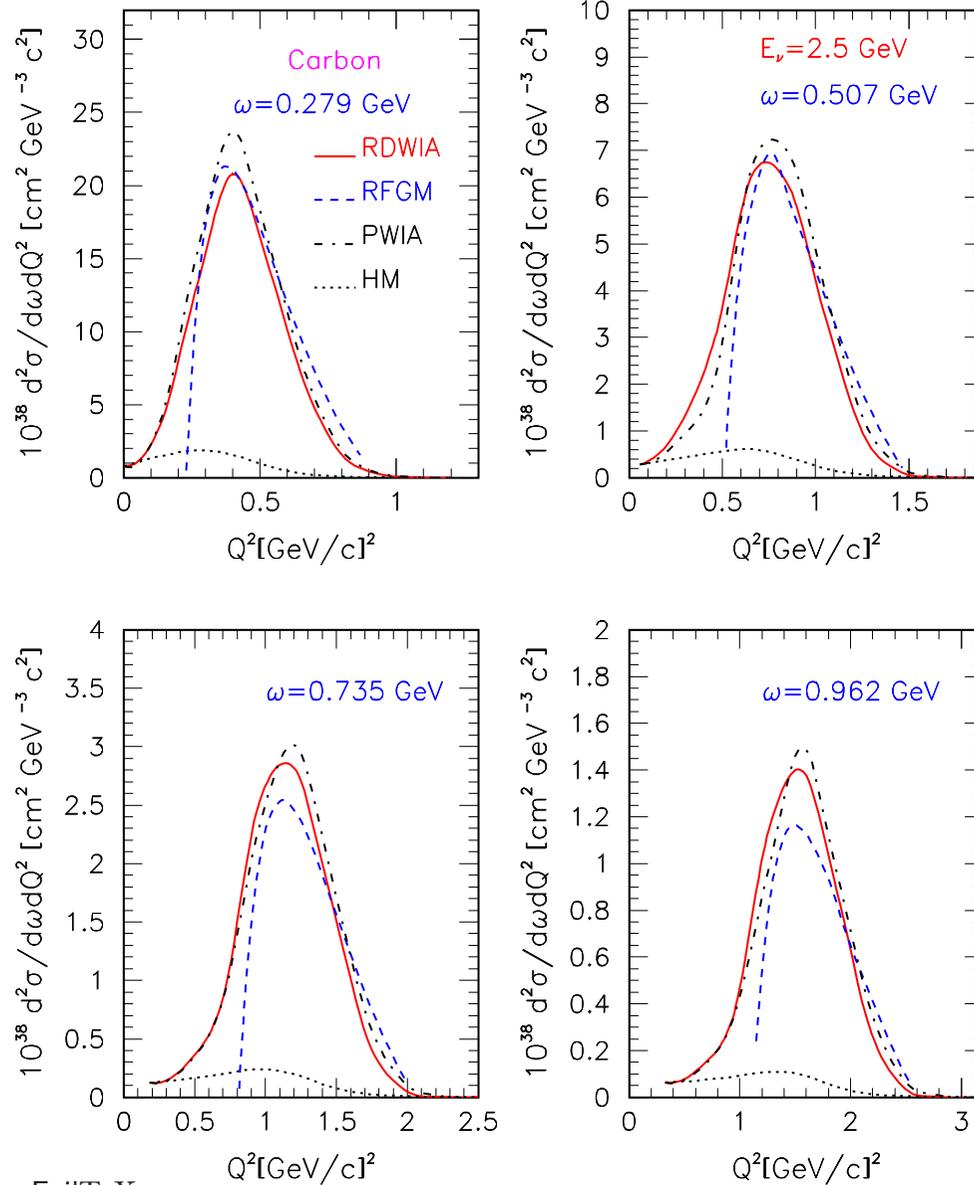
Inclusive cross section versus the energy transfer  $\omega$  for electron scattering on <sup>12</sup>C. The data are from SLAC (filled circles), JLab (stars) and Saclay (filled squares). SLAC data are for electron beam energy  $E_e=500$  MeV and scattering angle  $\theta=60^\circ$ . JLab data are for  $E_e=2020$  MeV and  $\theta=15^\circ$ . Saclay data are for  $E_e=620, 680$  MeV and  $\theta=36^\circ$ . The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.



Differences between the calculated and measured values of the inclusive cross sections at the maximum as a function of three-momentum transfer. Note that:  $Q^2 = \mathbf{q}^2 - \omega^2$



Inclusive cross section versus the four-momentum transfer  $Q^2$  for neutrino scattering on <sup>12</sup>C with energy  $\varepsilon_\nu = 0.7$  GeV and for the four values of energy transfer:  $\omega = 0.105, 0.159, 0.213$  and  $0.266$  GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the high-momentum component contribution to inclusive cross section.



Inclusive cross section versus  $Q^2$  for neutrino scattering on <sup>12</sup>C with energy  $\varepsilon_\nu = 2.5$  GeV and for the four values of energy transfer:  $\omega = 0.279$ , 0.507, 0.735 and 0.962 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the high-momentum component contribution to inclusive cross section.

$d\sigma/dQ^2$  cross sections

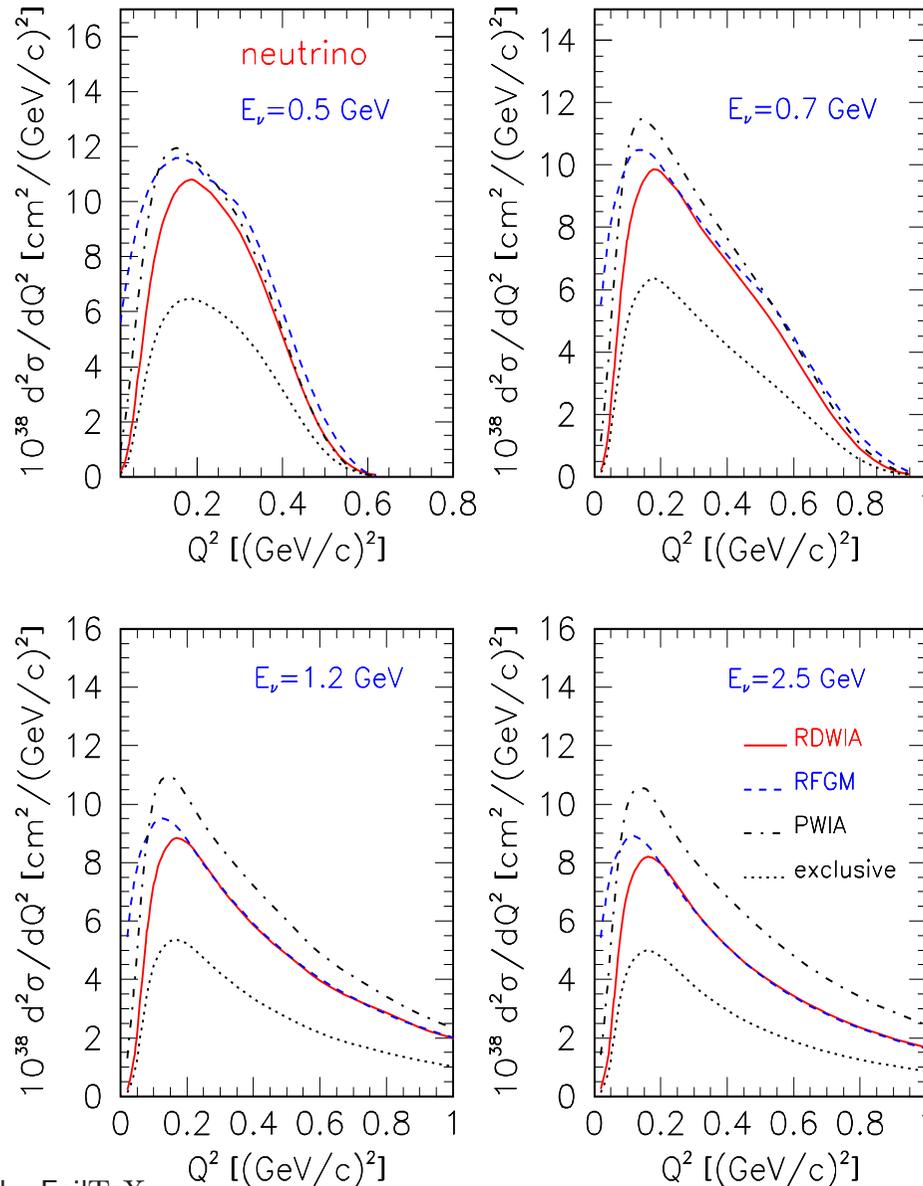
- $d^2\sigma/dQ^2$  vs  $Q^2$  at fixed neutrino energy

det: Ratio  $O/C = (d\sigma/dQ^2)_{neutron}(\text{*oxygen*}) / (d\sigma/dQ^2)_{neutron}(\text{*carbon*})$

- $M_A$  extraction and nuclear effect

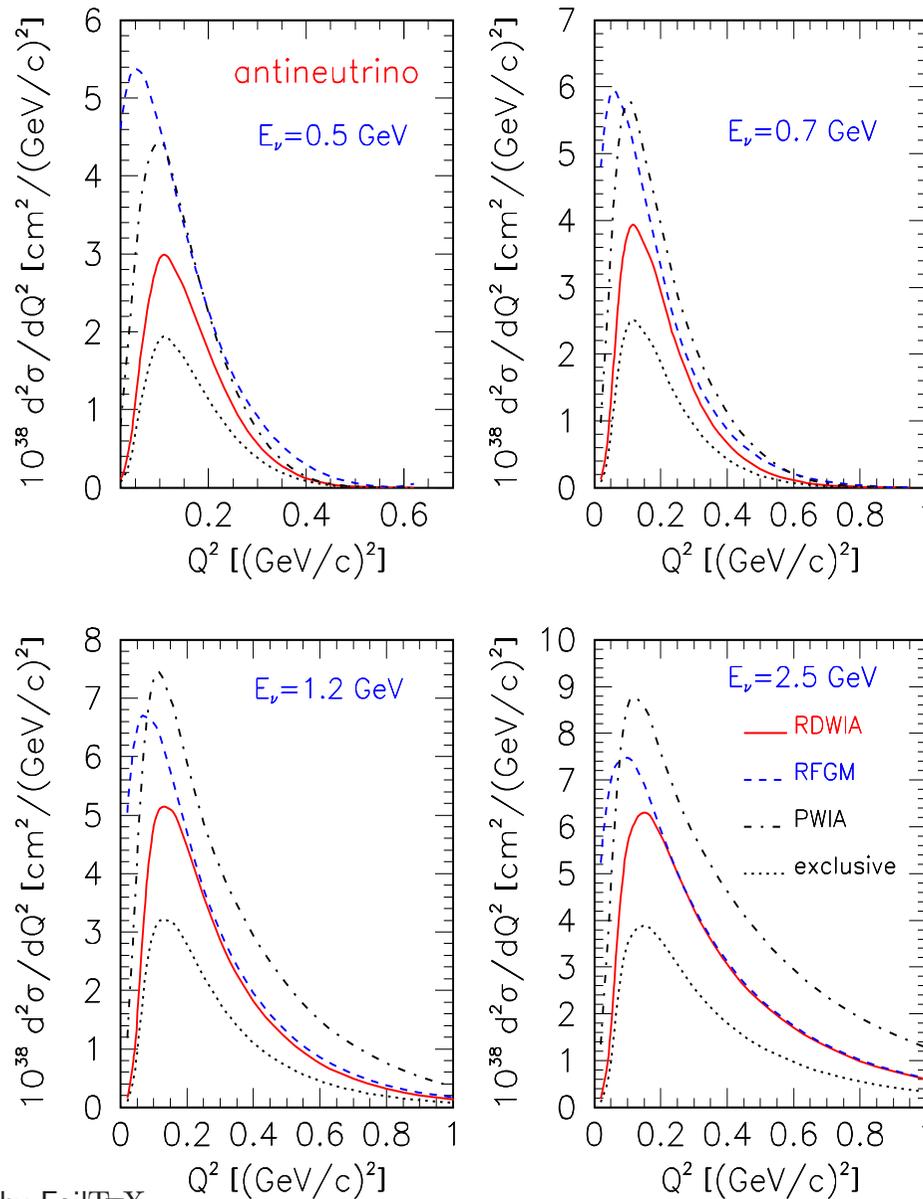
det:  $(d\sigma/dQ^2)_{nucl} = (d\sigma/dQ^2) / N(Z)$

det:  $R = (d\sigma/dQ^2)_{nuc} / (d\sigma/dQ^2)_{freenuc}$



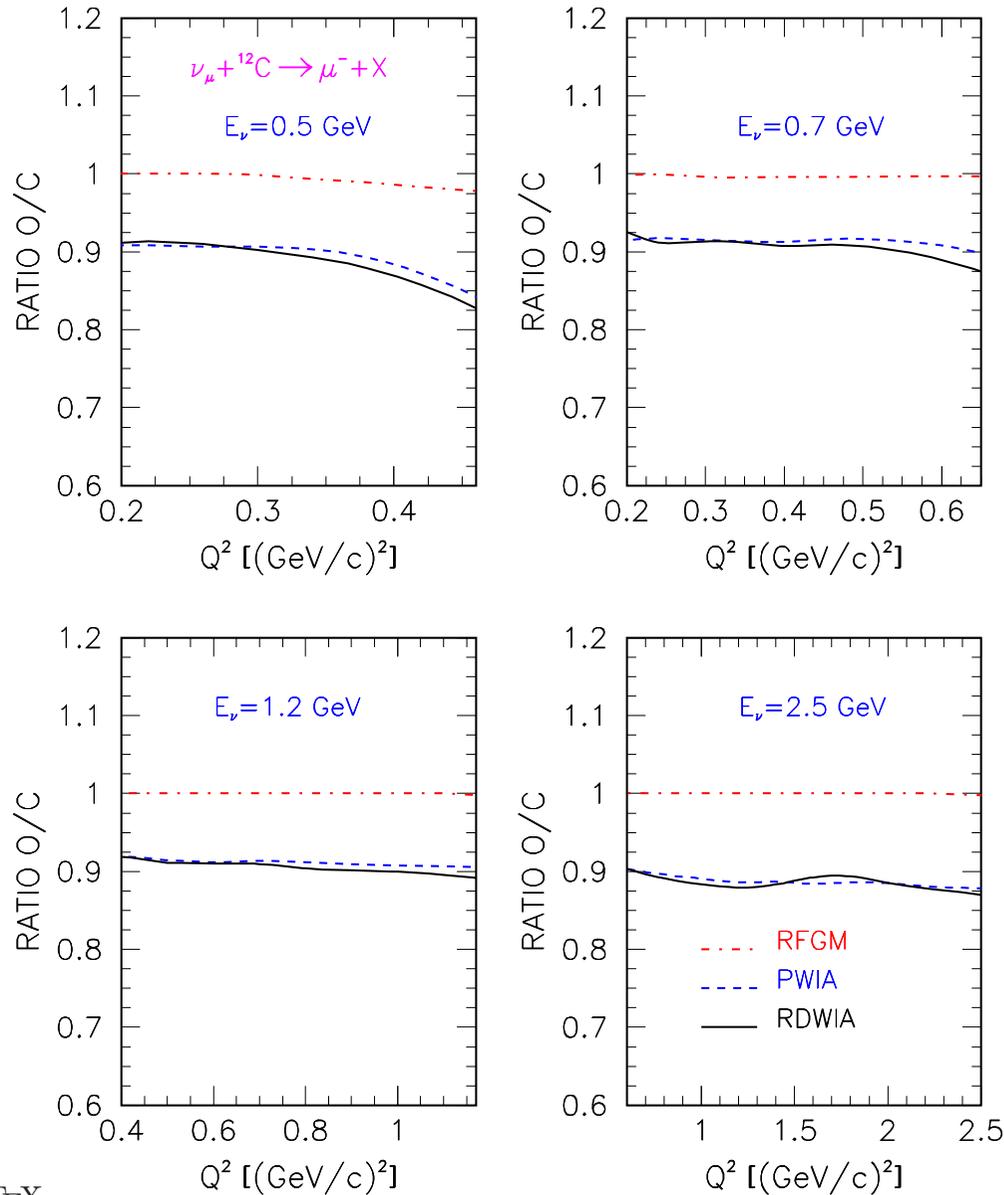
Inclusive cross section versus  $Q^2$  for neutrino scattering on  $^{12}\text{C}$  and for the four values of incoming neutrino energy:  $\varepsilon_\nu = 0.5, 0.7, 1.2$  and  $2.5$  GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the RDWIA result for the exclusive reaction (two-track events).

In the region  $< 0.2 (\text{GeV}/c)^2$  the RFGM results are higher than those obtained in the RDWIA and at  $Q^2 = 0.1 (\text{GeV}/c)^2$  the difference is about  $\sim 36\%$  for  $\varepsilon_\nu = 0.5$  GeV and  $\sim 27\%$  for  $\varepsilon_\nu = 2.5$  GeV.

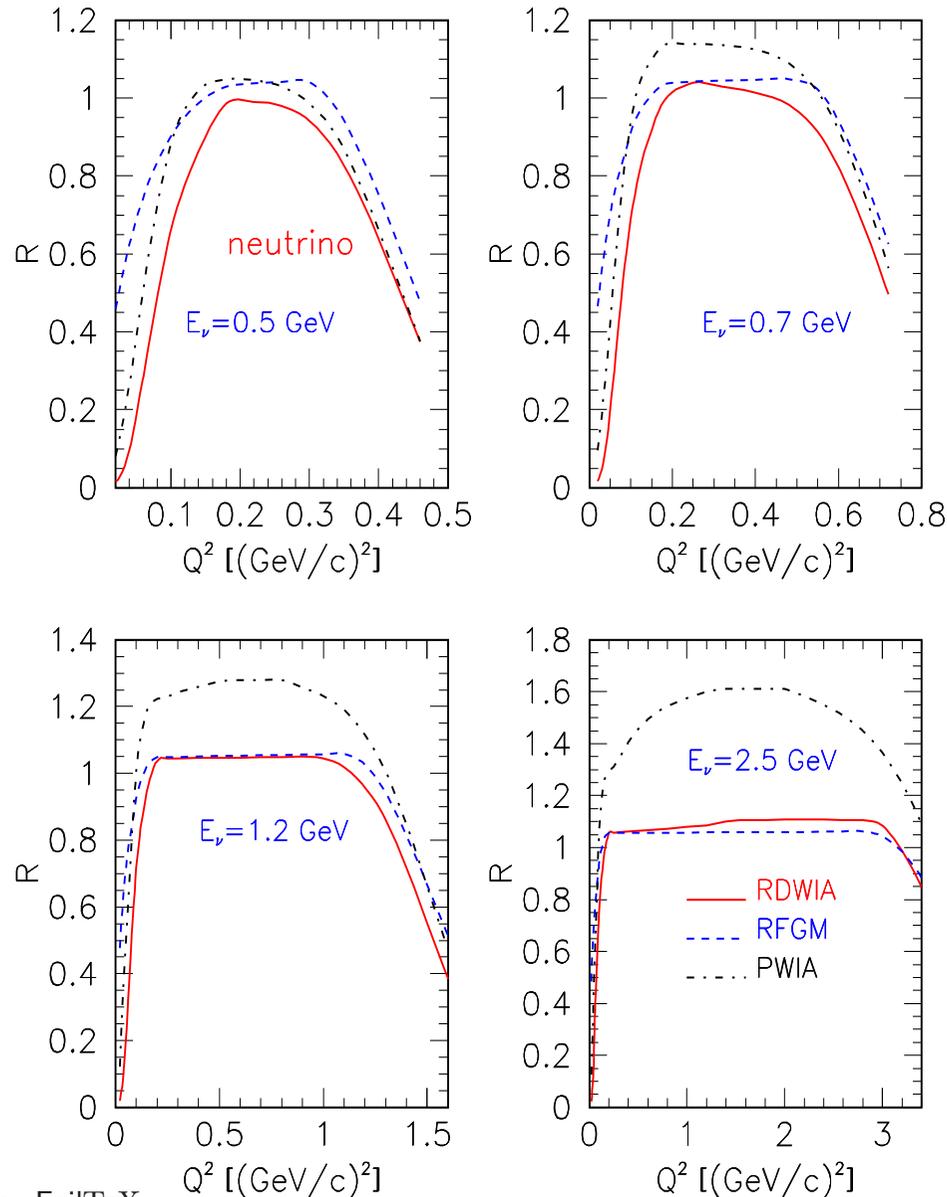


Inclusive cross section versus  $Q^2$  for antineutrino scattering on <sup>12</sup>C and for the four values of incoming neutrino energy:  $\varepsilon_\nu = 0.5, 0.7, 1.2$  and  $2.5$  GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the RDWIA result for the exclusive reaction (two-track events).

In the region  $< 0.2$  (GeV/c)<sup>2</sup> the RFGM results are higher than those obtained in the RDWIA and at  $Q^2 = 0.1$  (GeV/c)<sup>2</sup> the difference is about  $\sim 57\%$  for  $\varepsilon_\nu = 0.5$  GeV and  $\sim 33\%$  for  $\varepsilon_\nu = 2.5$  GeV.

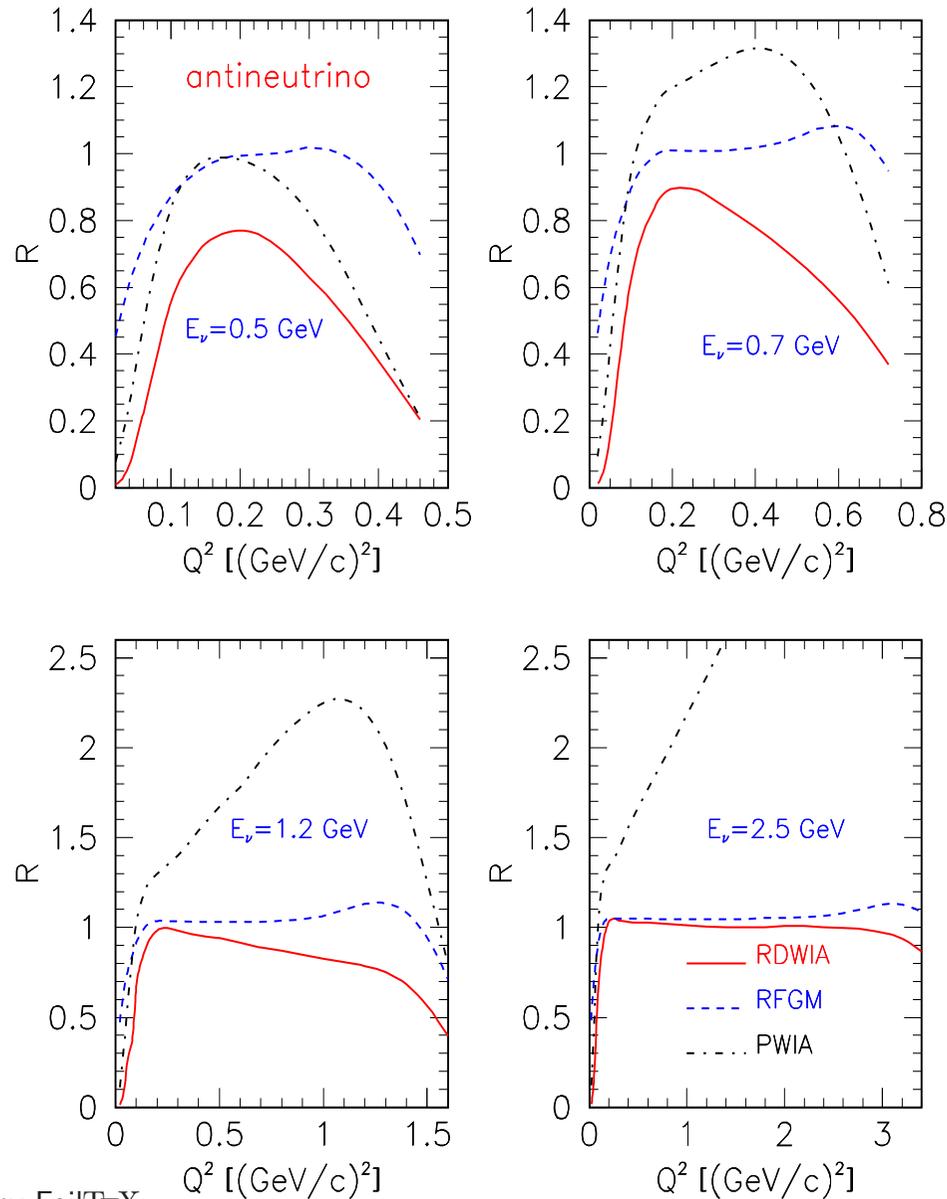


Ratio of the  $Q^2$  distribution per neutron  $R=O/C$  versus  $Q^2$  for neutrino scattering off  ${}^{16}\text{O}$  and  ${}^{12}\text{C}$  and for the four values of incoming neutrino energy:  $E_{\nu}=0.5, 0.7, 1.2$  and  $2.5 \text{ GeV}$ . The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the PWIA and RFGM calculations.



$R = (d\sigma/dQ^2)_{nuc}/(d\sigma/dQ^2)_{freenuc}$   
 versus  $Q^2$  for neutrino scattering on <sup>12</sup>C and for the four values of incoming neutrino energy:  $\varepsilon_\nu = 0.5, 0.7, 1.2$  and  $2.5$  GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations.

Range of  $Q^2$  where  $R \approx const$ , (i.e. nuclear effects cannot modify the value of  $M_A$ ) increases with neutrino energy



$R = (d\sigma/dQ^2)_{nuc}/(d\sigma/dQ^2)_{free\,nuc}$   
 versus  $Q^2$  for antineutrino scattering on <sup>12</sup>C  
 and for the four values of incoming neutrino  
 energy:  $\varepsilon_\nu = 0.5, 0.7, 1.2$  and  $2.5$  GeV. The  
 solid line is the RDWIA calculation while the  
 dashed and dashed-dotted lines are respectively  
 the RFGM and PWIA calculations.

$d\sigma/dQ^2$  cross sections averaged over (anti)neutrino flux

## $\nu$ -mode

- Normalization of the flux:  $N_{tot}^\nu = \int_{\varepsilon_{min}}^{\varepsilon_{max}} [I_\nu^\nu(\varepsilon) + I_{\bar{\nu}}^\nu(\varepsilon)] d\varepsilon$
- Weights:  $W_\nu^\nu(\varepsilon) = I_\nu^\nu(\varepsilon)/N_{tot}^\nu$  and  $W_{\bar{\nu}}^\nu(\varepsilon) = I_{\bar{\nu}}^\nu(\varepsilon)/N_{tot}^\nu$
- $\langle d\sigma/dQ^2 \rangle^\nu = \int_{\varepsilon_{min}(Q^2)}^{\varepsilon_{max}} [W_\nu^\nu(\varepsilon)(d\sigma^\nu/dQ^2)(Q^2, \varepsilon) + W_{\bar{\nu}}^\nu(\varepsilon)(d\sigma^{\bar{\nu}}/dQ^2)(Q^2, \varepsilon)] d\varepsilon$

$R = (d\sigma/dQ^2)_{nuc}/(d\sigma/dQ^2)_{freenuc}$  averaged over (anti)neutrino flux

- Normalization:  $C^\nu(Q^2) = \int_{\varepsilon_{min}(Q^2)}^{\varepsilon_{max}} [W_\nu^\nu(\varepsilon) + W_{\bar{\nu}}^\nu(\varepsilon)] d\varepsilon$
- Weights:  $V_\nu^\nu(\varepsilon, Q^2) = W_\nu^\nu(\varepsilon)/C^\nu(Q^2)$  and  $V_{\bar{\nu}}^\nu(\varepsilon) = W_{\bar{\nu}}^\nu(\varepsilon)/C^\nu(Q^2)$
- $\langle R^\nu(Q^2) \rangle = \int_{\varepsilon_{min}(Q^2)}^{\varepsilon_{max}} [V_\nu^\nu(\varepsilon, Q^2)R^\nu(Q^2, \varepsilon) + V_{\bar{\nu}}^\nu(\varepsilon, Q^2)R^{\bar{\nu}}(Q^2, \varepsilon)] d\varepsilon$

- $\langle \sigma_{tot}^\nu \rangle = \int_{Q_{min}^2}^{Q_{max}^2} \langle d\sigma/dQ^2 \rangle dQ^2$  and  $R^\nu = \langle \sigma_{tot}^\nu \rangle_{RDWA} / \langle \sigma_{tot}^\nu \rangle_{RFGM}$ ,  
 where  $Q_{min}^2 = 0.0156 \text{ (GeV/c)}^2$  and  $Q_{max}^2 = 1 \text{ (GeV/c)}^2$

## $\bar{\nu}$ -mode

The same as for  $\nu$ -mode, with  $\nu \iff \bar{\nu}$

### BooNE (anti)neutrino flux

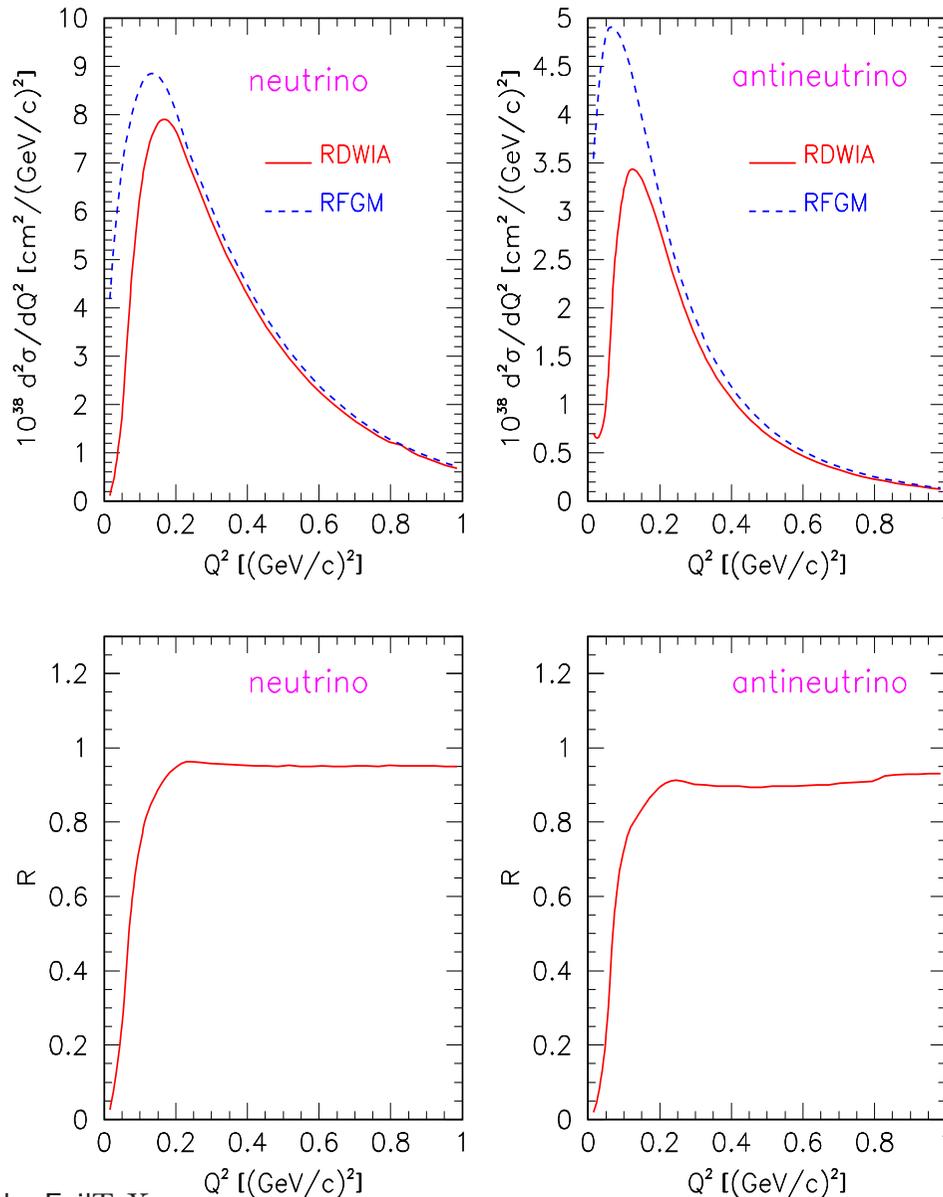
In the energy range  $0.275 \leq \varepsilon_{\nu(\bar{\nu})} \leq 2.6 \text{ GeV}$ :

- $\nu$ -mode:  $\approx 86\%$  of the total  $(\nu + \bar{\nu})$  flux
- $\bar{\nu}$ -mode:  $\approx 81\%$  of the total  $(\nu + \bar{\nu})$  flux

### NuMI neutrino flux

In the energy range  $0.3 \leq \varepsilon_{\nu(\bar{\nu})} \leq 14 \text{ GeV}$ :

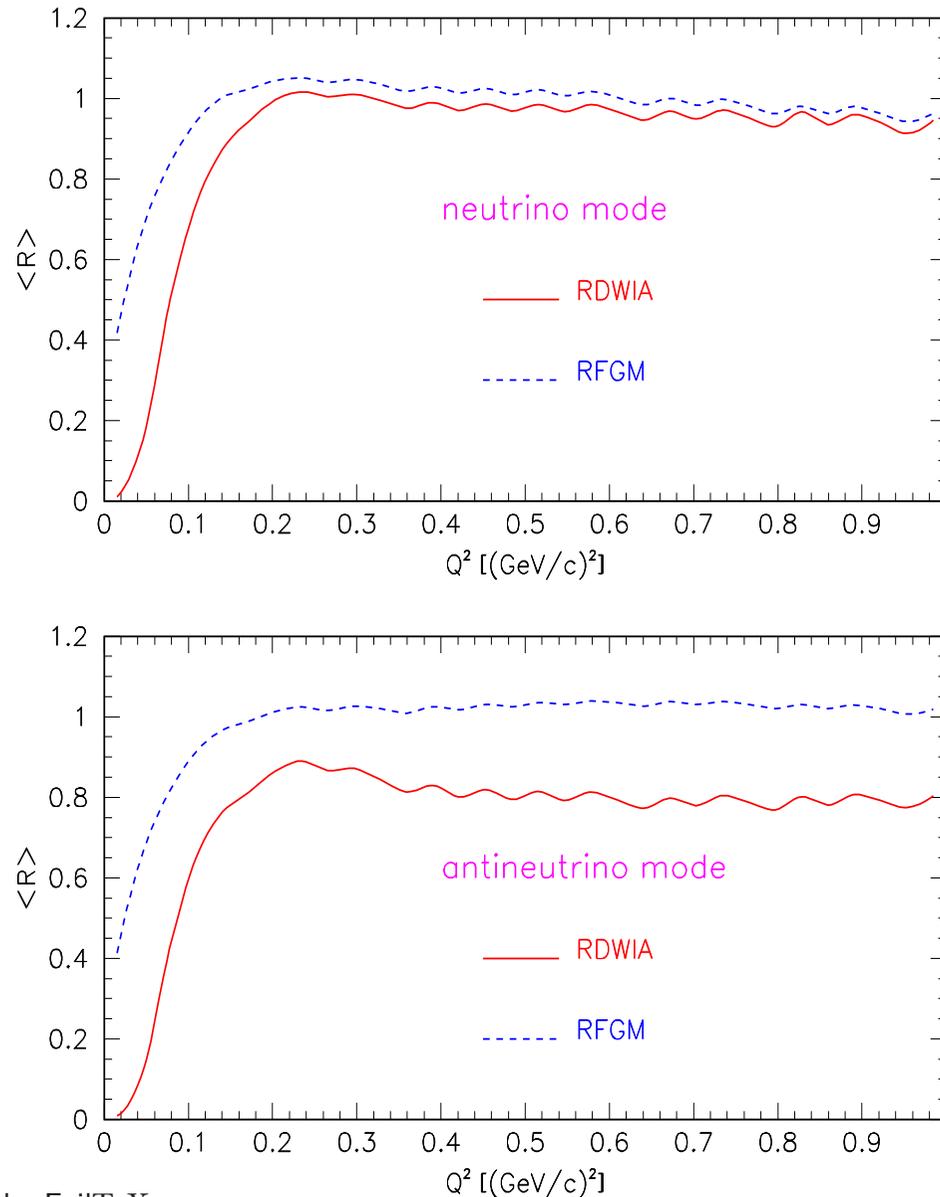
- $\nu$ -mode:  $\approx 95\%$  of the total  $(\nu + \bar{\nu})$  flux



## BooNe flux

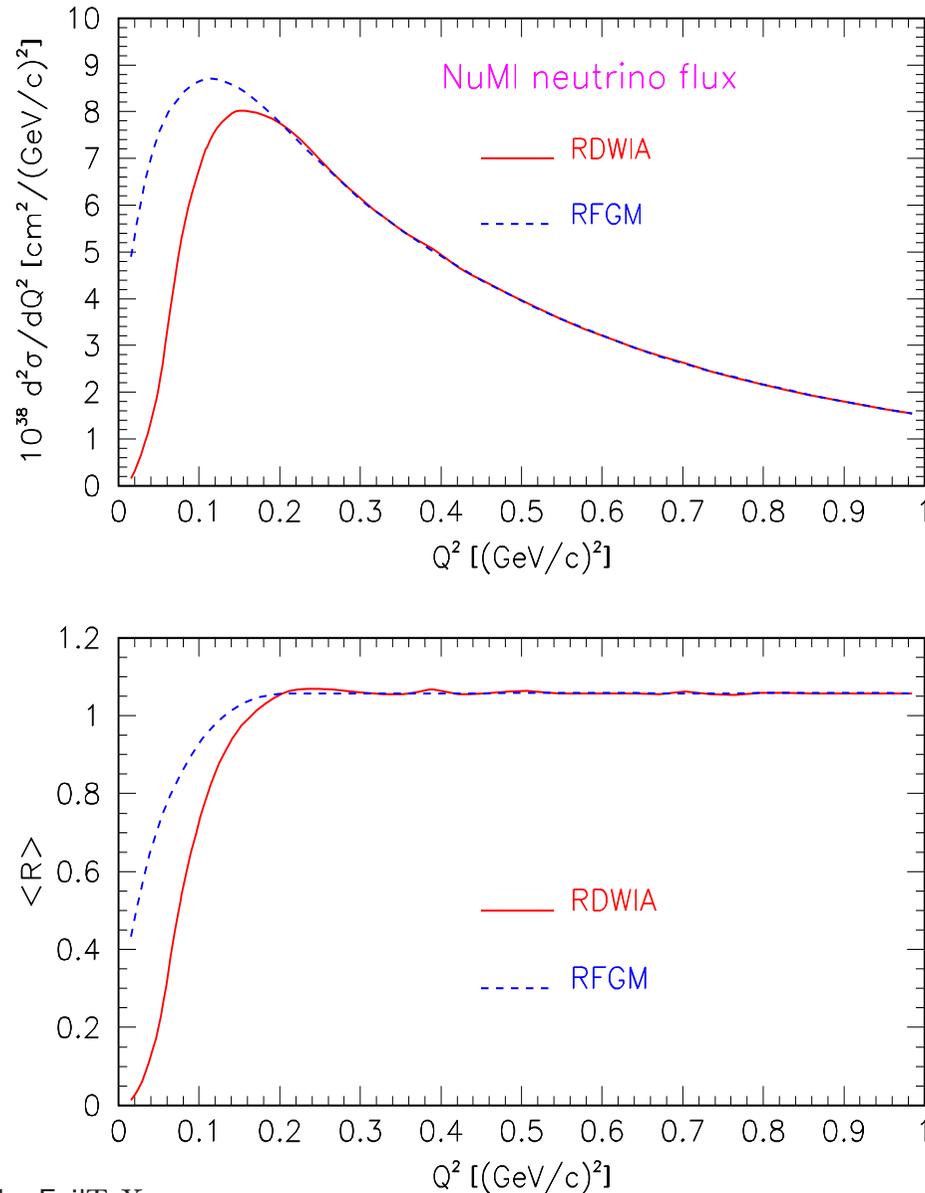
$\langle d\sigma/dQ^2 \rangle$  cross section vs  $Q^2$  for  $\nu$  - mode (top left) and  $\bar{\nu}$  - mode (top right) of the beam. The solid line is the RDWIA result and the dashed line is the RFGM result.  $R^\nu \approx 0.852$  and  $R^{\bar{\nu}} \approx 0.724$ . For neutrino the maximum of distribution calculated in the RDWIA (RFGM) is at  $Q^2 \approx 0.17(0.14) (\text{GeV}/c)^2$  and for antineutrino at  $\approx 0.14(0.08) (\text{GeV}/c)^2$

Ratio of the averaged cross sections  $R = \langle d\sigma/dQ^2 \rangle_{RDWIA} / \langle d\sigma/dQ^2 \rangle_{FG}$  vs  $Q^2$  for  $\nu$  - mode (bottom left) and  $\bar{\nu}$  - mode (bottom right).



## BooNe flux

$\langle R \rangle = (d\sigma/dQ^2)_{nuc}/(d\sigma/dQ^2)_{free}$   
 versus  $Q^2$  for  $\nu$ -mode (top panel) and for  $\bar{\nu}$ -mode (bottom panel). The solid line is the RDWIA calculation and the dashed line is the RFGM calculations. In the range of  $Q^2 \geq 0.3 (\text{GeV}/v)^2$   $\langle R \rangle \approx const$ , i.e. nuclear effects would not impact  $M_A$  fits.



## NuMI flux

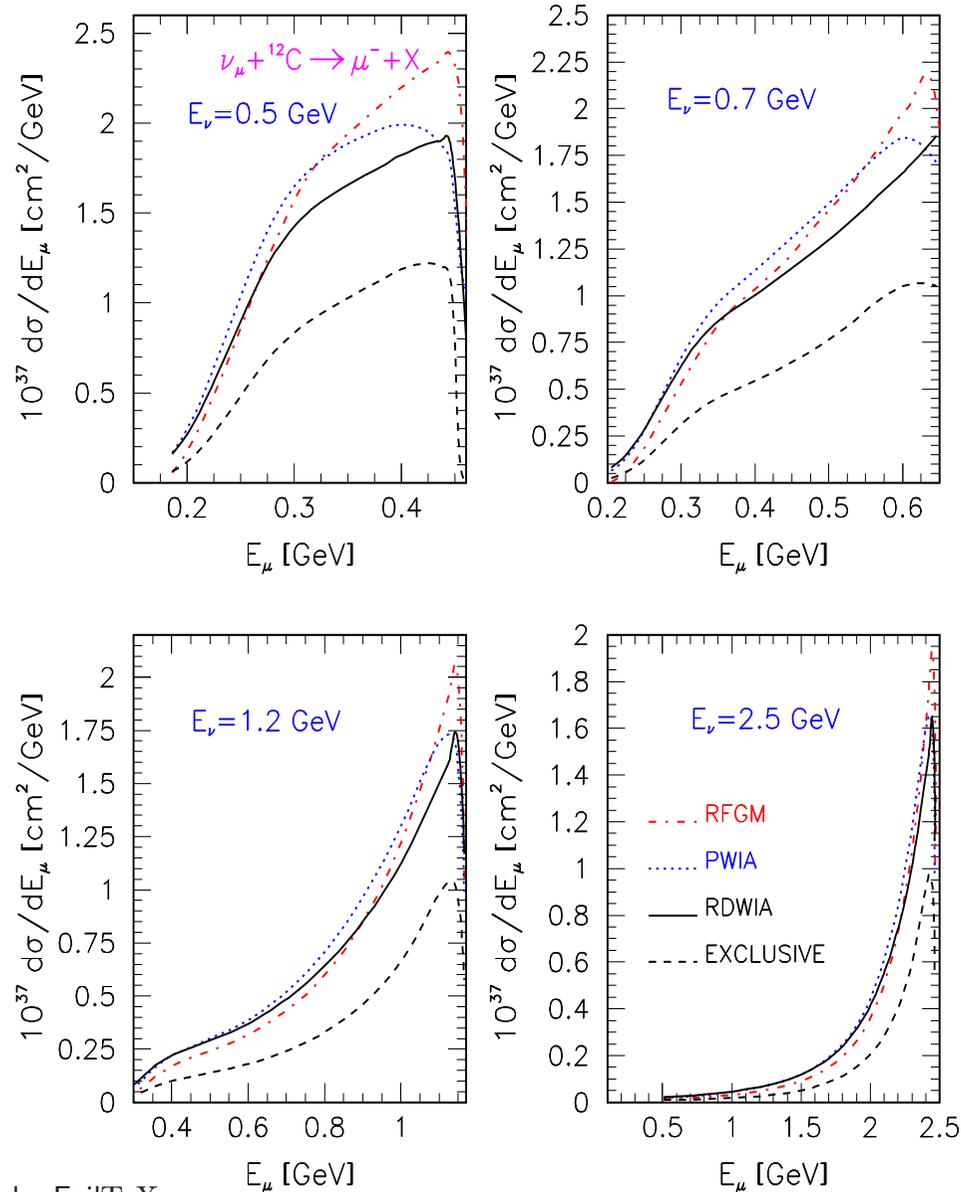
$\langle d\sigma/dQ^2 \rangle$  cross section vs  $Q^2$  for  $\nu$ -mode (top panel) of the beam. The solid line is the RDWIA result the dashed line is the RFGM result.  $R^\nu \approx 0.903$ . The maximum of distribution calculated in the RDWIA (RFGM) is at  $Q^2 \approx 0.17(0.12) (GeV/c)^2$ .

$\langle R \rangle = (d\sigma/dQ^2)_{nuc}/(d\sigma/dQ^2)_{free}$  versus  $Q^2$  for  $\nu$ -mode (bottom panel). The solid line is the RDWIA calculation and the dashed line is the RFGM calculations.

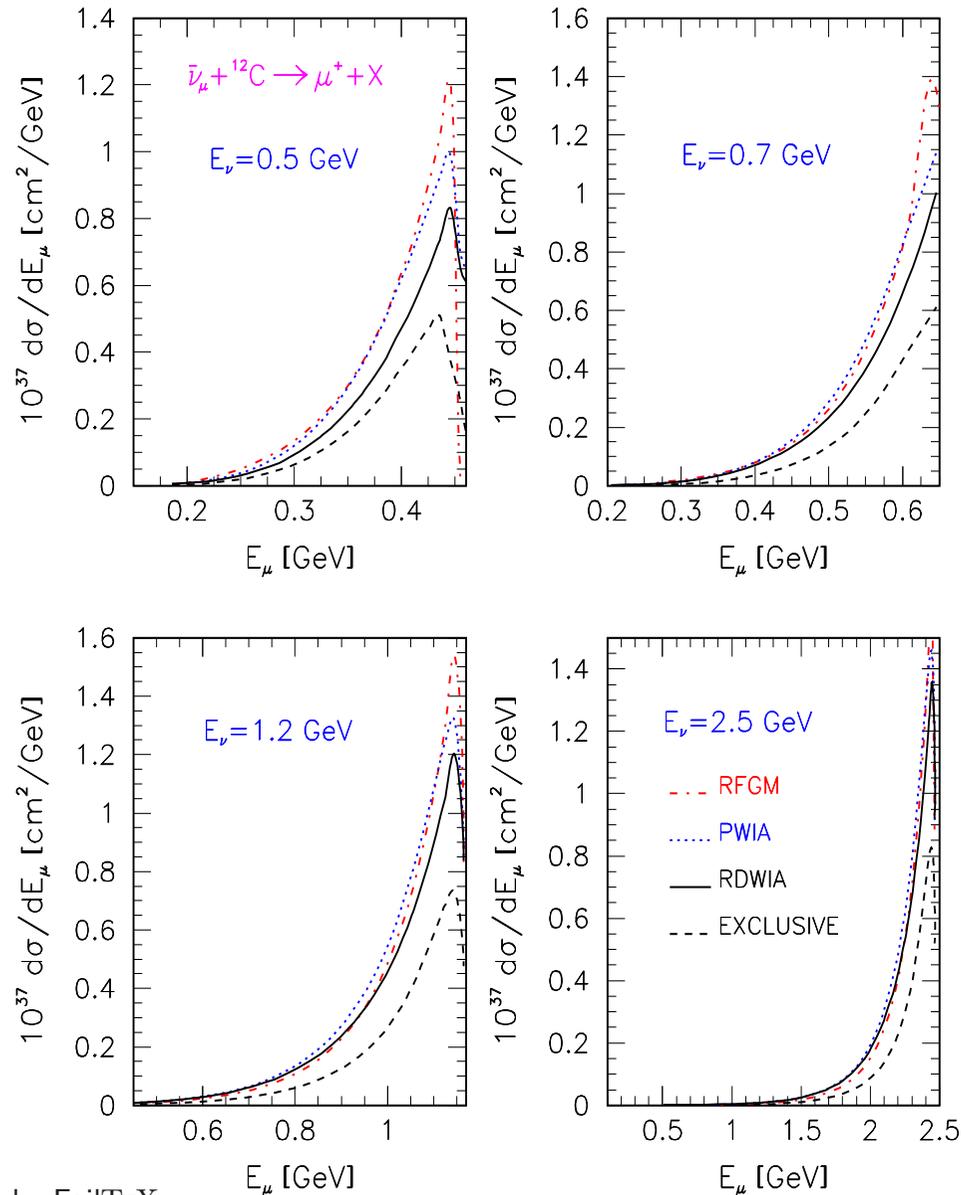
In the range of  $Q^2 \geq 0.2 (GeV/v)^2$   $\langle R \rangle \approx const$ , i.e. nuclear effects can not modify the result of  $M_A$  fits.

$d\sigma/d\varepsilon_\mu$  cross sections

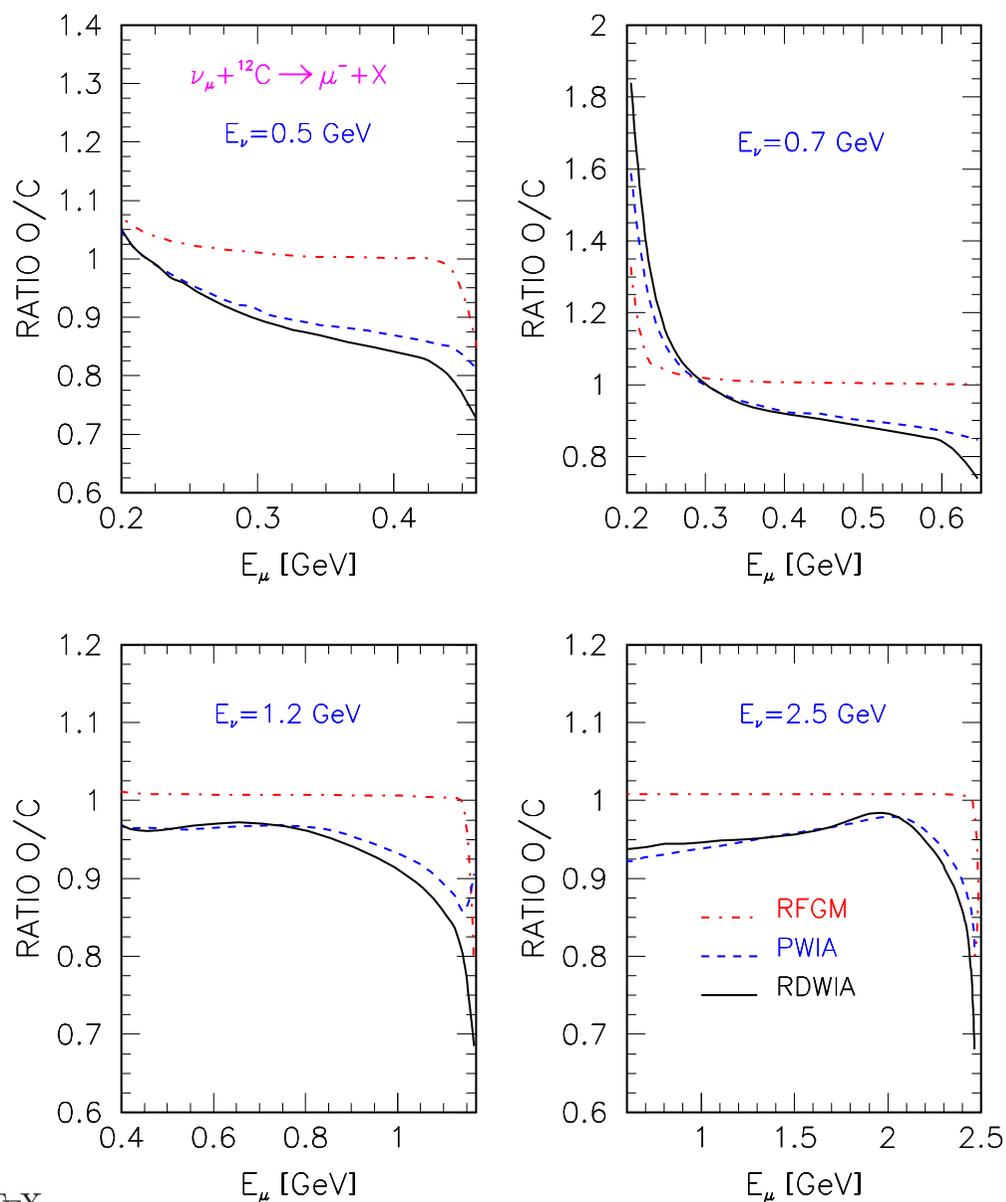
- $d\sigma/d\varepsilon_\mu$  vs  $\varepsilon_\mu$  at fixed neutrino energy
- Comparison of the  $d\sigma/d\varepsilon_\mu$  cross sections per nucleon for neutrino scattering off oxygen and carbon
  - det:  $(d\sigma/d\varepsilon_\mu)_{nuc} = (d\sigma/d\varepsilon_\mu)/N(Z)$
  - det:  $R = O/C = (d\sigma/d\varepsilon_\mu)_{nuc}(\text{oxygen})/(d\sigma/d\varepsilon_\mu)_{nuc}(\text{carbon})$



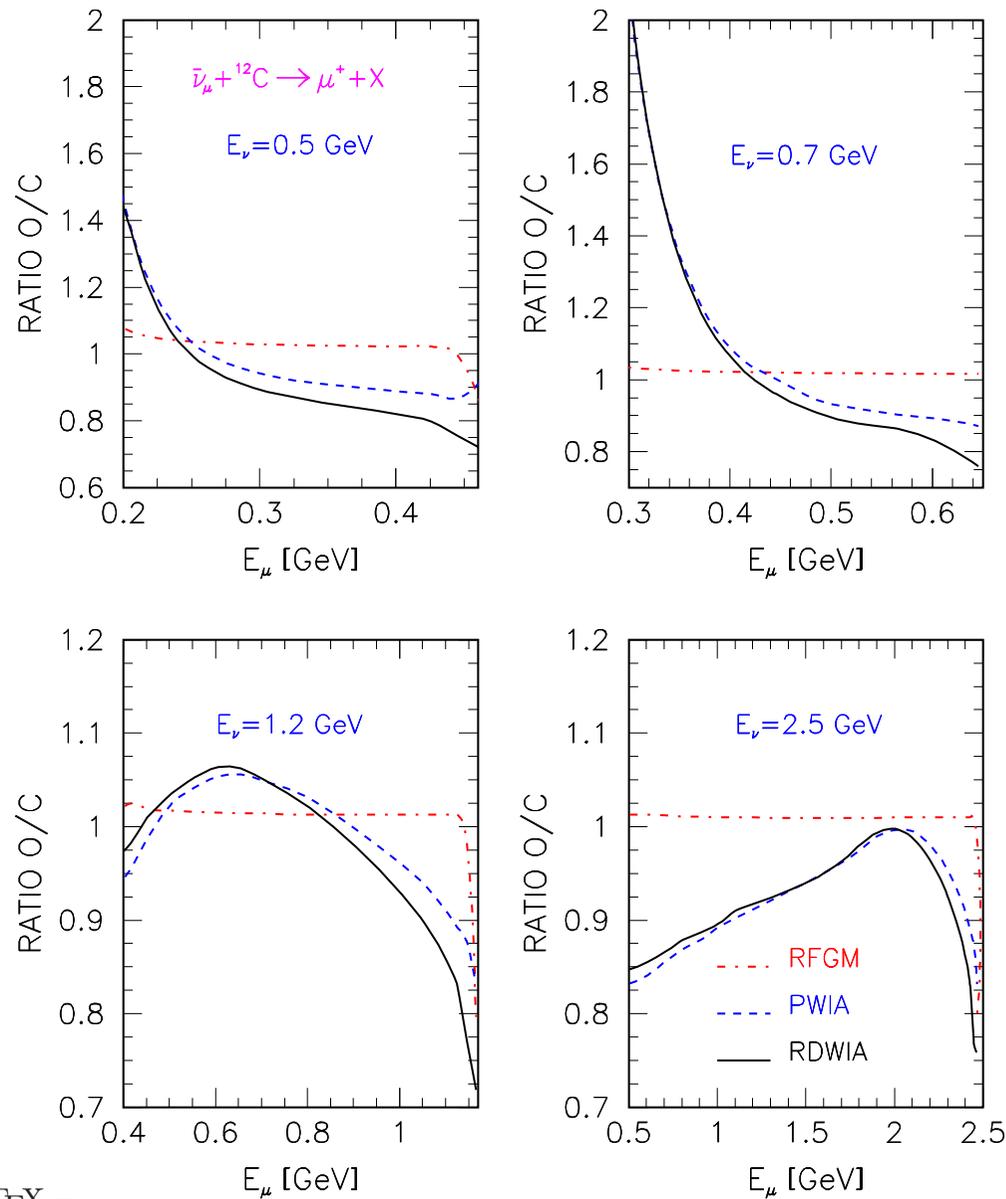
Inclusive cross section versus the muon energy for neutrino scattering on <sup>12</sup>C and for the four values of incoming neutrino energy:  $E_\nu = 0.5, 0.7, 1.2$  and  $2.5$  GeV. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA for the exclusive reaction (two-track events).



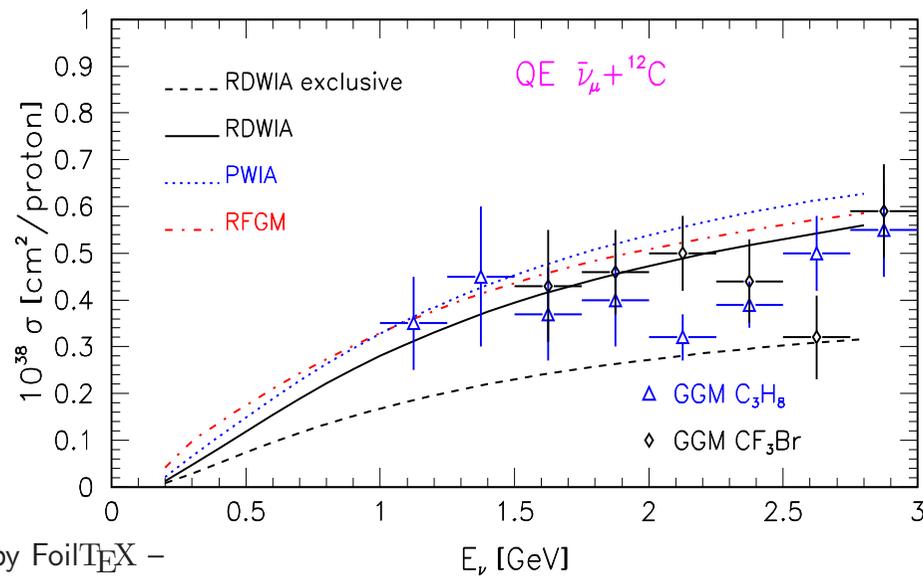
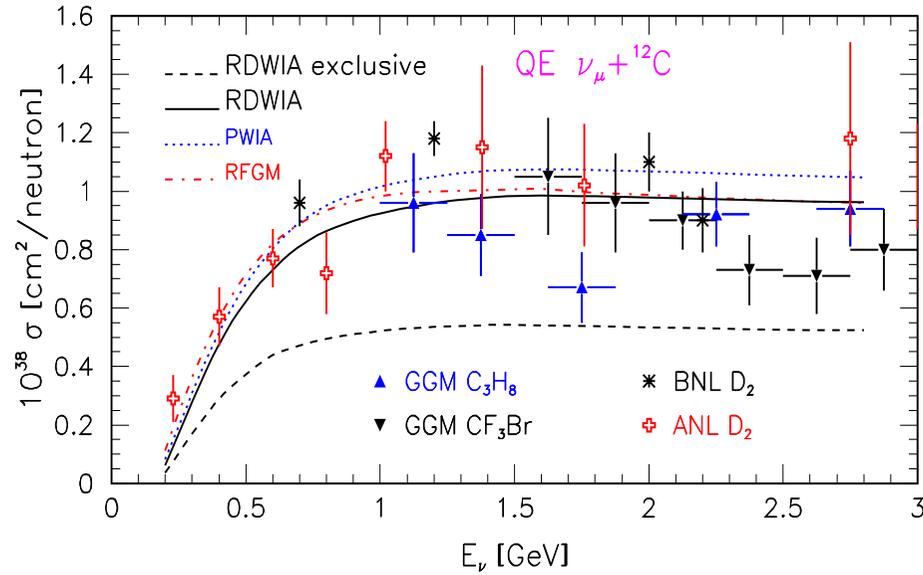
Inclusive cross section versus the muon energy for antineutrino scattering on  ${}^{12}\text{C}$  and for the four values of incoming neutrino energy:  $E_\nu = 0.5, 0.7, 1.2$  and  $2.5$  GeV. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA for the exclusive reaction (two-track events).



Ratio of the muon energy distribution per neutron  $R=O/C$  versus muon energy for neutrino scattering off  ${}^{16}\text{O}$  and  ${}^{12}\text{C}$  and for the four values of incoming neutrino energy:  $E_\nu=0.5$ ,  $0.7$ ,  $1.2$  and  $2.5$  GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the PWIA and RFGM calculations.

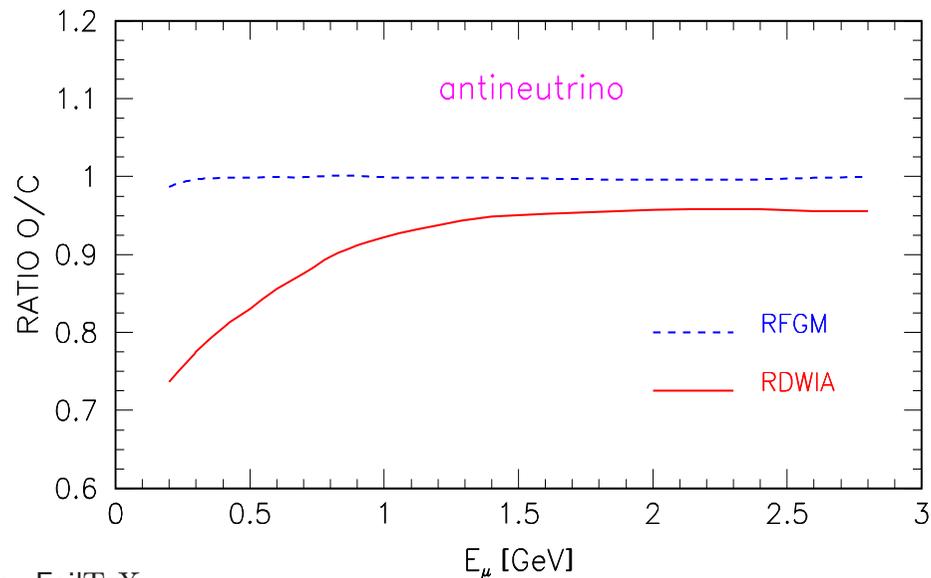
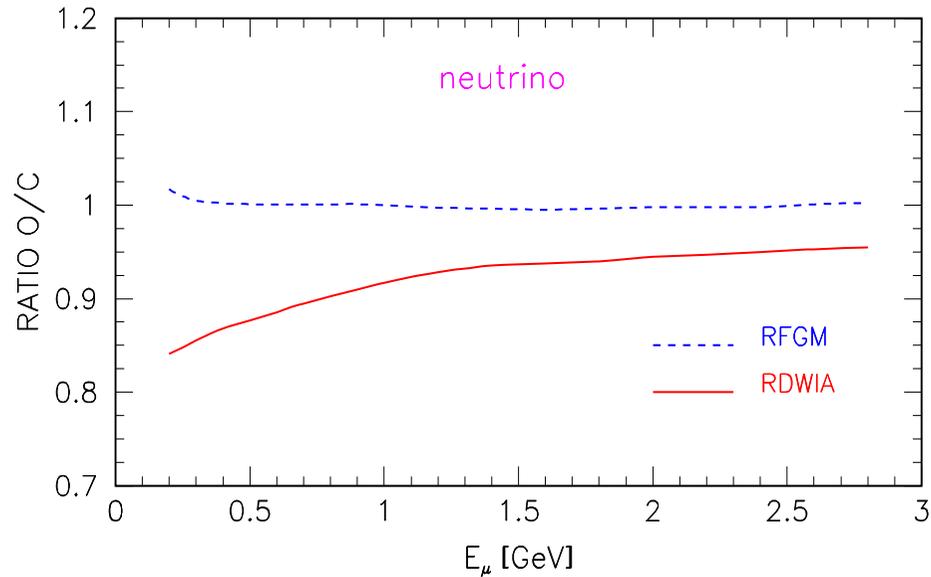


Ratio of the muon energy distribution per neutron  $R=O/C$  versus muon energy for antineutrino scattering off  ${}^{16}\text{O}$  and  ${}^{12}\text{C}$  and for the four values of incoming neutrino energy:  $E_\nu=0.5, 0.7, 1.2$  and  $2.5 \text{ GeV}$ . The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the PWIA and RFGM calculations.



Total cross section for CC QE scattering of muon neutrino (upper panel) and antineutrino (lower panel) on  ${}^{12}\text{C}$  as a function of incoming (anti)neutrino energy. The solid line is the RDWIA result while the dashed-dotted and dotted lines are respectively the RFGM and PWIA results. The dashed line is the RDWIA result for exclusive reaction (two-track events). In the region  $0.2 < \varepsilon_\nu < 2.6$  GeV the values of the RFGM cross sections are higher than those obtained within the RDWIA.

For energy  $\varepsilon_\nu = 0.5$  GeV this discrepancy is about 15% for  $\nu_\mu$  and  $\sim 46\%$  for  $\bar{\nu}_\mu$ . This difference decreases with energy and for  $\varepsilon_\nu = 2.6$  GeV is about 0.3% for  $\nu_\mu$  and  $\sim 6\%$  for  $\bar{\nu}_\mu$ . Data points for different targets are from ANL, BNL, and GGM



Ratio of the total cross section per neutron  $R=O/C$  versus neutrino energy for neutrino (top panel) and antineutrino (bottom panel) scattering off  $^{16}\text{O}$  and  $^{12}\text{C}$  and for the four values of incoming neutrino energy:  $E_\nu=0.5, 0.7, 1.2$  and  $2.5$  GeV. The solid line is the RDWIA calculation while the dashed line is RFGM calculations.

## Summary

QE CC  $\nu(\bar{\nu})^{12}\text{C}$  cross sections were studied in different approaches.

- The reduced cross sections for neutrino and electron scattering off  $^{12}\text{C}$  were tested against  $^{12}\text{C}(e, e'p)$  data.
  - ★ In RDWIA the cross sections for  $\nu(\bar{\nu})$  scattering are similar to those of electron scattering and in a good agreement with data.
  - ★ The RFGM fails completely when compared to exclusive cross section data.
- The inclusive and total cross sections were tested against  $^{12}\text{C}(e, e')$  scattering data.
  - ★ In the peak region RFGM overestimates the value of inclusive cross section at low momentum transfer ( $|\mathbf{q}| < 500 \text{ MeV}/c$ ). The discrepancy with data is about 20% at  $|\mathbf{q}| = 300 \text{ MeV}/c$  and decreases as momentum transfer increases.
  - ★ The RDWIA result is in agreement with data with accuracy  $\pm 10\%$ .
- The inclusive and total cross sections for  $\nu(\bar{\nu})^{12}\text{C}$  interaction were compared (cross section per neutron/proton with those for  $\nu(\bar{\nu})^{16}\text{O}$  reaction.
  - ★ In the RFGM the cross sections per nucleon for Carbon are practically similar to those for Oxygen.

- ★ In the RDWIA the cross sections for Oxygen are lower than those for Carbon and the difference is decreases as neutrino energy increases.
- ★ At  $Q^2 > 0.2 \text{ (GeV/c)}^2$  the nuclear effects slight change the slope in the  $Q^2$  distribution. The size of this range  $\Delta Q^2$  increases with neutrino energy and practically does not depend on the nuclear models.  
This range is preferable for the  $M_A$  analysis to reduce the uncertainty from the nuclear model.
- Perhaps the RDWIA will be able resolve the low  $Q^2$  problem. The test of the RDWIA results against neutrino data is necessary.
- At neutrino energy 0.7 GeV the RFGM result for neutrino (antineutrino) total cross sections is about 10% (30%) higher than RDWIA one and this difference reduces up to  $< 1\%$  (7%) at energy 2.5 GeV.
- Our results show that nuclear-model dependence of the inclusive and total cross sections weakens with neutrino energy but still remains significant for energy  $E_\nu \lesssim 1 \text{ GeV}$ .