

Progress in the description of neutrino-nuclear DIS

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Outline

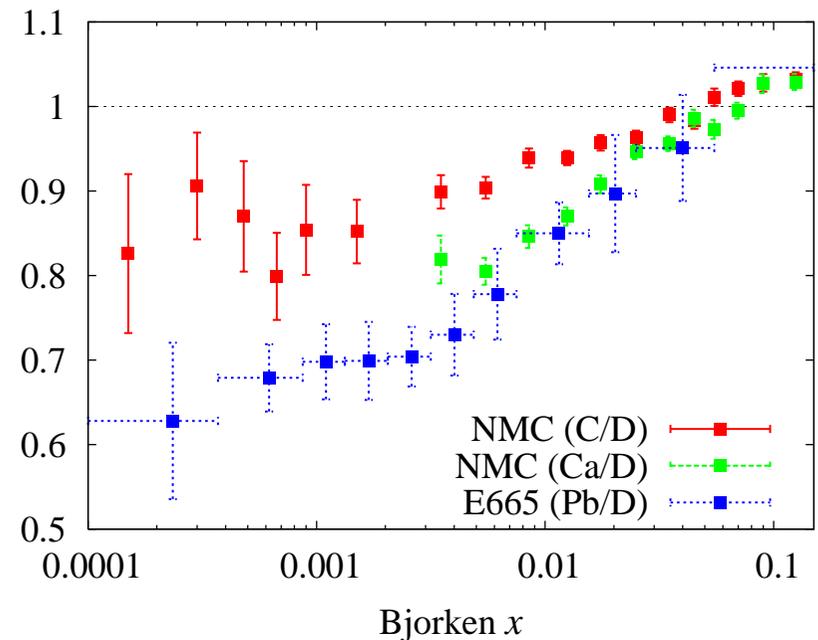
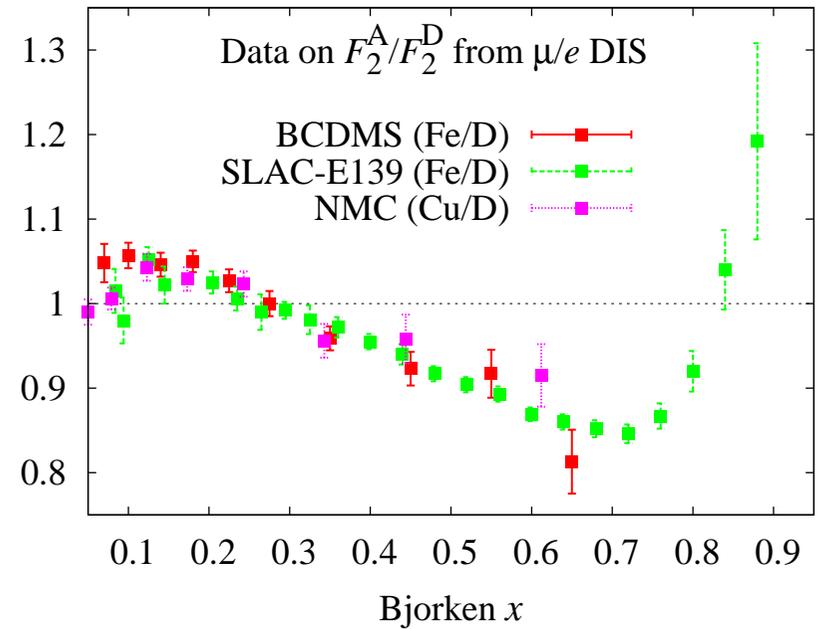
- Overview of charged-lepton data on the direct measurement of nuclear effects in DIS (EMC effect).
- Overview (brief) of a realistic model of nuclear effects in DIS.
- Recent results for neutrino-nuclear DIS: structure functions, sum rules, cross sections.

Experimental evidence of nuclear effects in DIS

- Data on nuclear effects in DIS are available in the form of the ratio $\mathcal{R}_2(A/B) = F_2^A/F_2^B$.
- Nuclear targets from ${}^2\text{D}$ to ${}^{208}\text{Pb}$
- Experiments:
 - Muon beam at CERN (EMC, BCDMS, NMC) and FNAL (E665).
 - Electron beam at SLAC (E139, E140), HERA (HERMES) and recently at JLab.
- Kinematics and statistics:
Data covers the region $10^{-4} < x < 0.9$ and $0 < Q^2 < 150 \text{ GeV}^2$. About 600 data points with $Q^2 > 1 \text{ GeV}^2$.
- Although most of the data come from CL DIS measurements, additional information is available from Drell-Yan experiments (E772, E866).

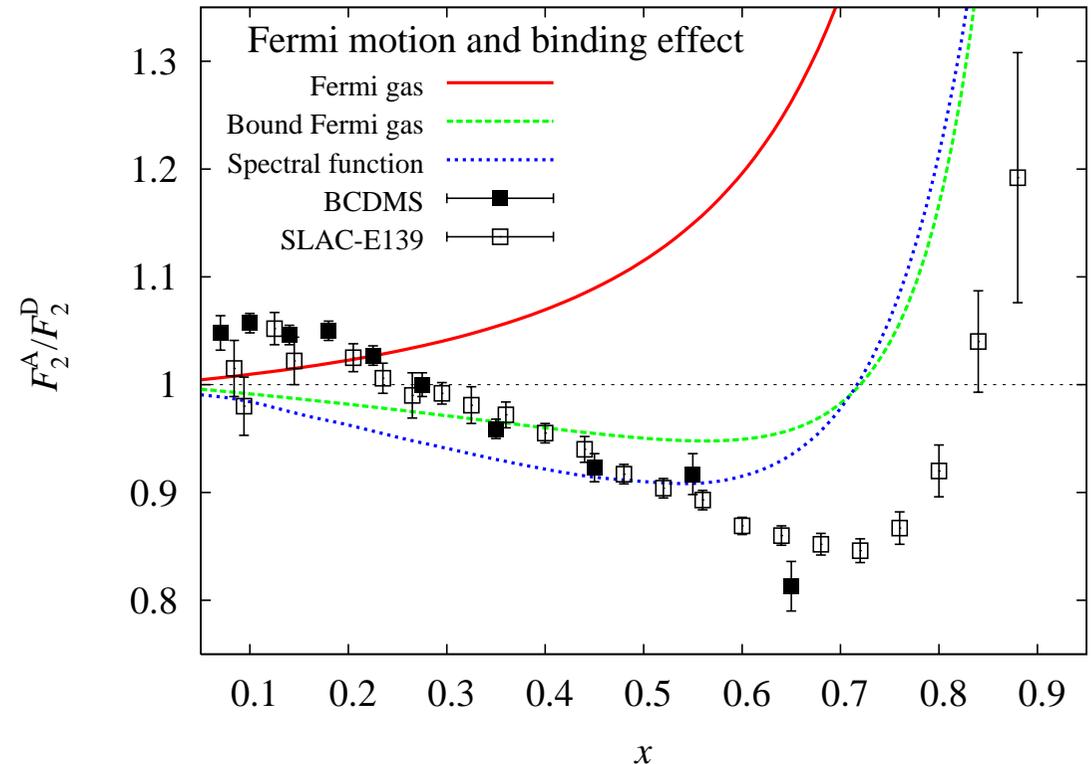
Data on the EMC ratios show a weak Q^2 nuclear effects. Characteristic nuclear effects in different regions of the Bjorken x .

- Nuclear shadowing at small values of x ($x < 0.05$).
- Antishadowing at $0.1 < x < 0.25$.
- A well with a minimum at $x \sim 0.6 \div 0.75$ (EMC effect).
- Enhancement at large $x > 0.75 \div 0.8$ (Fermi motion region).



Understanding nuclear effects

At large x ($x > 0.1$) the major mechanism is incoherent scattering from bound nucleons (Impulse approximation, or IA). Within this approximation it is important to treat Fermi motion and nuclear binding effects properly. However, it is not enough for quantitative understanding of data. IA should be corrected for a number of effects.



Nucleon off-shell effect

Bound nucleons are off-mass-shell $p^2 = (M + \varepsilon)^2 - \mathbf{p}^2 \neq M^2$. In off-shell region nucleon structure functions and form factors generally depend on additional variable p^2 :

$$F_2^N(x, Q^2, p^2) \approx F_2^N(x, Q^2) \left(1 + \delta f(x) \frac{p^2 - M^2}{M^2} \right)$$

- The off-shell function $\delta f(x)$ makes sense of the response of nucleon parton distributions to variation of the nucleon mass, $\delta f = \partial \ln q(x, p^2) / \partial \ln p^2$.
- Off-shell dependence is closely related to idea of modification of nucleon in nuclear environment. In a simple model $\delta f(x)$ can be related to the variation of the nucleon core radius in nuclear environment.
- We extract $\delta f(x)$ from analysis of data on nuclear EMC effect [S.K. & R.Petti, NPA765(2006)126].

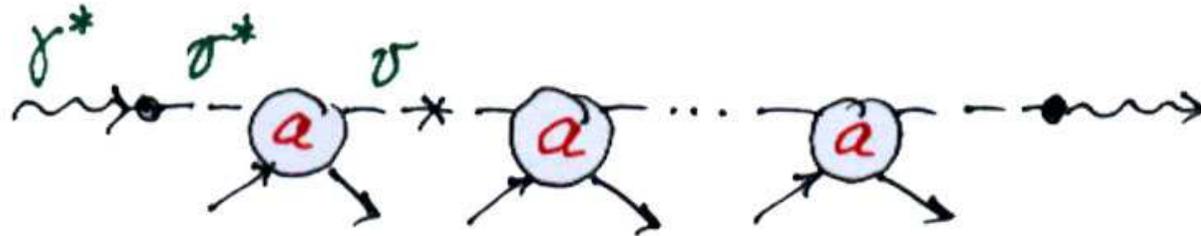
Nuclear pion effect

Leptons can interact with nuclear meson field which mediate interaction between bound nucleons (meson exchange currents, or MEC).

- Nuclear momentum sum rule is violated in IA. Nuclear pion correction is important to balance missing light-cone momentum $\langle y \rangle_\pi + \langle y \rangle_N = 1$.
- The correction is localized in a region $x \lesssim p_F/M \sim 0.25$.
- The magnitude of the correction is driven by average number of “pions” $n_\pi = \int dy f_{\pi/A}(y)$. By order of magnitude $n_\pi/A \sim 0.1$ for ^{56}Fe .
- Nuclear pion correction effectively leads to enhancement of nuclear sea quark distribution and does not affect the valence quark distribution (for isoscalar nuclear target).

Nuclear DIS in coherent regime: shadowing

At small x DIS is driven by $\gamma^* \rightarrow v^*$ conversions into virtual hadronic states. Nuclear effects come from multiple interactions of hadronic states during the propagation through matter.



To find characteristic region of x compare an average time of life (coherence length) of hadronic fluctuation $\tau = [Mx(1 + m_v^2/Q^2)]^{-1}$ with average internucleon distance $r \sim 1.5 \text{ Fm}$: $\tau > r \implies x < 0.15$. Developed shadowing effect at $x < 10^{-2}$.

The amplitude a describes interaction of hadronic component of the virtual boson with a nucleon. The magnitude of coherent effects is driven by effective v^*N cross section ($\text{Im } a$). Behavior in transitional region is also affected by $\text{Re } a$.

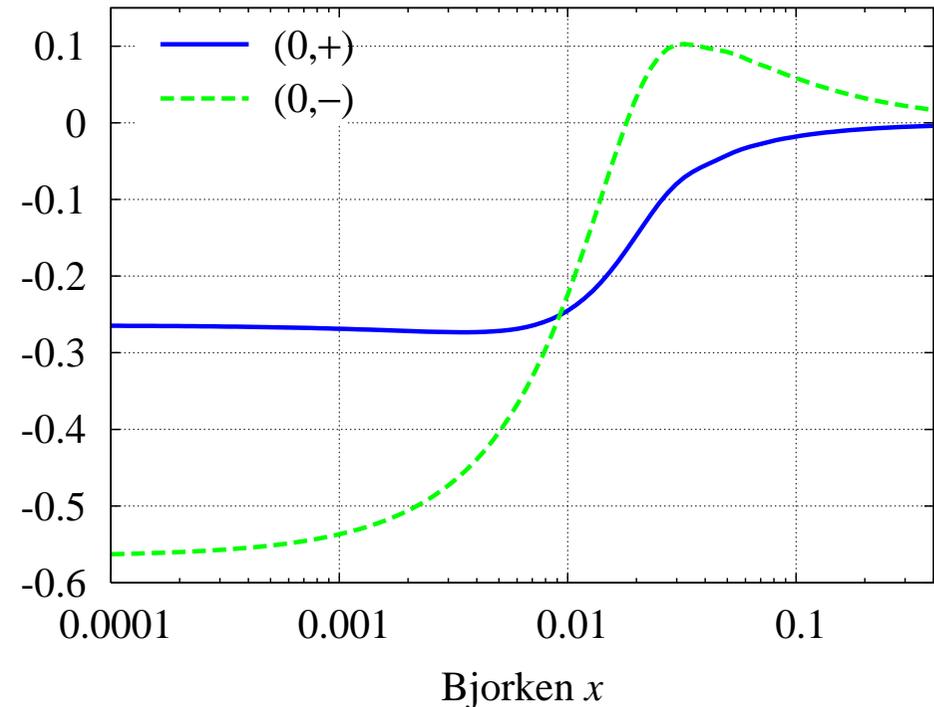
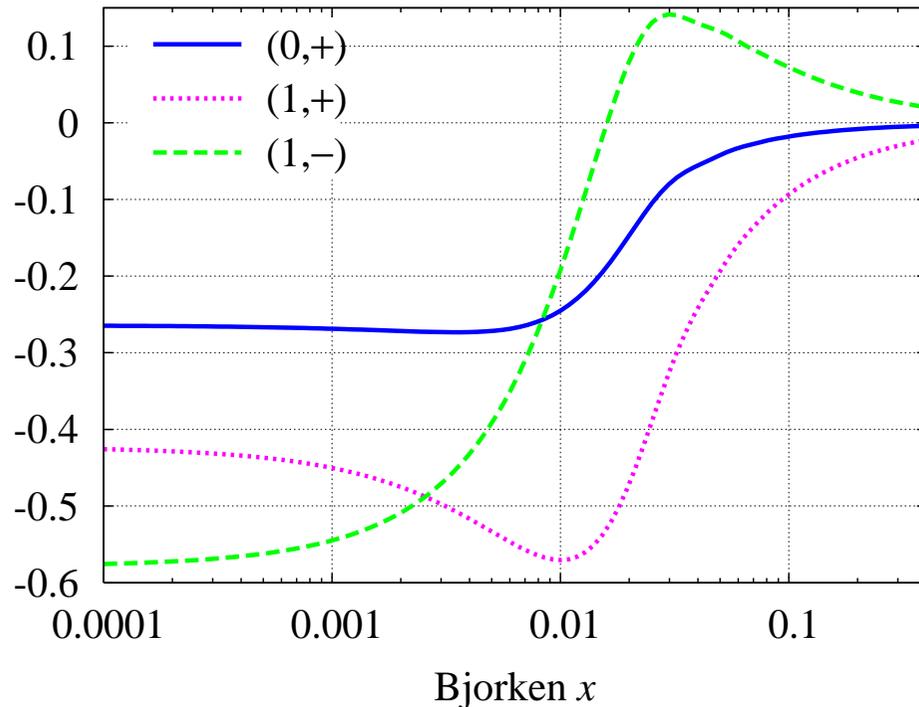
The amplitude a is characterized by the helicity state h of the boson ($h = \pm 1$ and $h = 0$ for transverse and longitudinal polarization, respectively). In addition the amplitude depends on the isospin I (proton and neutron dependence) and C -parity (ν and $\bar{\nu}$ dependence),

$a_h^{(I,C)}$. Note that interaction of virtual photon γ^* is described by a C -even amplitude, and (anti)neutrino interaction involve both C -even and C -odd amplitudes.

Correspondence between $a_h^{(I,C)}$ and the structure functions. Define average transverse amplitude $a_T^I = (a_{+1}^I + a_{-1}^I)/2$ and asymmetry $a_\Delta^I = (a_{+1}^I - a_{-1}^I)/2$

$$\begin{aligned}
 a_T^{(0,+)} &\rightarrow F_1^{\mu(p+n)} \text{ and } F_1^{(\nu+\bar{\nu})(p+n)} , & a_\Delta^{(0,-)} &\rightarrow F_3^{(\nu+\bar{\nu})(p+n)} \\
 a_T^{(1,+)} &\rightarrow F_1^{\mu(p-n)} \text{ and } F_1^{(\nu+\bar{\nu})(p-n)} , & a_\Delta^{(1,-)} &\rightarrow F_3^{(\nu+\bar{\nu})(p-n)} \\
 a_T^{(0,-)} &\rightarrow F_1^{(\nu-\bar{\nu})(p+n)} , & a_\Delta^{(0,+)} &\rightarrow F_3^{(\nu-\bar{\nu})(p+n)} \\
 a_T^{(1,-)} &\rightarrow F_1^{(\nu-\bar{\nu})(p-n)} , & a_\Delta^{(1,+)} &\rightarrow F_3^{(\nu-\bar{\nu})(p-n)}
 \end{aligned}$$

Coherent multiple scattering nuclear corrections depend on quantum numbers (h, C, I) .



The relative nuclear correction to transverse effective cross section σ_T calculated for different isospin and C -parity scattering states for ^{208}Pb at $Q^2 = 1 \text{ GeV}^2$. The labels on the curves mark the values of the isospin I and C -parity, (I, C) .

Model

Taking into account major nuclear corrections we build a quantitative model for nuclear structure functions (for more detail see [S.K. & R.Petti, NPA765\(2006\)126](#))

$$F_i^A = F_i^{p/A} + F_i^{n/A} + \delta_\pi F_i + \delta_{\text{coh}} F_i$$

- * $F_i^{p/A}$ and $F_i^{n/A}$ are bound proton and neutron structure functions with Fermi motion, binding and off-shell effects calculated using realistic nuclear spectral function.
- * $\delta_\pi F_i^A$ and $\delta_{\text{coh}} F_i^A$ are nuclear pion and shadowing corrections.

In actual calculations we use:

- Proton and neutron SFs computed in NNLO pQCD + TMC + HT using phenomenological PDFs and HTs from fits to DIS data ([Alekhin](#)).
- Realistic nuclear spectral function which includes the mean-field as well as the correlated part.
- Nuclear pion correction as a convolution of nuclear pion distribution function with pion PDFs.
- Coherent nuclear corrections are calculated using Glauber multiple scattering theory in terms of effective amplitude a_T .

Analysis of EMC effect

⇒ Parameterize unknown off-shell correction function $\delta f(x)$ and effective scattering amplitude a_T responsible for nuclear shadowing. Calculate nuclear structure functions, test with data and extract parameters from data.

⇒ We study the data from e/μ DIS in the form of ratios $\mathcal{R}_2(A/B) = F_2^A/F_2^B$ for a variety of targets. The data are available for A/D and $A/^{12}\text{C}$ ratios.

⇒ In our analysis we perform a fit to minimize $\chi^2 = \sum_{\text{data}} (\mathcal{R}_2^{\text{exp}} - \mathcal{R}_2^{\text{th}})^2 / \sigma^2(\mathcal{R}_2^{\text{exp}})$ with σ the experimental uncertainty using data with $Q^2 > 1 \text{ GeV}^2$ (overall about 560 points).

⇒ Verify the model by comparing the calculations with data not used in analysis.

Parametrization of off-shell function and effective amplitude:

$$\delta f(x) = C_N(x - x_1)(x - x_0)(x_2 - x)$$

$$a_T = \sigma_T(i + \alpha)/2, \quad \sigma_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

Not all parameters are free or independent.

- * We fix $\sigma_0 = 27$ mb and $\alpha = -0.2$ to have the correspondence with VMD model at $Q^2 \rightarrow 0$.
- * From preliminary trials, the parameter x_2 turned out fully correlated with x_0 , $x_2 = 1 + x_0$ fixed in the final fit.
- * Best fit gives $\sigma_1 \approx 0$. The correlations between σ_1 and off-shell parameters are negligible. We fix $\sigma_1 = 0$ in the final fits.

Results

The model leads to a very good agreement with data on nuclear EMC effect. The x and Q^2 dependence of the EMC ratios is reproduced for all studied nuclei (${}^4\text{He}$ to ${}^{208}\text{Pb}$) in a 4-parameter fit with $\chi^2/\text{d.o.f.} = 459/556$. For detailed discussion and comparison with data see S.K. & R.P., NPA765(2006)126.

Off-shell function

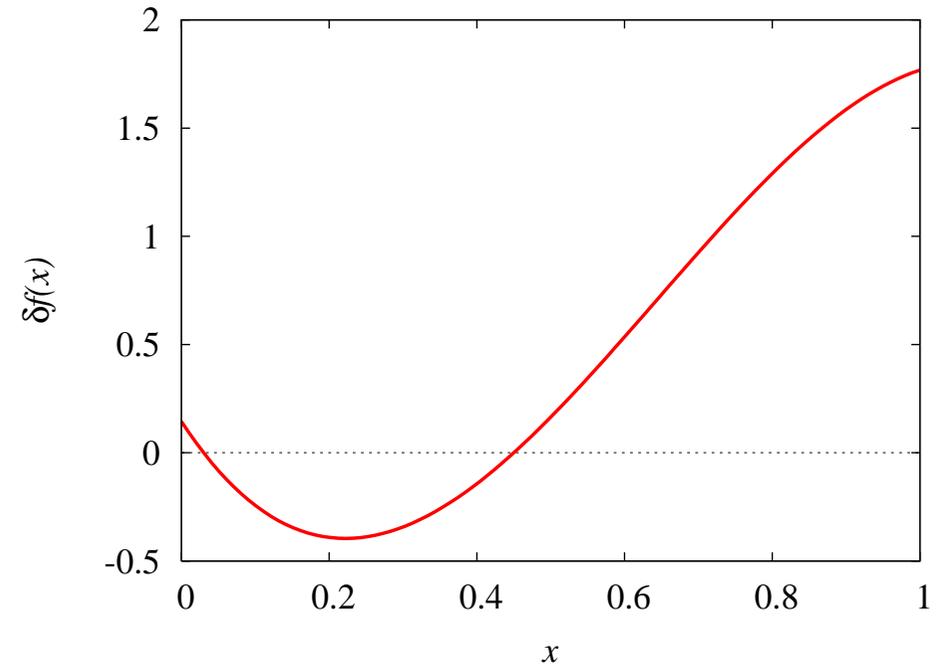
The function $\delta f(x)$ provides a measure of modification of quark distributions in bound nucleon.

$$\delta f(x) = C_N(x - x_1)(x - x_0)(1 + x_0 - x)$$

$$C_N = 8.1 \pm 0.3 \pm 0.5$$

$$x_0 = 0.448 \pm 0.005 \pm 0.007$$

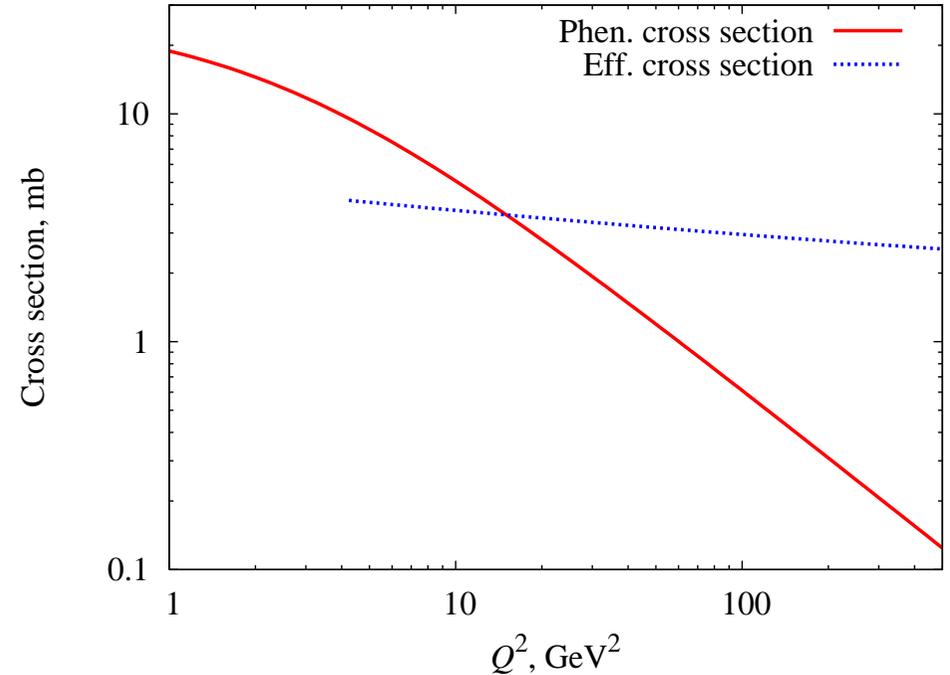
$$x_1 = 0.05$$

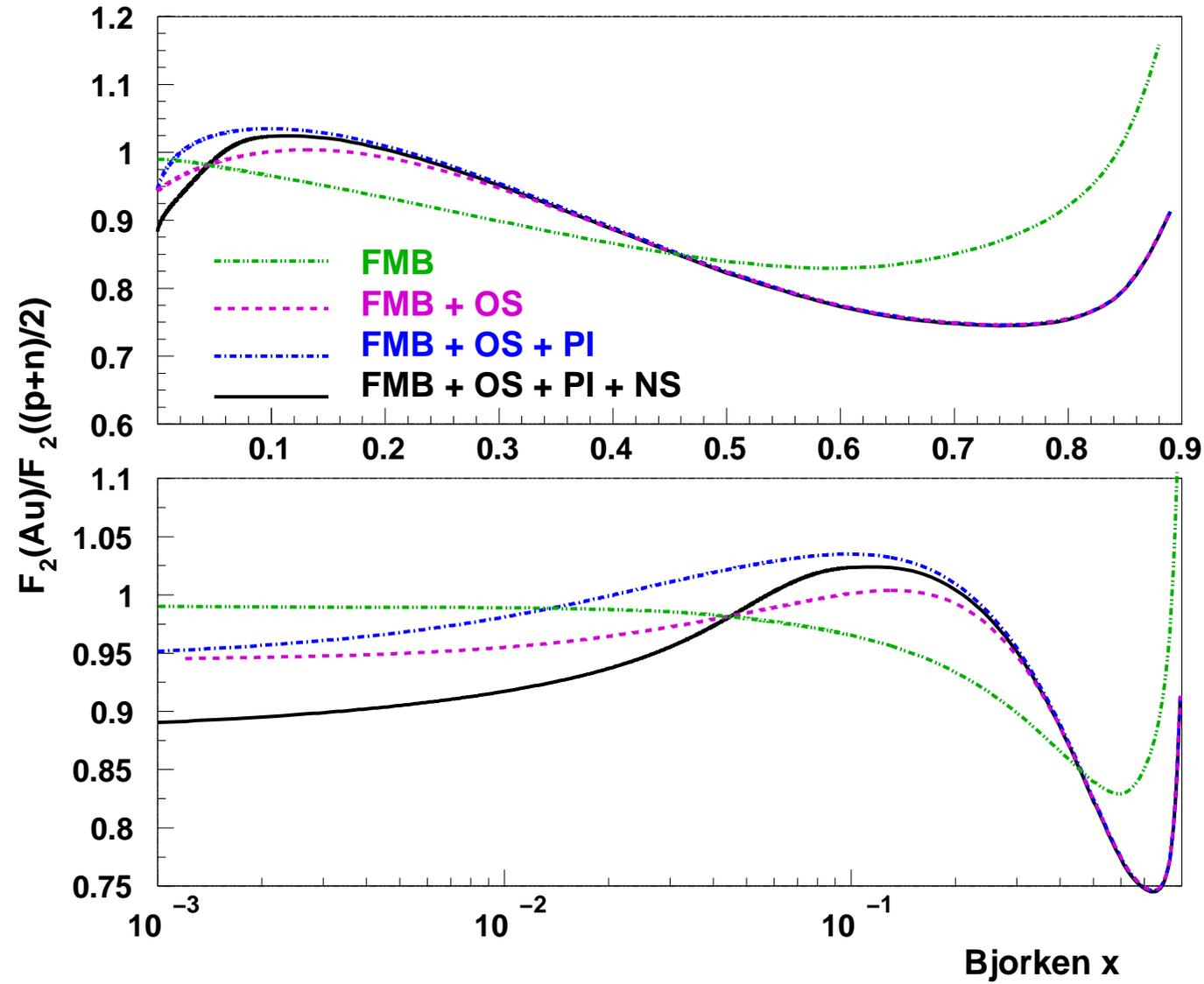


- Parameters from the global fit (all nuclei) are consistent with independent fits to different subsets of nuclei
- The off-shell effect results in the enhancement of the structure function for $x_1 < x < x_0$ and depletion for $x < x_1$ and $x > x_0$.

Effective cross section

- The monopole form $\sigma_T = \sigma_0 / (1 + Q^2/Q_0^2)$ with $\sigma_0 = 27 \text{ mb}$ and $Q_0^2 = 1.43 \pm 0.06 \pm 0.195 \text{ GeV}^2$ provides a good fit to existing DIS data on nuclear shadowing for $Q^2 < 20 \text{ GeV}^2$.
- Cross section at high Q^2 is not constrained by data. But we can calculate it! To do that we apply the model to nuclear valence quark distribution and require exact cancellation between off-shell (OS) and shadowing (NS) contributions to normalization: $\delta N_{\text{val}}^{\text{OS}} + \delta N_{\text{val}}^{\text{NS}} = 0$.





Different nuclear effects calculated for ^{197}Au at $Q^2 = 10 \text{ GeV}^2$.

Summary for the CL DIS analysis

- The model is in a very good agreement with data for the entire kinematical region of x and Q^2 and for all studied nuclei from ${}^4\text{He}$ to ${}^{207}\text{Pb}$ with overall $\chi^2/\text{d.o.f} = 459/556$ for a 4-parameter fit.
- The fit parameters are common to all nuclei. The parameters are stable for the fits with different subsets of nuclei (see Table 2 in NPA765(2006)126).
- The off-shell effect is a way to describe modification of the nucleon structure in nuclear environment. The extracted parameters of $\delta f(x)$ suggest an increase in the size of the bound nucleon valence region (nucleon core) (in ${}^{56}\text{Fe}$ by $\sim 10\%$ on average).
- Predictions for the deuteron are in a good agreement with E665 D/p data and also with Gomez et.al. extraction of $D/(p+n)$ ratio (except for the region $x > 0.7$).

Nuclear effects in neutrino DIS

- We apply the model developed for CL nuclear scattering for neutrino-nuclear interactions (for more details see S.K. & R.Petti, PRD46(2007)094023).
- Additional input is required to treat nuclear effects for νA scattering.
 - ⇒ Off-shell corrections for different structure functions (F_2 and F_3) and its dependence on ν and $\bar{\nu}$.
 - ⇒ Calculation of nuclear shadowing for $F_2^{\nu, \bar{\nu}}$ and $F_3^{\nu, \bar{\nu}}$ requires the amplitudes $a^{(I,C)}$ for different C -parity and isospin I . (the latter is important for accurate evaluation of isovector contributions, the neutron excess correction).
- DIS sum rules for nuclei (Adler and GLS) help to fix unknown amplitudes $a^{(0,-)}$ and $a^{(1,-)}$ responsible for (anti)shadowing corrections for $x F_3^{\nu+\bar{\nu}}$ and $F_2^{\bar{\nu}-\nu}$ combinations.

The Adler sum rule (ASR)

$$S_A = \int_0^{M_A/M} dx (F_2^{\bar{\nu}} - F_2^{\nu}) / (2x) = 2I_z$$

- In parton model S_A is the difference between number of valence u and d in the target. $S_A(p) = +1$, $S_A(n) = -1$.
- ASR is independent of Q^2 and survives strong interaction effects because of CVC.
- Nonconservation of axial current is neglected in derivation of ASR. Therefore, ASR is good for sufficiently high Q^2 but violated at low Q^2 (e.g. by PCAC terms).
- For generic nucleus of Z protons and N neutrons $S_A/A = \beta = (Z - N)/A$.

Explicit calculation of different nuclear effects:

- FMB correction cancels out. So does the pion correction.
- Both off-shell (OS) and nuclear shadowing (NS) corrections are nonzero. The requirement of exact cancellation between OS and NS corrections fixes $\text{Re } a^{(1,-)} / \text{Im } a^{(1,-)}$ for a state $(I = 1, C = -1)$.

Gross–Llewellyn-Smith sum rule (GLS)

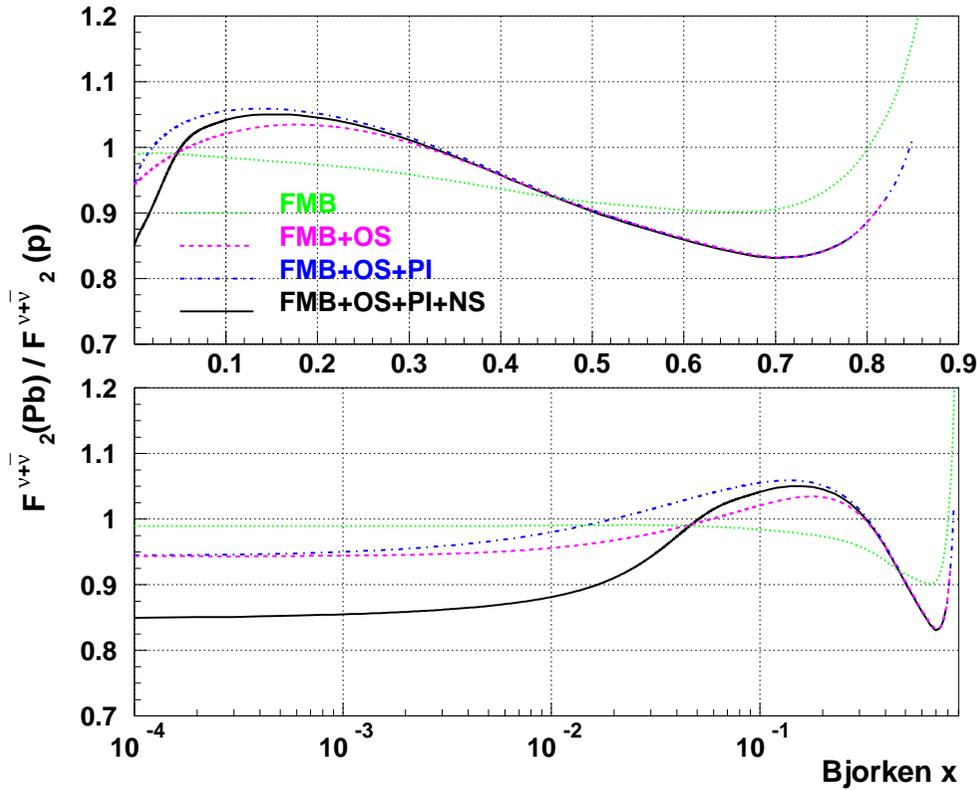
$$S_{\text{GLS}} = \frac{1}{2} \int_0^{M_A/M} dx (F_3^{\bar{\nu}} + F_3^{\nu})$$

- In parton model $S_{\text{GLS}} = 3$ is the number of valence u and d quarks in the target.
- In QCD the relation between S_{GLS} and the baryon number only holds at asymptotic Q^2 . S_{GLS} is affected by QCD radiative corrections and HT effects $S_{\text{GLS}} = 3(1 - \alpha_S/\pi - 3.25(\alpha_S/\pi)^2 + \dots) + \text{HT}_{\text{GLS}}$.

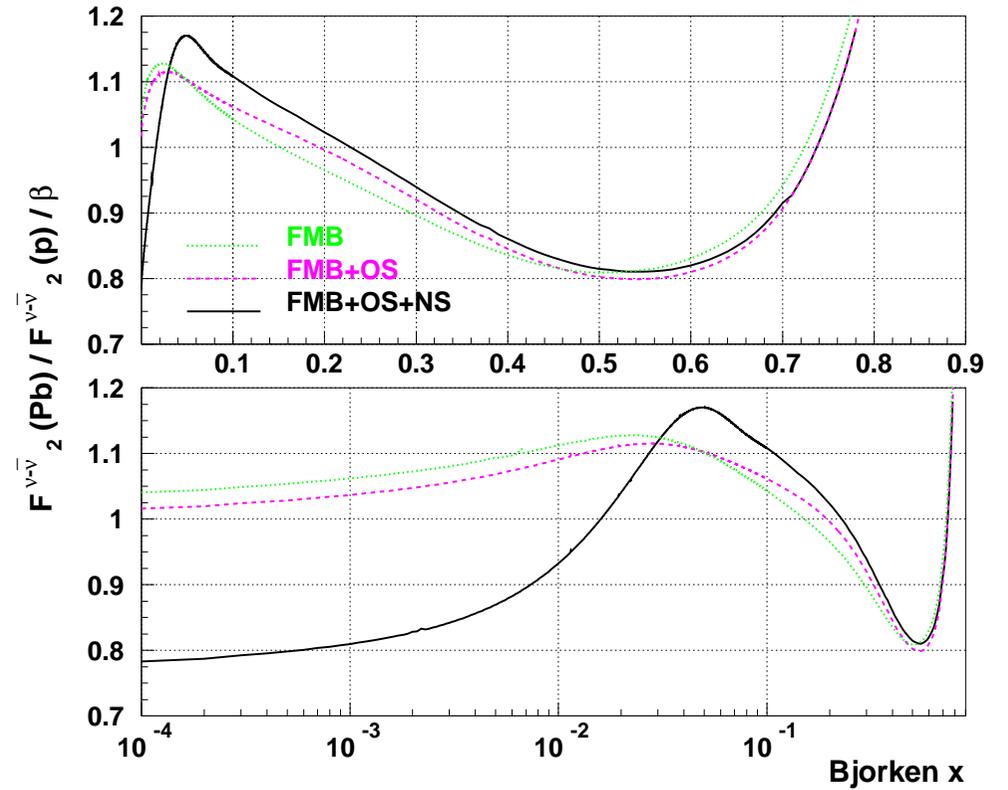
Explicit calculation of different nuclear effects:

- FMB correction to S_{GLS} cancels out. Nuclear pion correction to $F_3^{\nu+\bar{\nu}}$ vanishes.
- Both off-shell (OS) and nuclear shadowing (NS) corrections are nonzero. The requirement of exact cancellation between these corrections fixes $\text{Re } a^{(0,-)} / \text{Im } a^{(0,-)}$ for a $(I = 0, C = -1)$ state.

Dependence of nuclear effects on C -parity

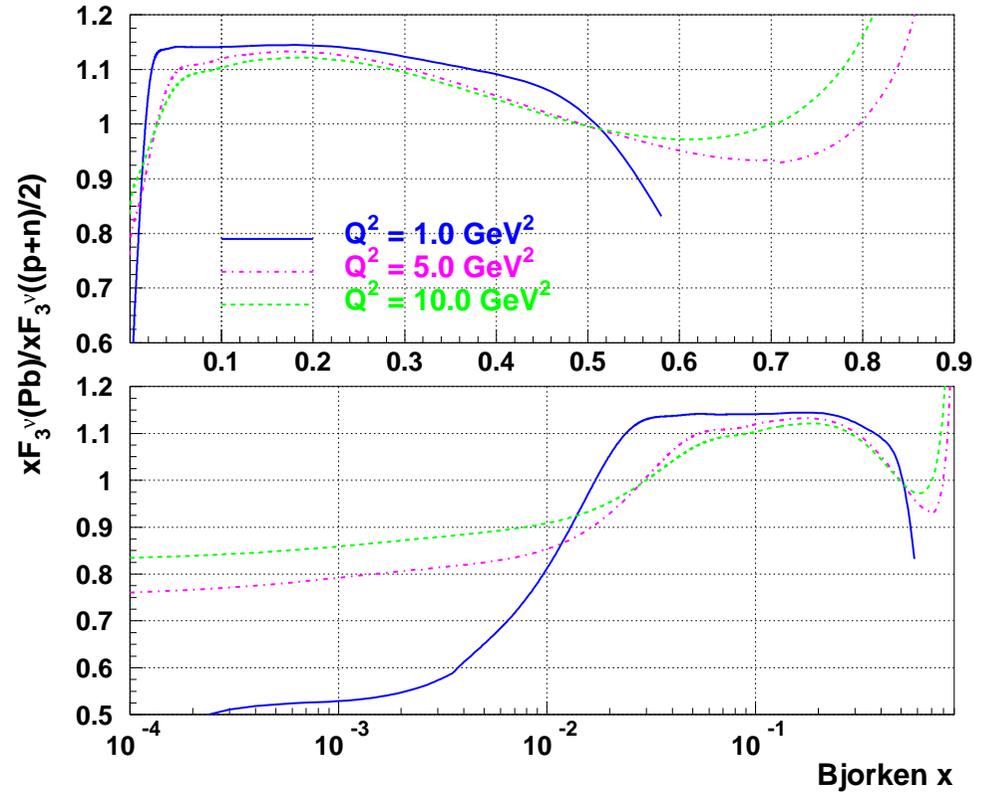
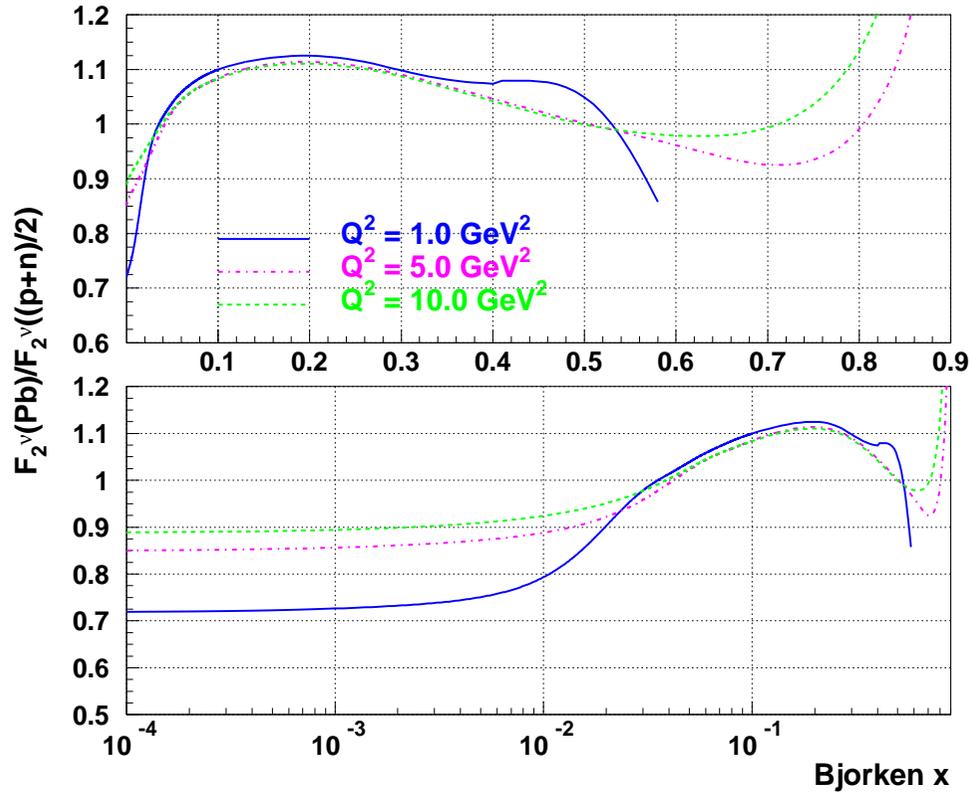


The ratio $\frac{1}{A}F_2^{(\nu+\bar{\nu})A}/F_2^{(\nu+\bar{\nu})p}$ calculated for ^{207}Pb at $Q^2 = 5 \text{ GeV}^2$. The labels on the curves correspond to effects due to Fermi motion and nuclear binding (FMB), off-shell correction (OS), nuclear pion excess (PI) and coherent nuclear processes (NS).



The ratio $\frac{1}{A}F_2^{(\nu-\bar{\nu})A}/(\beta F_2^{(\nu-\bar{\nu})p})$ calculated for ^{207}Pb at $Q^2 = 5 \text{ GeV}^2$. The labels on the curves correspond to effects due to Fermi motion and nuclear binding (FMB), off-shell correction (OS) and coherent nuclear processes (NS).

Nuclear Effects for F_2^ν and F_3^ν



The ratios of CC neutrino structure functions for ^{207}Pb normalized to one nucleon and those of the isoscalar nucleon $(p+n)/2$ (left panel for F_2 and right panel for xF_3).

Neutrino cross sections

Recently published cross section data:

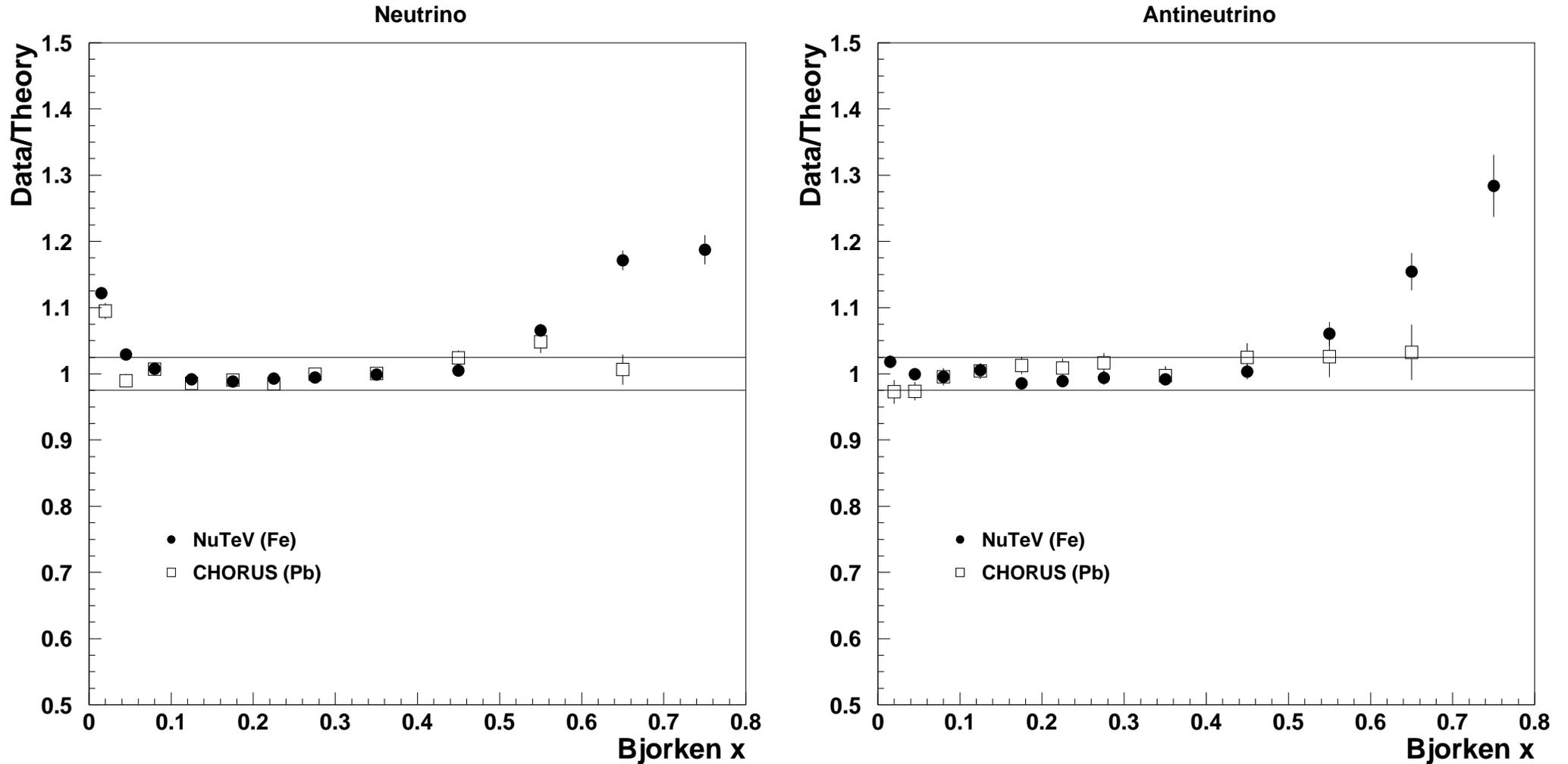
NuTeV data on ^{56}Fe :

about $1400\nu + 1200\bar{\nu}$ data points for $35 < E < 340$ GeV,
 $0.015 < x < 0.75$, $0.05 < y < 0.95$.

CHORUS data on ^{208}Pb :

about $600\nu + 600\bar{\nu}$ data points for $25 < E < 170$ GeV,
 $0.02 < x < 0.65$, $0.1 < y < 0.8$.

Data/Theory pulls for cross sections



The ratio of the measured differential cross-section and our calculation vs. x for neutrino and antineutrino interactions. The x -point is the weighted average over available E and y . The solid horizontal lines indicate a $\pm 2.5\%$ band.

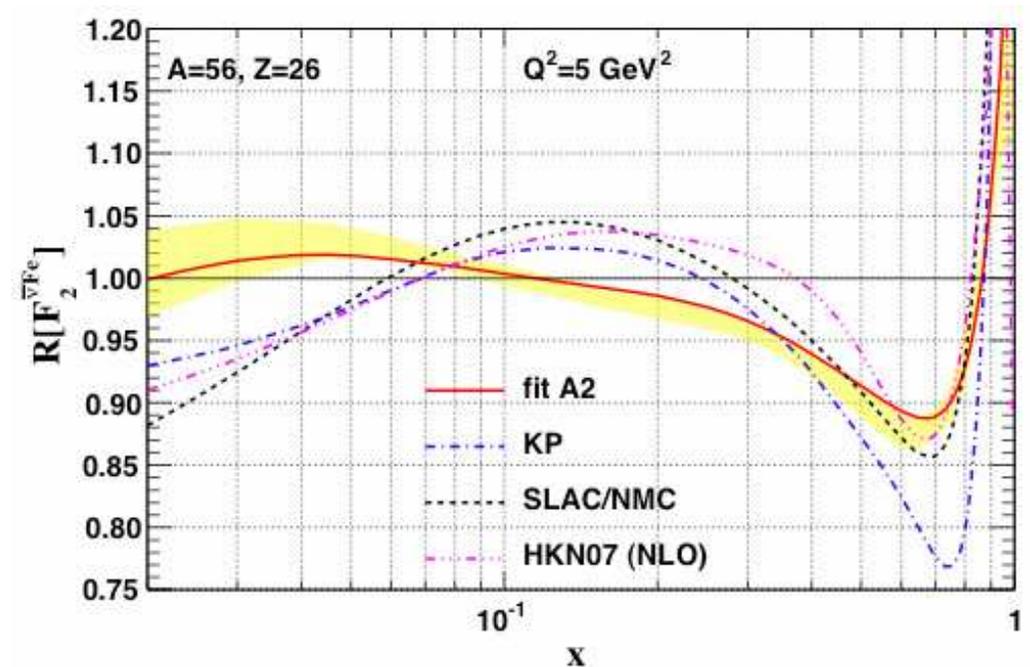
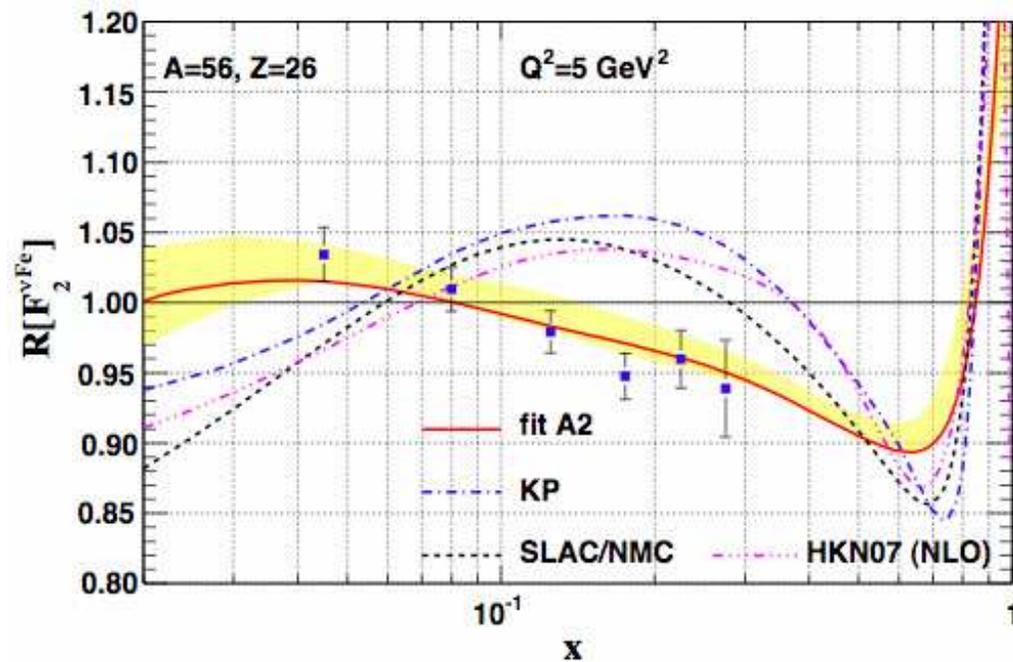
χ^2 analysis

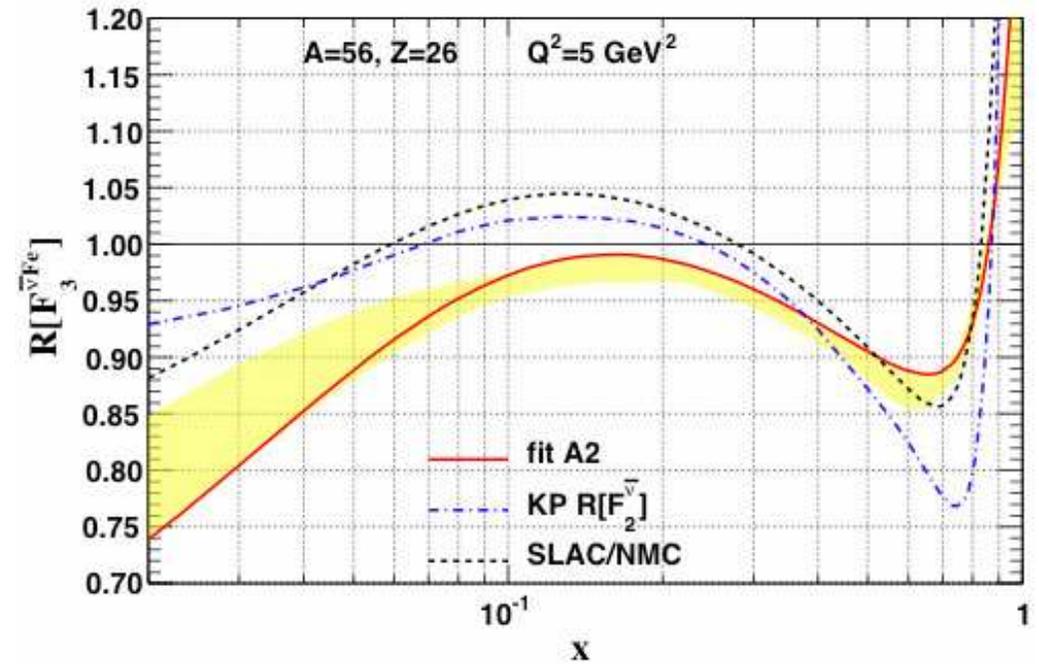
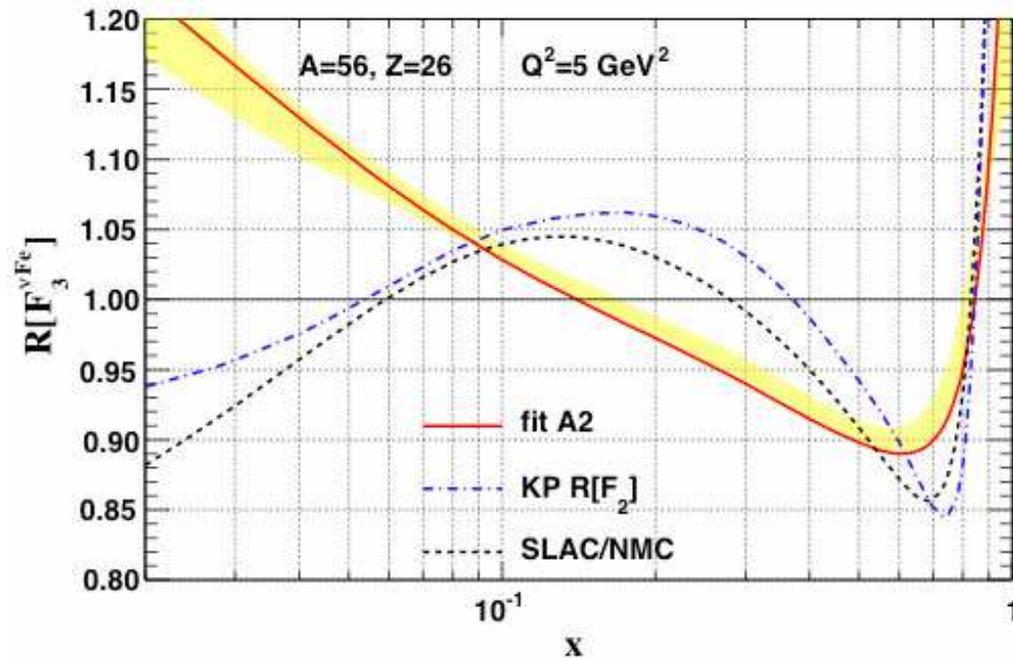
Cut	No. of data points		χ^2 /d.o.f.	
	Neutrino	Antineutrino	Neutrino	Antineutrino
NuTeV (Fe)				
No cut	1423	1195	1.36	1.10
$x > 0.015$	1324	1100	1.15	1.08
$x < 0.55$	738	671	1.16	1.02
$0.015 < x < 0.55$	686	620	0.97	1.01
CHORUS (Pb)				
No cut	607	607	0.68	0.84
$x > 0.02$	550	546	0.55	0.83
$x < 0.55$	506	507	0.74	0.83
$0.02 < x < 0.55$	449	447	0.60	0.83

Values of χ^2 obtained from comparison of NuTeV and CHORUS cross section data with our calculations (not a fit).

Comparison with other studies

Recently CTEQ group (Schienbein et.al. PRD77(2008)054013) presented the results of analysis of NuTeV neutrino cross sections aiming to extract PDFs in iron. The presented ratios $F_2^\nu(\text{Fe})/F_2^\nu(\text{N})$ for neutrino and antineutrino are significantly different from that measured with charged leptons, as well as from our calculations.





The presented figures are taken from Jorge Morfin's talk at Nufact'08 Workshop.

Nuclear quark distributions in iron extracted from neutrino data seems to be in a conflict with those from CL data. IMHO, this is difficult to reconcile with a common idea of universal and process-independent PDFs of hadrons (nuclei).

Summary

- A quantitative model was discussed, which describes x , Q^2 and A dependence of nuclear EMC effect observed for CL scattering.
- The approach was applied to calculate neutrino-nuclear DIS structure functions and cross sections. We performed a study of GLS and Adler sum rules for nuclear targets and dependence of nuclear effects on the structure function type and its C -parity.
- Our calculations of (anti)neutrino inelastic differential cross sections agree well with data on all studied targets [^{12}C (NOMAD), ^{56}Fe (NuTeV), ^{207}Pb (CHORUS)] for intermediate region of x .
 - NuTeV data show excess over theory at large $x > 0.5$ for both ν and $\bar{\nu}$. However, this is not supported by CHORUS(Pb) and NOMAD(C) data [and also NOMAD(Fe) data – Roberto Petti, private communication].
 - Both, NuTeV and CHORUS data show some excess over theory at small x (0.015 – 0.025) [also supported by preliminary NOMAD(Fe) data – Roberto Petti, private communication].