"Analysis of quasi-elastic neutrino charged-current scattering off ^{16}O and neutrino energy reconstruction"

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- Motivations.
- Formalism of the charged-current (CC) quasi-elastic (QE) scattering.
- Analysis of the CCQE interaction and neutrino energy reconstruction.
- Results.
- Summary.

For more details of the present approach see:
A.Butkevich, S.Mikheyev,Phys.Rev.C72:025501,2005;
A.Butkevich, S.Kulagin,Phys.Rev.C76:045502,2007;
A.Butkevich,Phys.Rev.C78:015501,2008

Motivations

- To study the neutrino oscillation effects on the terrestrial distance scale the intense neutrino beam cover the energy range from a few 100 MeV to a few GeV.

- In this energy range the neutrino nuclear scattering is dominated by (QE) scattering from bound nucleons and resonance production. An important source of systematic uncertainties is due to nuclear effects in neutrino interactions.

- Most event generators to model the scattering from nuclei are based on the Relativistic Fermi Gas Model (RFGM). High-precision QE electron and neutrinos scattering data are show that the accuracy of the RFGM prediction becomes poor at low Q^2 where the nuclear effects are largest.

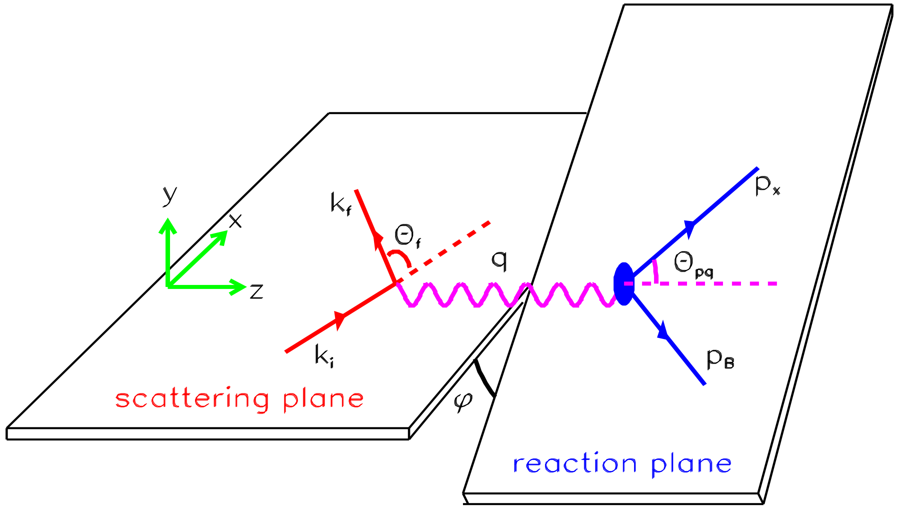
- Within the Relativistic Distorted-Wave Impulse Approximation (RDWIA) we study neutrino-nuclear scattering and analyses:

  (⋆) CCQE inclusive cross section $d\sigma/d\omega dQ^2$, $d\sigma/dQ^2$, and $d\sigma/d\cos\theta$
  (⋆) nuclear-model dependence of the two-track CCQE events selection efficiency

- Systematic uncertainties of the reconstructed neutrino energy

  (⋆) kinematic and calorimetric methods
  (⋆) kinematic method: nucleon Fermi motion effect and nuclear-model dependence of the reconstructed neutrino energy

Formalism of the quasi-elastic scattering



Kinematic definitions for A$(l, l'N)$B reactions.

In the lab frame the differential cross section for exclusive (anti)neutrino CC scattering can be written as

$$\frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} = R\frac{|\boldsymbol{p}_x|\varepsilon_x}{(2\pi)^5}\frac{|\boldsymbol{k}_f|}{\varepsilon_i}\frac{G^2\cos^2\theta_c}{2}L_{\mu\nu}W^{\mu\nu},$$

where Ω_f is the solid angle for the lepton momentum, Ω_x is the solid angle for the ejectile nucleon momentum, $G \simeq 1.16639 \times 10^{-11}$ MeV^{-2} is the Fermi constant, θ_C is the Cabbibo angle $(\cos\theta_C \approx 0.9749)$. The recoil factor R is given by

$$R = \left|1 - \frac{\varepsilon_x}{\varepsilon_B}\frac{\boldsymbol{p}_x\cdot\boldsymbol{p}_B}{\boldsymbol{p}_x\cdot\boldsymbol{p}_x}\right|^{-1},$$

and ε_x is solution to equation

$$\varepsilon_x + \varepsilon_B - m_A - \omega = 0,$$

where $\varepsilon_B = \sqrt{m_B^2 + \boldsymbol{p}_B^2}$, $\boldsymbol{p}_B = \boldsymbol{q} - \boldsymbol{p}_x = -\boldsymbol{p}_m$, $\boldsymbol{p}_x = \sqrt{\varepsilon_x^2 - m^2}$, and m_A, m_B, and m are masses of the target, recoil nucleus, and nucleon respectively.

The lepton tensor can be written as the sum of symmetric $L_S^{\mu\nu}$ and antisymmetric $L_A^{\mu\nu}$ tensors

$$
\begin{aligned}
L^{\mu\nu} &= L_S^{\mu\nu} + L_A^{\mu\nu} \\
L_S^{\mu\nu} &= 2\left(k_i^{\mu} k_f^{\nu} + k_i^{\nu} k_f^{\mu} - g^{\mu\nu} k_i k_f\right) \\
L_A^{\mu\nu} &= h 2i\epsilon^{\mu\nu\alpha\beta}(k_i)_{\alpha}(k_f)_{\beta},
\end{aligned}
$$

where h is $+1$ for positive lepton helicity and -1 for negative lepton helicity, $\epsilon^{\mu\nu\alpha\beta}$ is the antisymmetric tensor

The weak CC hadronic tensors $W_{\mu\nu}$ is given by bilinear products of the transition matrix elements of the nuclear CC operator J_{μ} between the initial nucleus state $|A\rangle$ and the final state $|B_f\rangle$ as

$$
W_{\mu\nu} = \sum_f \langle B_f, p_x | J_{\mu} | A\rangle \langle A | J_{\nu} | B_f, p_x\rangle
$$

where the sum is taken over undetected states.

In the inclusive reactions only the outgoing lepton is detected and the differential cross sections can be written as

$$\frac{d^3\sigma}{d\varepsilon_f d\Omega_f} = \frac{1}{(2\pi)^2}\frac{|\boldsymbol{k}_f|}{\varepsilon_i}\frac{G^2\cos^2\theta_c}{2}L_{\mu\nu}\mathcal{W}^{\mu\nu},$$

where $\mathcal{W}^{\mu\nu}$ is inclusive hadronic tensor.

- We describe the lepton-nucleon scattering in the Impulse Approximation (IA), in which only one nucleon of the target is involved in reaction and the nuclear current is written as the sum of single-nucleon currents. Then, the nuclear matrix element takes the form

$$\langle p, B|J^\mu|A\rangle = \int d^3r\ \exp(i\boldsymbol{t}\cdot\boldsymbol{r})\overline{\Psi}^{(-)}(\boldsymbol{p},\boldsymbol{r})\Gamma^\mu\Phi(\boldsymbol{r}),$$

  where Γ^μ is the vertex function, $\boldsymbol{t} = \varepsilon_B\boldsymbol{q}/W$ is the recoil-corrected momentum transfer, $W = \sqrt{(m_A+\omega)^2 - \boldsymbol{q}^2}$ is the invariant mass, Φ and $\Psi^{(-)}$ are relativistic bound-state and outgoing wave functions.

- The single-nucleon charged current has $V{-}A$ structure $J^\mu = J^\mu_V + J^\mu_A$. For a free nucleon vertex function $\Gamma^\mu = \Gamma^\mu_V + \Gamma^\mu_A$ we use CC2 vector current vertex function

$$\Gamma^\mu_V = F_V(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_M(Q^2)$$

and the axial current vertex function

$$\Gamma_A^\mu = F_A(Q^2)\gamma^\mu\gamma_5 + F_P(Q^2)q^\mu\gamma_5.$$

We use the MMD approximation [P.Mergell et al (1996)] of the nucleon form factors.

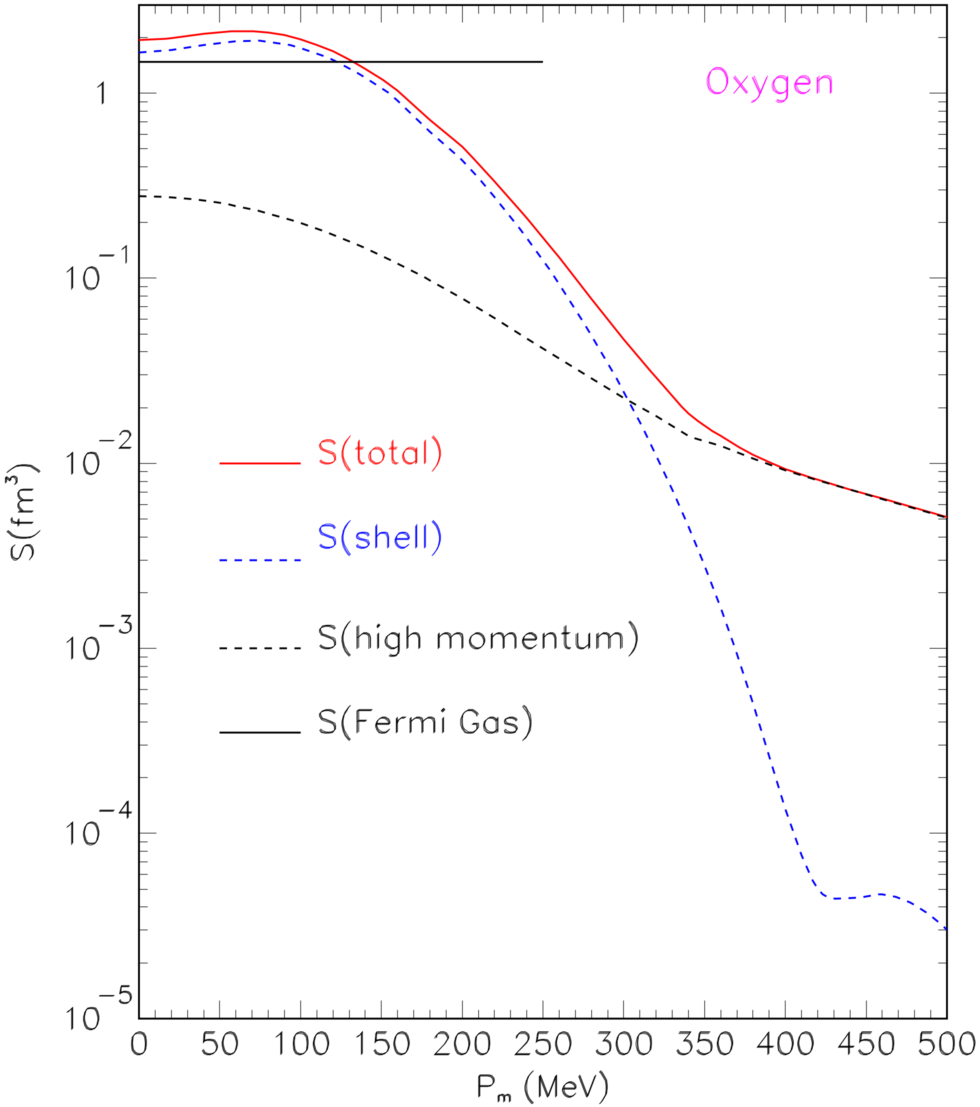
- The axial F_A and psevdoscalar F_P form factors in the dipole approximation are parameterized as

$$F_A(Q^2) = \frac{F_A(0)}{(1+Q^2/M_A^2)^2}, \quad F_P(Q^2) = \frac{2mF_A(Q^2)}{m_\pi^2 + Q^2},$$

where $F_A(0) = 1.267$, m_π is the pion mass, and $M_A \simeq 1.032$ GeV is the axial mass.

- In independ particle shell model the relativistic bound-state functions Φ are obtained within the Hartree–Bogolioubov approximation in the $\sigma - \omega$ model [B.Serot et al. 1986]. According to JLab data [K. Fissum (2004)] the occupancy of the IPSM orbitals of ^{16}O is approximately 75% on average. We assume that the missing strength can be attributed to the short-range NN-correlations in the ground state.
  - ⋆ We consider a phenomenological model in PWIA, which incorporates high-energy and high-momentum component P_{HM} due to NN-correlations [C.Ciofi degli Atti et al (1996), S.Kulagin et al (2006)].

⋆ In our calculations the spectral function P_{HM} incorporates 25% of the total normalization of the spectral function.



- In the RDWIA the final state interaction between the outgoing nucleon and the residual nucleus is taking into account and the ejectile wave function Ψ is solution of a Schrödinger equation containing equivalent central and spin-orbit potentials, which are functions of the scalar and vector potentials S and V, and are energy dependent.

- We use the LEA program [J.J. Kelly (1995)] for numerical calculation of the distorted wave functions with EDAD1 SV relativistic optical potential [E. Cooper (1993)].

- In the PWIA the final state interaction between the outgoing nucleon and the residual nucleus is neglected and the ejectile wave function Ψ is plane wave.

- In Fermi Gas model for oxygen we use p_F=250 MeV/c and ε=27 MeV. The RFGM does not account nuclear shell structure, FSI effect, and the presence of NN-correlations.

Analysis and Results

Low Q^2 problem

Charged-current QE events distributions as a function of Q^2 or $\cos\theta$ were measured by K2K and MiniBoone experiments. High statistic data show a disagreement with the RFGM prediction, $i.e.$ the data samples exhibit significant deficit in the region of low $Q^2 \leq$0.2 GeV^2 and small muon scattering angles.

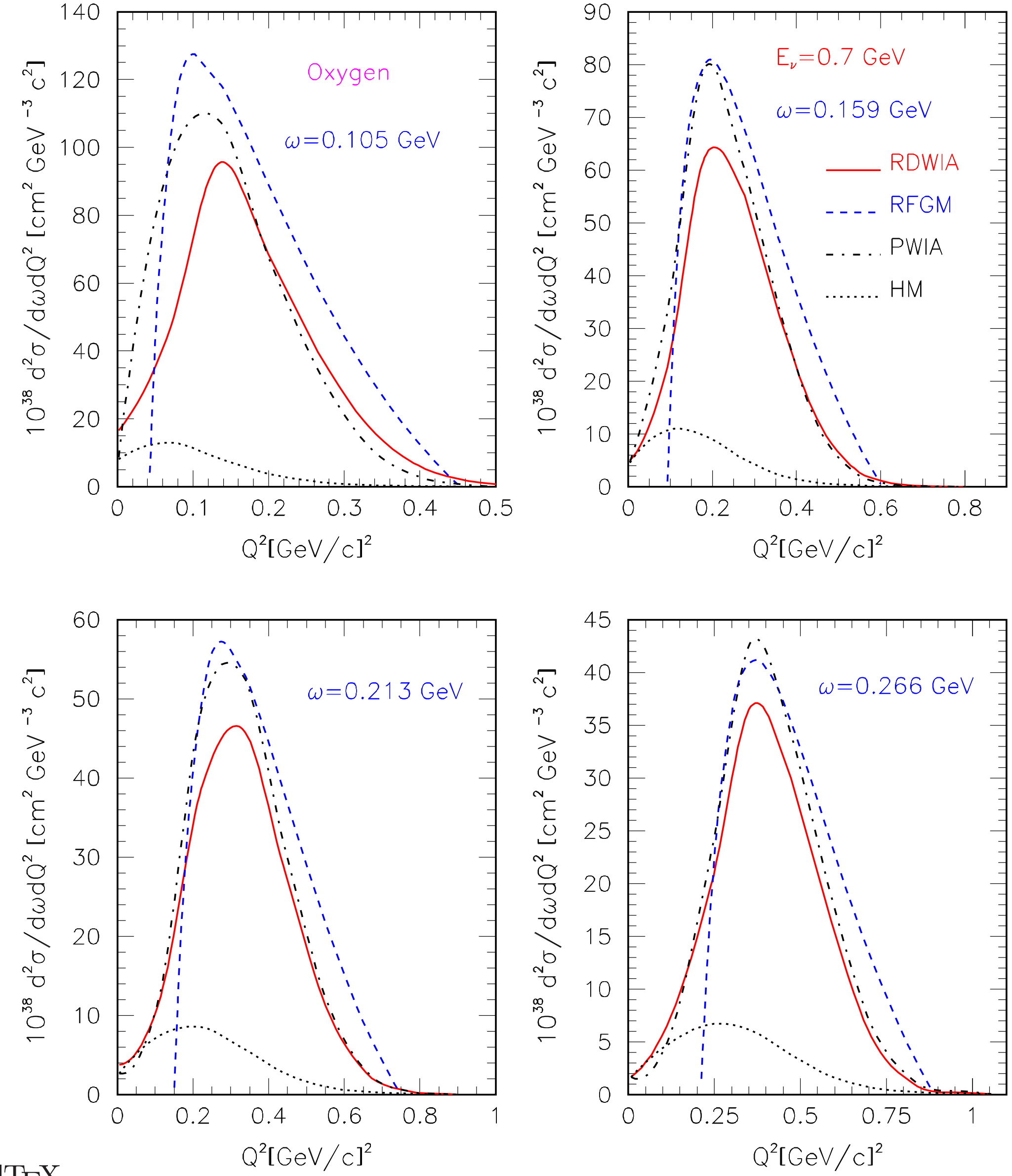
Note that in the case of (anti)neutrino scattering off free nucleon CCQE differential cross sections $d\sigma^{\nu,\overline{\nu}}/dQ^2$ at $Q^2 \longrightarrow 0$ can be written as

$$\frac{d\sigma^{\nu,\overline{\nu}}}{dQ^2} = \frac{G^2}{2\pi}\cos^2\theta_c[F_V^2(0) + F_A^2(0)]$$

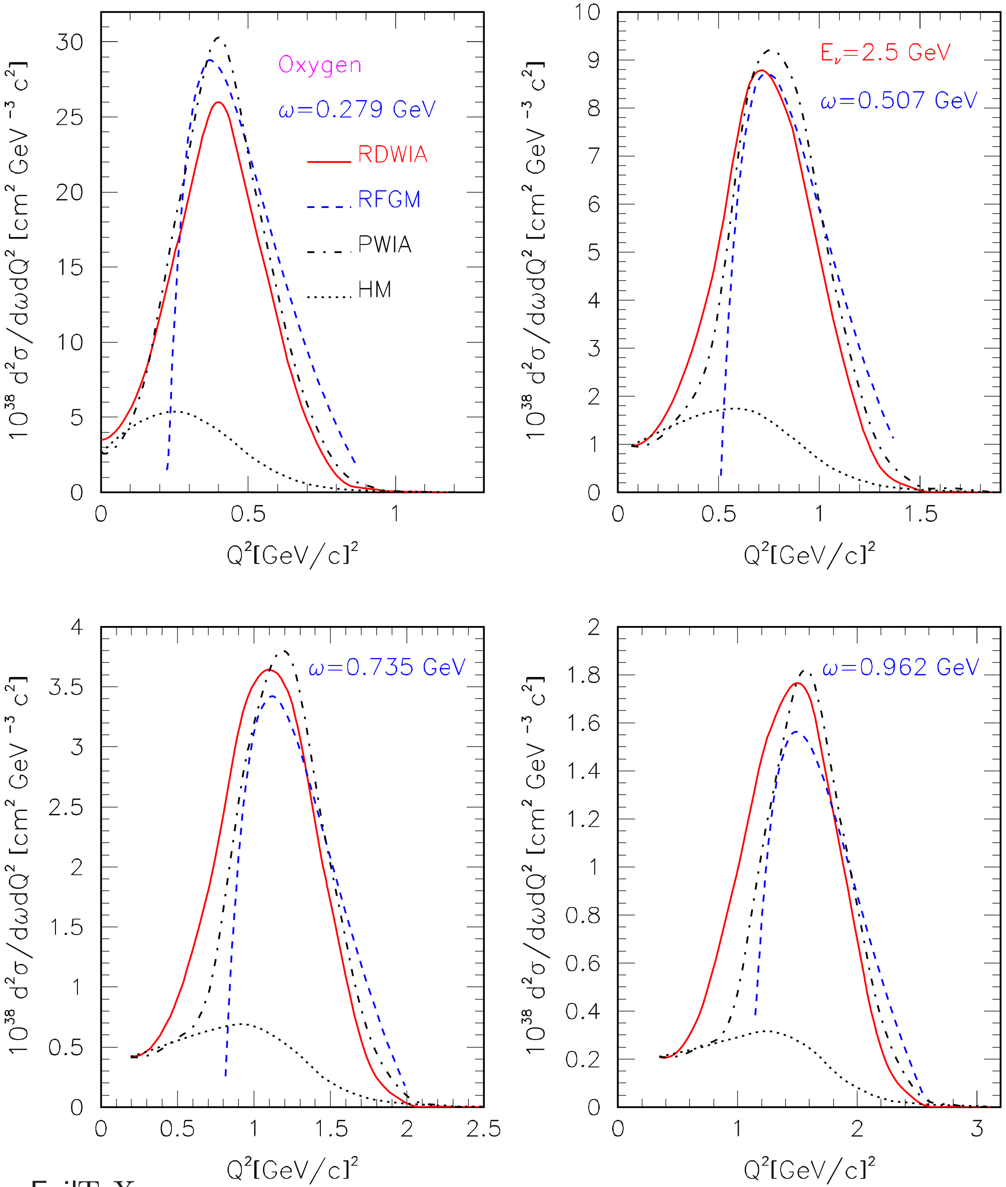
and do not depend on neutrino energy. The difference

$$\frac{d\sigma^{\nu}}{dQ^2} - \frac{d\sigma^{\overline{\nu}}}{dQ^2} = \frac{G^2}{\pi}\cos^2\theta_c\frac{Q^2}{m\varepsilon_i}\left(1 - \frac{Q^2}{4m\varepsilon_i}\right)(F_V + F_M)F_A$$

is proportional to F_A and decreases with neutrino energy. In the range of $\varepsilon_i \sim$ 0.5 $\div$ 1 GeV it can be used for measuring of the axial form factor F_A.



Inclusive cross section versus the four-momentum transfer Q^2 for neutrino scattering on ^{16}O with energy ε_ν=0.7 GeV and for the four values of energy transfer: ω=0.105, 0.159, 0.213 and 0.266 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the high-momentum component contribution to inclusive cross section.

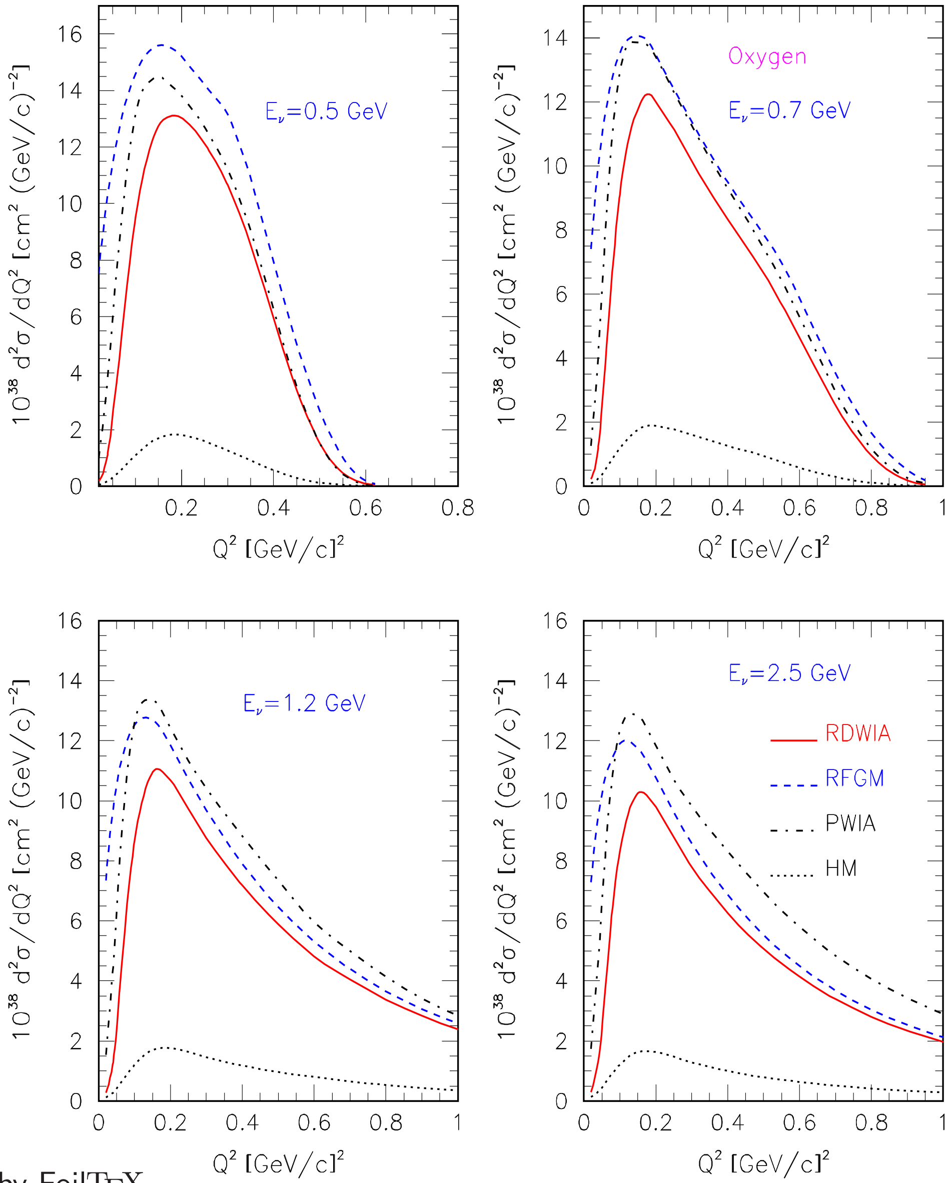


Inclusive cross section versus Q^2 for neutrino scattering on ^{16}O with energy ε_ν=2.5 GeV and for the four values of energy transfer: ω=0.279, 0.507, 0.735 and 0.962 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the high-momentum component contribution to inclusive cross section.

Comments

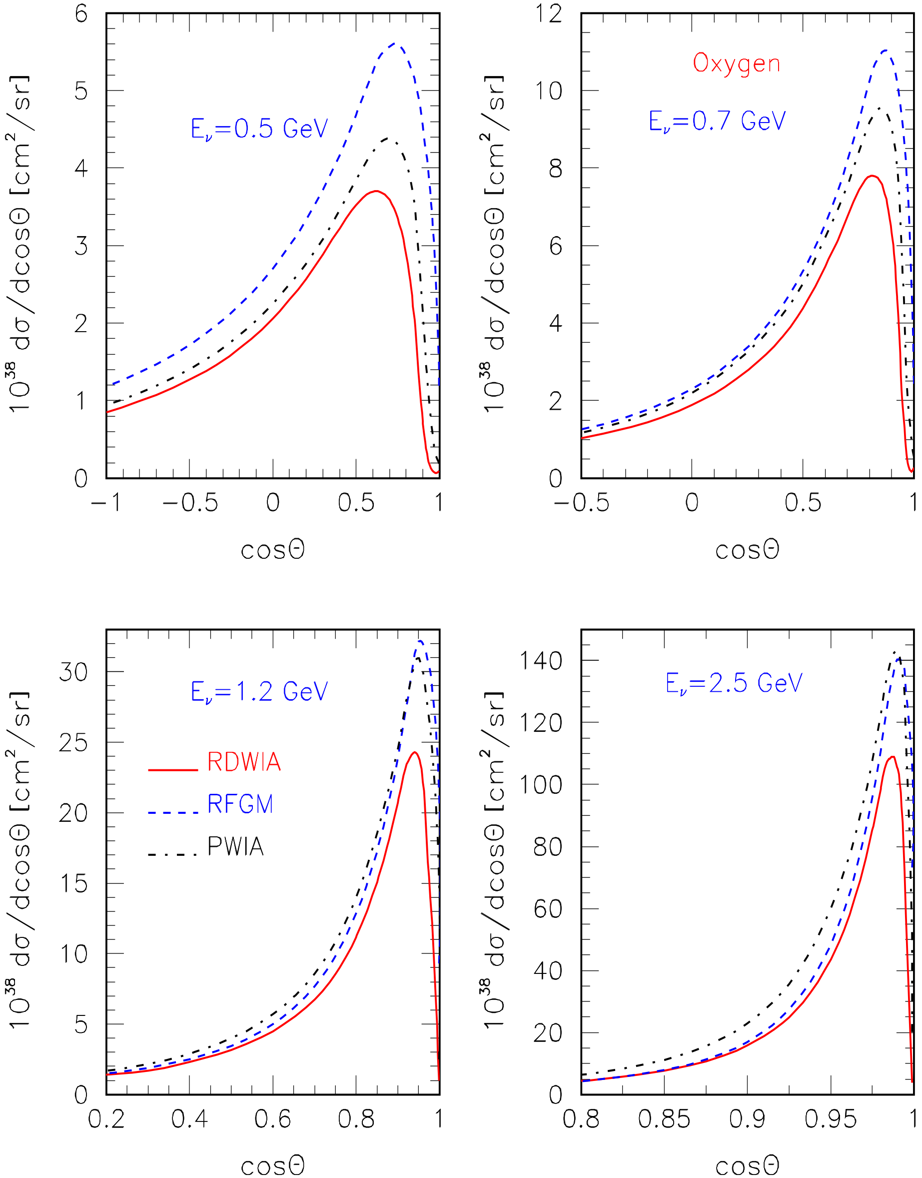
- Theoretical uncertainties of the correlated NN-pairs contribution to the inclusive cross section are higher as compared to the shell-nucleon contribution. The electron nucleus scattering data K.G. Fissum et al. 2004(JLab), D.Rohe et al. (2006)(JLab), and M.Iodice et al. (2007)(Frascati) show that more complicated configurations than simple hard interaction between two nucleons are involved in this case.

- The off-shell ambiguities will be important for the high-momentum component, and one might expect the details of the off-shell extrapolation to became critical.



Inclusive cross section versus Q^2 for neutrino scattering on ^{16}O and for the four values of incoming neutrino energy: ε_ν=0.5, 0.7, 1.2 and 2.5 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. The dotted line is the high-momentum component contribution to inclusive cross section. The RFGM results are higher than those obtained in the RDWIA.

In the region around the maximum $Q^2 = 0.2$ (GeV/c)^2 the difference is about $\sim$ 18% for $\varepsilon_\nu = 0.5$ GeV and $\sim$ 11% for $\varepsilon_\nu = 2.5$ GeV. At $Q^2 = 0.05$ (GeV/c)^2 the contribution of the HM-component increases with energy from $\sim 15\%$ up to 23% in the energy range $0.5 \div 2.5$ GeV.



Inclusive cross section versus the muon scattering angle for the four values of incoming neutrino energy: ε_ν=0.5, 0.7, 1.2 and 2.5 GeV. The solid line is the RDWIA calculation while the dashed and dashed-dotted lines are respectively the RFGM and PWIA calculations. In the region $0.8 < cos\theta < 1$ the value of the RFGM cross sections are higher than those obtained within the RDWIA.

For energy $\varepsilon_\nu = 0.5$ GeV ($\varepsilon_\nu = 2.5$ GeV) this discrepancy is about 25 times ($\sim 11\%$) at $\cos\theta = 0.95$ and $\sim 89\%$ ($\sim 2\%$) at $\cos\theta = 0.8$.

Selection of CCQE two-track events: nuclear-model dependence of the cut

- The two-track events are divided into two samples: QE and non QE enriched samples. Depending on detector capabilities dE/dx, information is applied to the second track for π/p separation.
- The measurement of the muon momentum and angle allows predict the angle of recoil proton assuming neutrino scattering off nucleon at rest.
- If the measured second track agrees with this prediction within $\Delta\theta$, it is likely a CCQE event.
- Using MC simulation based on the Fermi gas model the value of $\Delta\theta$ is chosen to give a reliable separation between the QE and non QE events.

We regard the angle θ_{pq} between the direction of outgoing proton and momentum transfer assuming that neutrino energy is reconstructed.

- For neutrino QE scattering on nucleon at rest $\boldsymbol{q} = \boldsymbol{p}_x$ and $\cos\theta_{pq}$ =1.
- For scattering off bound nucleon with momentum $\boldsymbol{p}_m$, $\boldsymbol{p}_x = \boldsymbol{p}_m + \boldsymbol{q}$ and

$$\cos\theta_{pq} = \frac{\boldsymbol{p}_x^2 + \boldsymbol{q}^2 - \boldsymbol{p}_m^2}{2|\boldsymbol{p}_x||\boldsymbol{q}|}.$$

- The maximal value of θpq corresponds to scattering on nucleon with maximal momentum $\boldsymbol{p}_{max}$, i.e.

$$\cos\theta^m_{pq} = \frac{\boldsymbol{p}_x^2 + \boldsymbol{q}^2 - \boldsymbol{p}_{max}^2}{2|\boldsymbol{p}_x||\boldsymbol{q}|}$$

  and $\cos\theta^m_{pq} \leq \cos\theta_{pq} \leq 1$.

- In the RFGM, $\varepsilon_m = \sqrt{\boldsymbol{p}_m^2 + m^2} - \epsilon_b$ and the recoil proton energy $\varepsilon_x = \sqrt{\boldsymbol{p}_m^2 + m^2} - \epsilon_b + \omega$ and for $|\boldsymbol{p}_{max}| = p_F$ we have

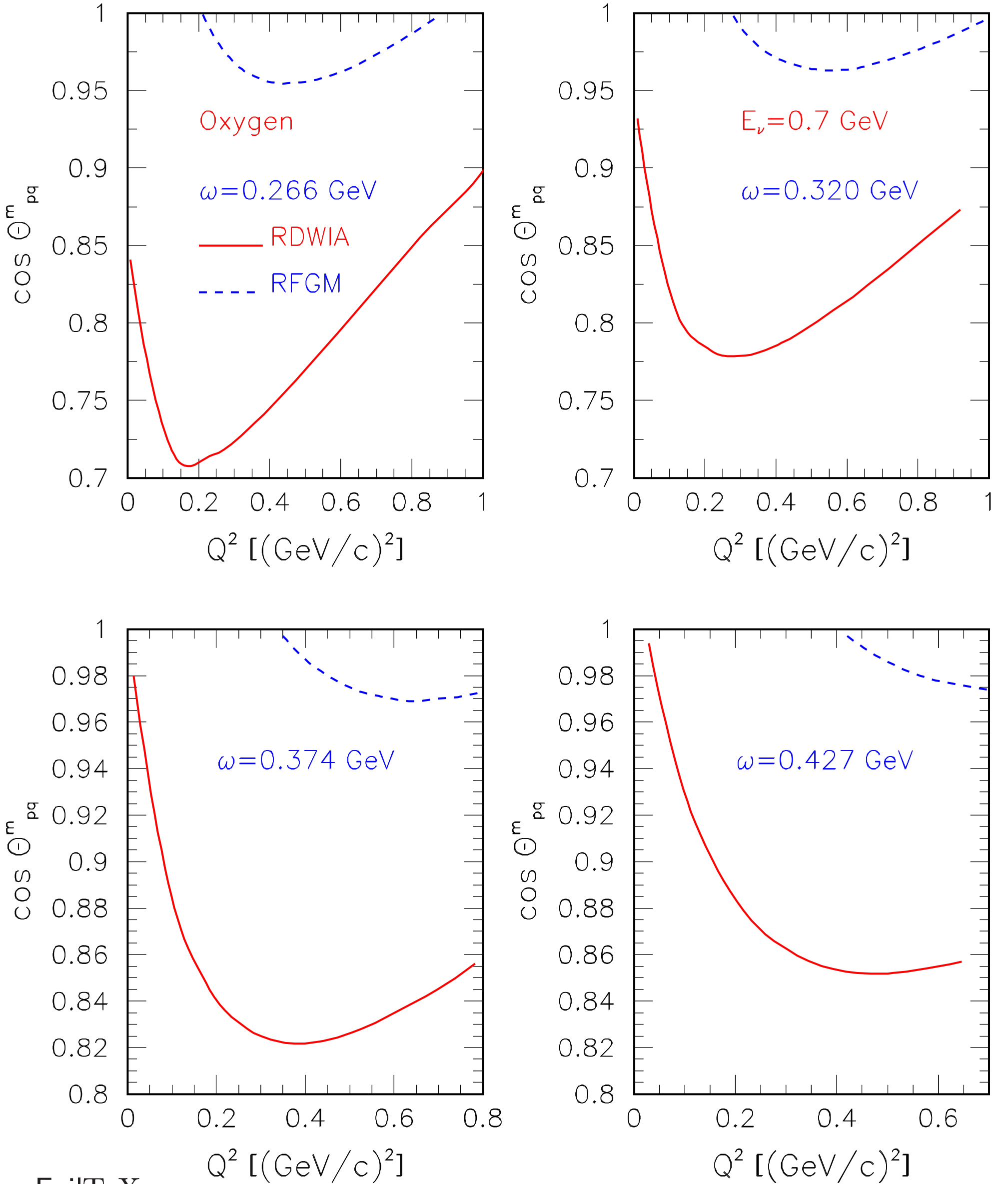
$$\boldsymbol{p}_x^2 = p_F^2 + \tilde{\omega}^2 + 2\tilde{\omega}\sqrt{p_F^2 + m^2},$$

  where $\tilde{\omega} = \omega - \epsilon_b$.

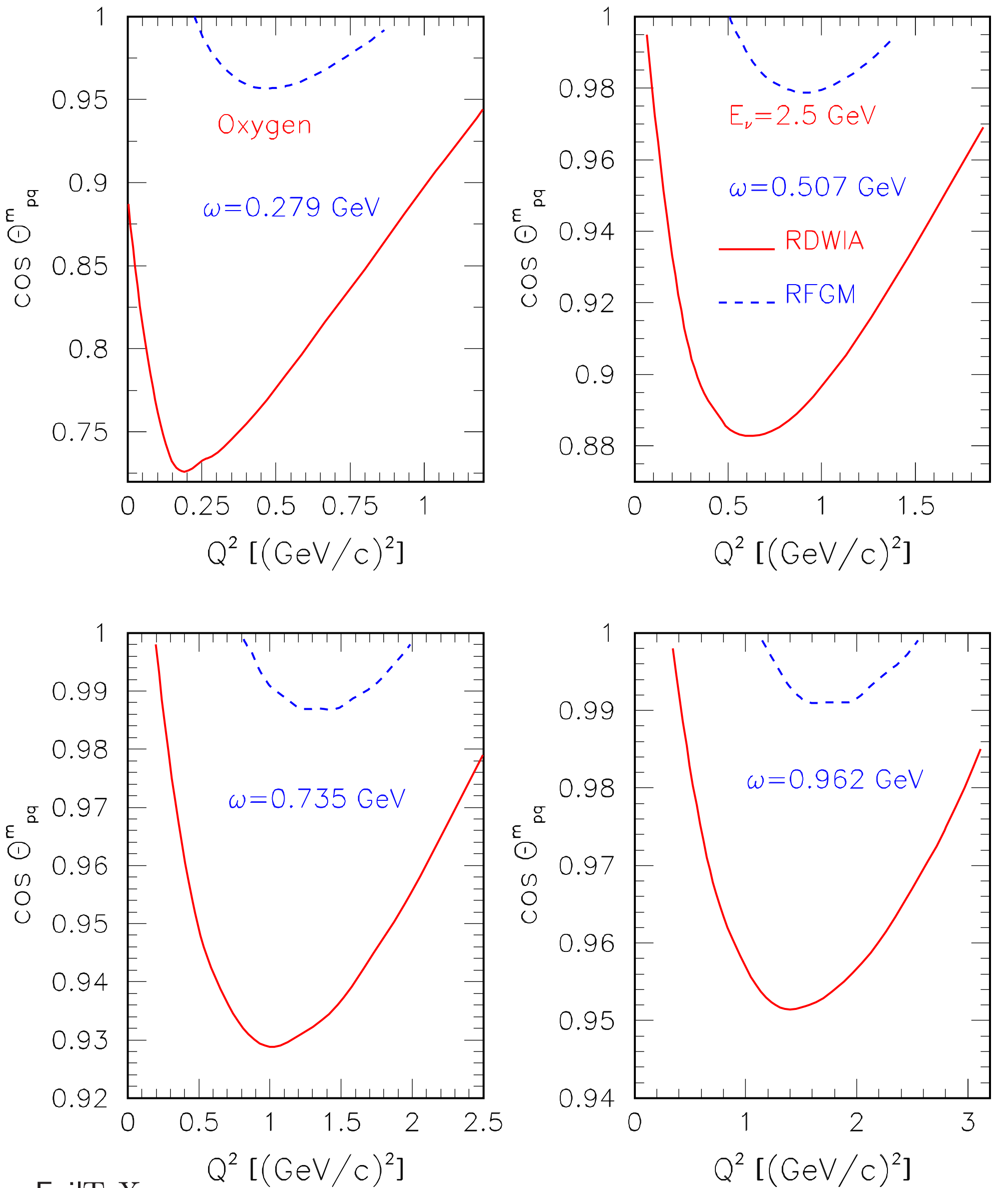
- In the RDWIA the energy of an outgoing nucleon can be written as follow $\varepsilon_x = \omega + m_A - \varepsilon_B$ and we have

$$\cos\theta^m_{pq} = \frac{\bar{\omega}(2m + \bar{\omega}) + (Q^2 - m^2) - \boldsymbol{p}_{max}^2}{\sqrt{\bar{\omega}(2m + \bar{\omega})(Q^2 + m^2)}}, \tag{2}$$

  where $\bar{\omega} = \omega - \langle\varepsilon_m\rangle - \boldsymbol{p}_{max}^2/2m^\star_B$, $m^\star_B = m_A - m + \langle\varepsilon_m\rangle$, $|\boldsymbol{p}_{max}|$=500 MeV/c, and mean missing energy for the shell nucleons $\langle\varepsilon_m\rangle = 27.1$ MeV.



Contours of the phase volume in the $(\cos\theta_{pq}, Q^2)$ coordinates for neutrino scattering off ^{16}O with energy ε_ν=0.7 GeV and for the four values of energy transfer: ω=0.288, 0.320, 0.374 and 0.427 GeV. The solid line is the RDWIA calculation whereas the dashed line is the RFGM calculation.



Contours of the phase volume in the $(\cos\theta_{pq}, Q^2)$ coordinates for neutrino scattering off ^{16}O with energy ε_ν=2.5 GeV and for the four values of energy transfer: ω=0.279, 0.507, 0.735 and 0.962 GeV. The solid line is the RDWIA calculation whereas the dashed line is the RFGM calculation. In the RDWIA kinematics the phase volume is larger then in the RFGM and the difference decreases with ω and neutrino energy.

Neutrino energy reconstruction

To evaluate the neutrino mass squared difference in the muon neutrino disappearance experiments, the probability of ν_μ disappearance versus neutrino energy is measured.

Apparently that $\delta(\Delta m^2) \approx \delta\varepsilon_\nu / L$

CCQE scattering

- Because the CCQE interaction represent a two-particle scattering process, it forms a good signal sample, and neutrino energy may be estimated using the kinematic of this reaction.
- There are two ways to measure the neutrino energy using CCQE events: kinematic or calorimetric reconstruction.

Kinematic reconstruction method

- In detectors with the energy threshold for proton detection $\varepsilon^p_{th} \geq 1$ GeV (Cherenkov detectors) the muon neutrino CCQE interactions will produce the one-track events. The kinematic reconstruction is applied for these events.

- Target nucleon is in rest

  The method is based on the assumption that the target nucleon to be at rest inside the nucleus and the correlation between the incident neutrino energy and a reconstructed muon momentum and scattering angle is used in this method.

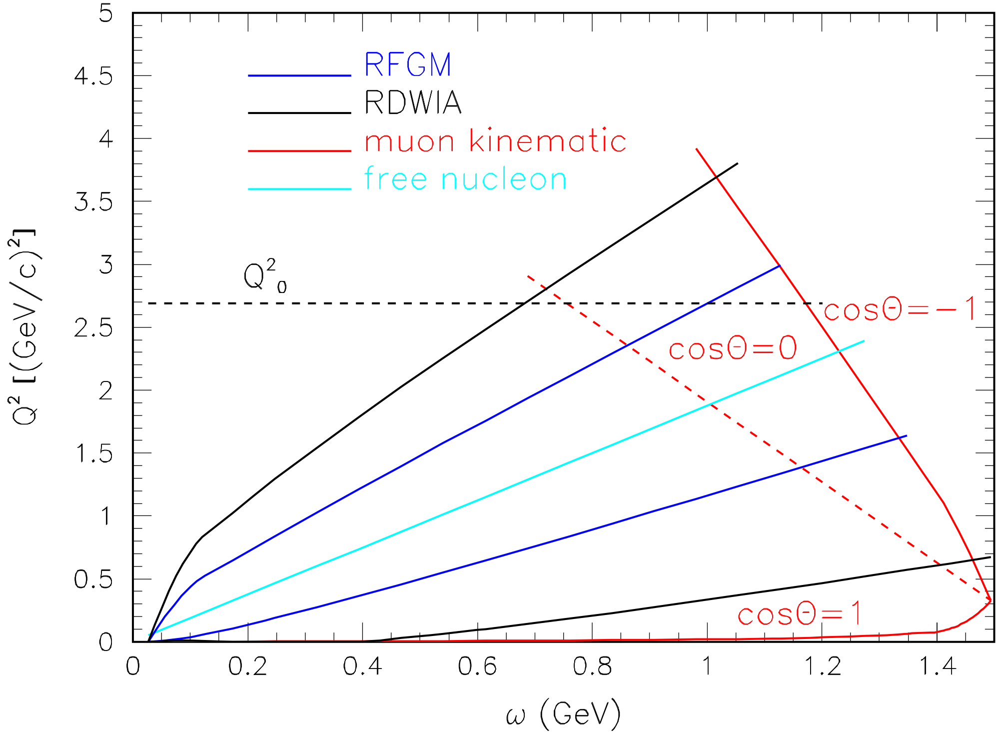
$$\varepsilon_r = \frac{\varepsilon_f(m - \epsilon_b) - (\epsilon_b^2 - 2m\epsilon_b + m_\mu^2)/2}{(m - \epsilon_b) - \varepsilon_f + k_f \cos\theta}.$$

Note

Formula can not be used for neutrino energy reconstruction at $\varepsilon_f \geq (m - \epsilon_b) + k_f \cos\theta$ or $Q^2 = Q_0^2 \geq 2m\varepsilon_i - m_\mu^2$, because the value of resulting ε_r is negative in this region. In terms of energy transfer, it is corresponds to the range $\omega_1 \leq \omega \leq \omega_2$, where ω_2 is the solution to the equation

$$Q_0^2 = 2\varepsilon_i(\varepsilon_f - k_f \cos\theta) - m_\mu^2. \tag{3}$$

with $\tilde{\omega} = \omega - \langle\varepsilon_m\rangle - \boldsymbol{p}_{max}^2/2m_B^\star$,



and in the RFGM, ω_1 is the solution to the equation In the RDWIA, ω_1 is the value of ω at which

$$Q_0^2 = \left[|\boldsymbol{p}_{max}| + \sqrt{(\tilde{\omega}^2 + 2m\tilde{\omega})} \right]^2 - \omega^2 \tag{4}$$

$$Q_0^2 = \left[p_F + \sqrt{(p_F^2 + 2\epsilon_F\tilde{\omega} + \tilde{\omega}^2)} \right]^2 - \omega^2, \tag{5}$$

where $\tilde{\omega} = \omega - \epsilon_b$ and $\epsilon_F^2 = p_F^2 + m^2$.
The size of this range $\Delta\omega = \omega_2 - \omega_1$ is proportional to $|\boldsymbol{p}_{max}|$ (p_F) and reduces with increase $\cos\theta$.

- Nucleon Fermi motion effect

Using momentum $\boldsymbol{p}_x = \boldsymbol{p}_m + \boldsymbol{q}$ and energy balance in the RFGM

$$\varepsilon_i + \sqrt{\boldsymbol{p}_m^2 + m^2} - \epsilon_b = \varepsilon_f + \varepsilon_x,$$

or

$$\varepsilon_i + m_A = \varepsilon_x + \varepsilon_f + \varepsilon_B.$$

in the RDWIA for shell nucleon and

$$\varepsilon_i + \varepsilon_N = \varepsilon_f + \varepsilon_x.$$

for nucleons with energy ε_N in the correlated NN-pair, we obtain the second order equation for neutrino energy which takes into account the bound nucleon momentum and energy distributions

$$A\varepsilon_r^2 - B\varepsilon_r + C = 0.$$

Now the reconstructed energy ε_r is a function of $(\boldsymbol{p}_m, \varepsilon_m, \cos\tau)$, where $\cos\tau = \boldsymbol{p}_m \cdot \boldsymbol{q}/|\boldsymbol{p}_m \cdot \boldsymbol{q}|$

- Moments of the reconstructed neutrino energy

The distribution $\varepsilon_r(\boldsymbol{p}_m, \varepsilon_m, \cos\tau)$ corresponds to measured values of $(k_f, \cos\theta)$ and at $\varepsilon_m, \boldsymbol{p}_m \rightarrow 0$ has an asymptotic form given by the formula for nucleon at rest.

- The n-th moment of $\varepsilon_r(k_f, \cos\theta, \boldsymbol{p}_m, \varepsilon_m)$ distribution versus of k_f and $\cos\theta$ can be written as

$$\langle\varepsilon_r^n(k_f, \cos\theta)\rangle = \int_{p_{min}}^{p_{max}} d\boldsymbol{p} \int_{\varepsilon_{min}}^{\varepsilon_{max}} S(\boldsymbol{p}, \varepsilon)[\varepsilon_r(k_f, \cos\theta, \boldsymbol{p}, \varepsilon)]^n d\varepsilon,$$

where $S(\boldsymbol{p}, \varepsilon)$ is the probability density function (pdf) for the target nucleon momentum and energy distribution being normalized with respect to the unit area.

- The pdf can be written as follow:

  ⋆ In the RFGM - $S(\boldsymbol{p}, \varepsilon_m) = 3\delta(\varepsilon - \epsilon_b)/4\pi p_F^3$

  ⋆ In the RDWIA - $S(\boldsymbol{p}, \varepsilon) = \sum_\alpha v_\alpha S_\alpha(\boldsymbol{p})\delta[\varepsilon - (\varepsilon_m)_\alpha] + v_{NN} S_{NN}(\boldsymbol{p}, \varepsilon)$,
  where S_α and S_{NN} are correspondingly the pdf for the momentum and energy of nucleons on the shell α and in the correlated NN-pairs.

- The mean of $\varepsilon_r(k_f, \cos\theta)$ and its variance $\sigma^2(\varepsilon_r)$ are defined by

$$\bar{\varepsilon}_r(k_f, \cos\theta) = \langle \varepsilon_r(k_f, \cos\theta) \rangle,$$
$$\sigma^2(\varepsilon_r) = \langle \varepsilon_r^2(k_f, \cos\theta) \rangle - \bar{\varepsilon}_r^2(k_f, \cos\theta)$$

- In principle the cut $R = \sigma(\varepsilon_r)/\bar{\varepsilon}_r \le \delta$ may be imposed (event by event) to select the events with well reconstructed energy.

Accuracy of reconstructed neutrino energy

- The accuracy of reconstructed energy $\varepsilon_r(\varepsilon_i)$ as a function of ε_i can be estimated using the moments of $\varepsilon_r(k_f, \cos\theta)$ distribution

$$\langle \varepsilon_r^n(\varepsilon_i) \rangle = \int dk_f \int W(k_f, \cos\theta)[\varepsilon_r(k_f, \cos\theta)]^n d\cos\theta,$$

where $W(k_f, \cos\theta)$ is the pdf of the muon momentum and scattering angle, i.e.,

$$W(k_f, \cos\theta) = \frac{1}{\sigma_{tot}(\varepsilon_i)} \frac{d^2\sigma}{dk_f d\cos\theta},$$

and

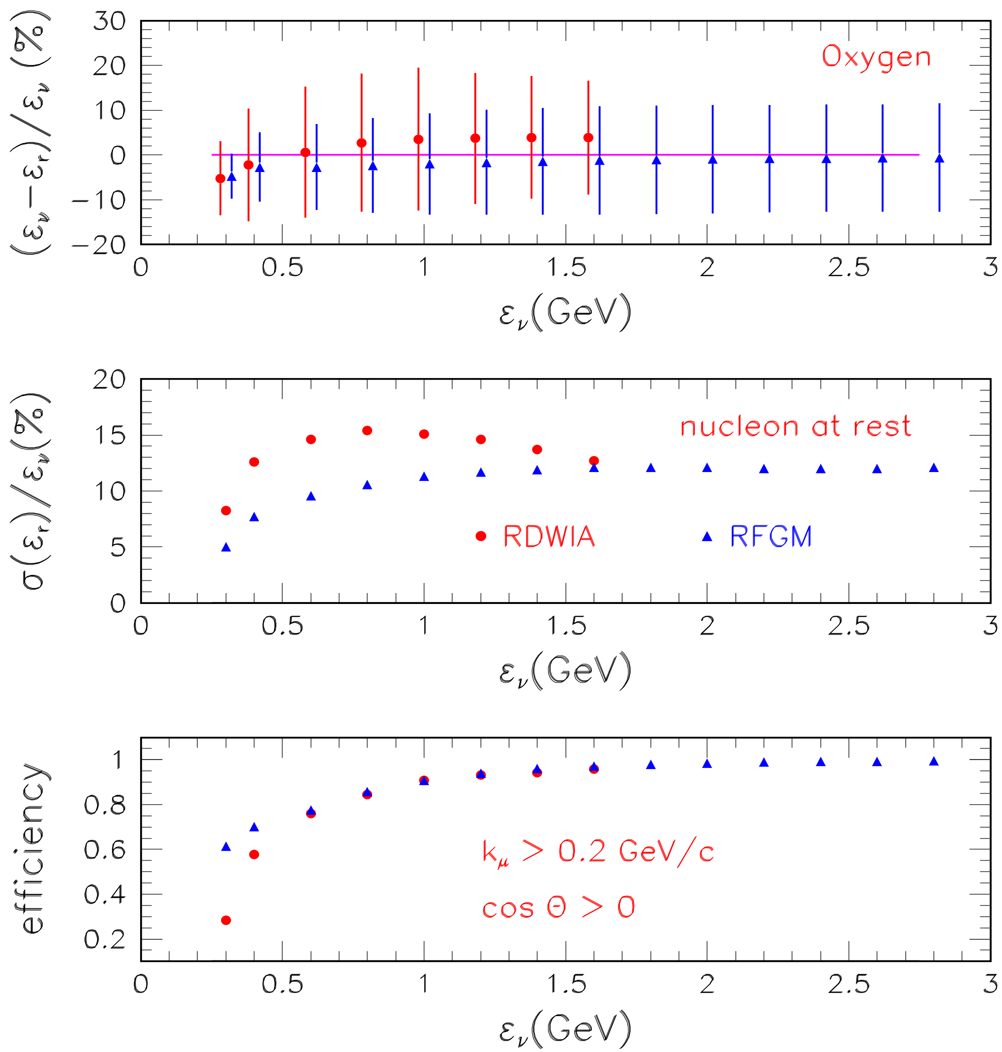
$$\sigma_{tot}(\varepsilon_i) = \int \frac{d^2\sigma}{dk_f d\cos\theta} dk_f d\cos\theta.$$

- Usually, to select CC events, k_f and $\cos\theta$ cuts are applied: $k_f \geq k_{cut}$ and $\cos\theta \geq (\cos\theta)_{cut}$. The lower limits of integration are $(k_f)_{min} = k_{cut}$, $(\cos\theta)_{min} = (\cos\theta)_{cut}$.

- $[\varepsilon_r(k_f, \cos\theta)]^n = \langle\varepsilon^n(k_f, \cos\theta)\rangle$ if the nucleon Fermi motion effect is taken into account, or $\varepsilon_r(k_f, \cos\theta)$ is given by formula for nucleon which is at rest, if this effect is neglected.

- The reconstructed neutrino energy $\bar{\varepsilon}_r = \langle\varepsilon_r\rangle$ is smeared with variance $\sigma^2(\varepsilon_i) = \langle\varepsilon_r^2(\varepsilon_i)\rangle - \bar{\varepsilon}_r^2(\varepsilon_i)$ and biased with $\Delta(\varepsilon_i) = \varepsilon_i - \bar{\varepsilon}_r$

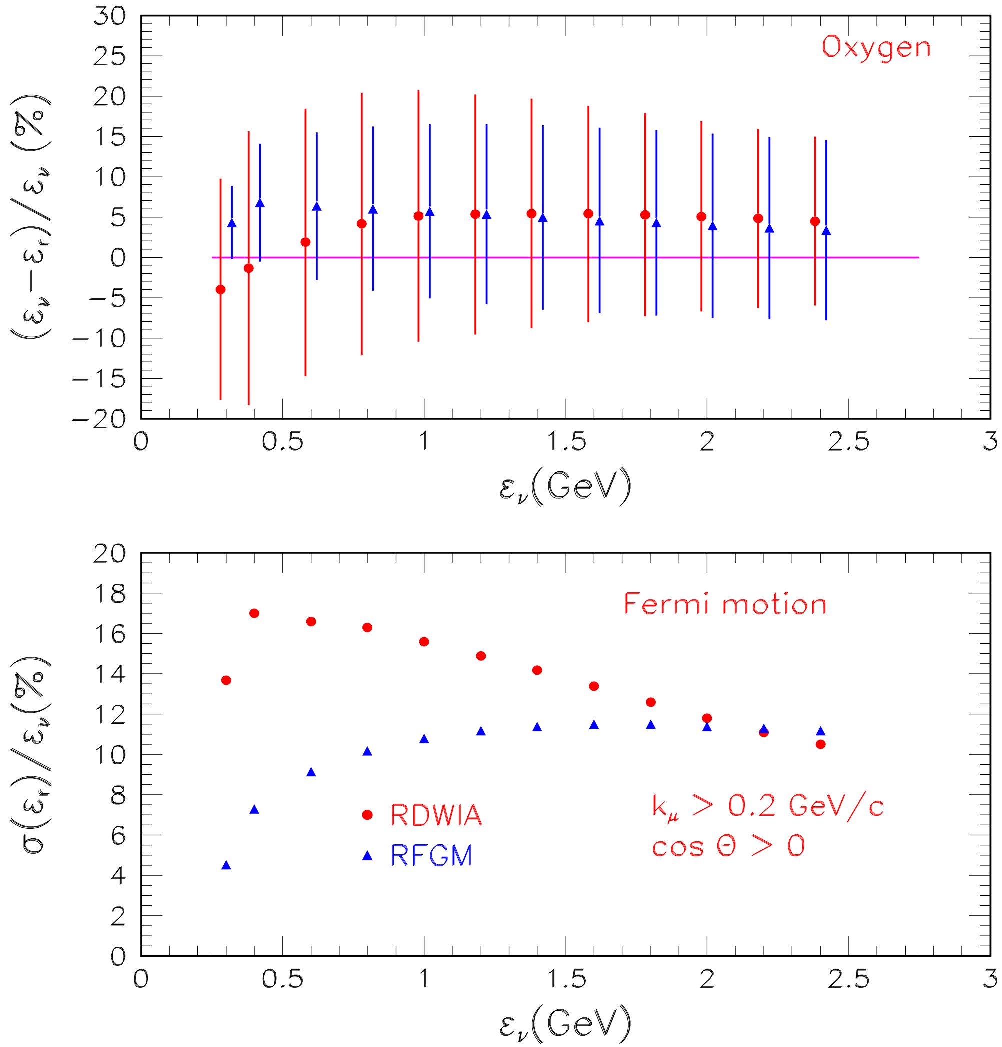
- The detailed description of this approach is given in [A.Butkevich (2008)].



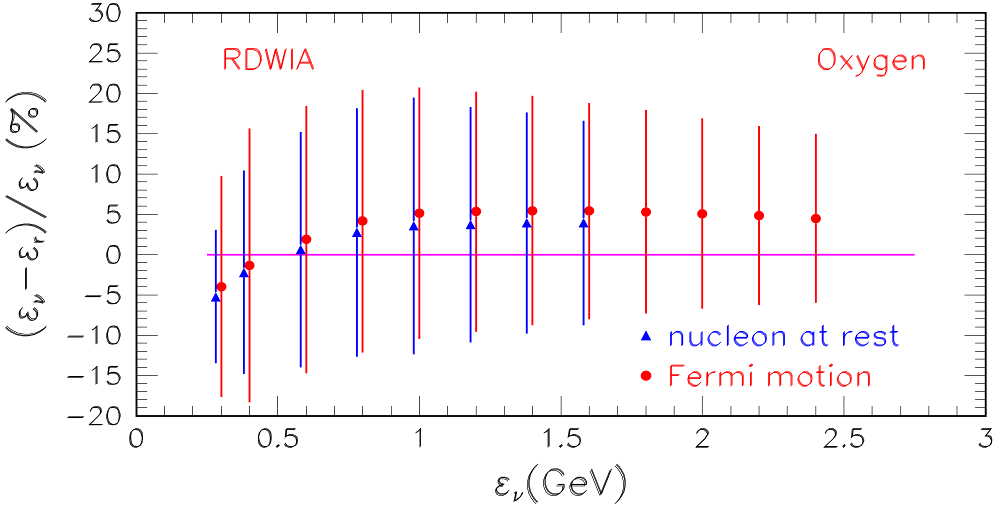
Bias (top panel), variance (middle panel) of the reconstructed neutrino energy, and efficiency (bottom panel) of the one-track events detection with $k_f \geq$0.2 (GeV/c) and $\cos\theta \geq$0 as functions of neutrino energy. The Neutrino energy reconstruction was formed assuming the target nucleon is at rest inside nucleus. The vertical bars show $\sigma[(\varepsilon_i - \varepsilon_r)/\varepsilon_i]$. As displayed in the key, biases, variances, and efficiencies were calculated in the RDWIA and RFGM.

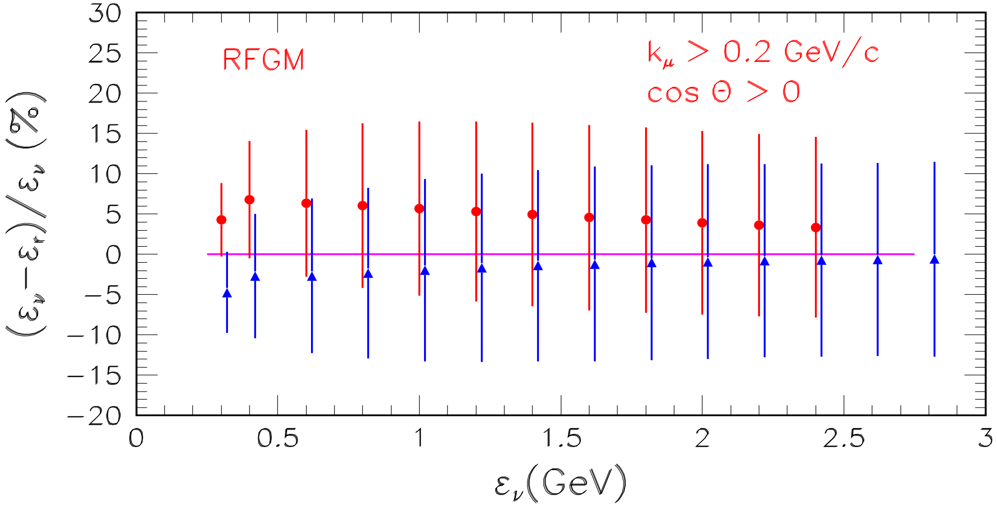
RFGM: in range $0.3 \div 2.5$ $\Delta = -4.7\% \rightarrow \Delta = -0.7\%$ and $\sigma/\varepsilon_i = 5.4\% \rightarrow \sigma/\varepsilon_i = 12\%$.

RDWIA: in range $0.3 \div 1.6$ $\Delta = -5.2\% \rightarrow \Delta = 3.9\%$ and $\sigma/\varepsilon_i = 8.3\% \rightarrow \sigma/\varepsilon_i = 12.7\%$.



Bias (top panel) and variance (bottom panel) of the reconstructed neutrino energy as functions of neutrino energy. The energy reconstruction was formed taking into account the nucleon momentum distribution in the target with $E_{max} = 10$ GeV. As displayed in the key, biases and variances were calculated in the RDWIA and RFGM. RFGM: in range $0.3 \div 2.5$ $\Delta = 4.3\% \to \Delta = 3\%$ and $\sigma/\varepsilon_i = 4.6\% \to \sigma/\varepsilon_i = 11\%$. RDWIA: in range $0.3 \div 2.5$ $\Delta = -4\% \to \Delta = 4.5\%$ and $\sigma/\varepsilon_i = 14.3\% \to \sigma/\varepsilon_i = 10.5\%$. Note: Bias may depend on the value of E_{max}.





Biases calculated in the RDWIA (top panel) and RFGM (bottom panel) as functions of neutrino energy. As displayed in the key, energy reconstructions were formed with and without the nucleon momentum distribution. The biases calculated within the RDWIA and RFGM assuming that nucleon is at rest (Δ_{fr}) and using the mean energy method (Δ_{me}) are presented as functions of neutrino energy. In the RDWIA approach the nucleon Fermi motion effect leads to increase the bias by about 1.2%. In the Fermi gas model with this effect ε_r is overestimated and $\Delta_{fr}(\Delta_{me}) = -4.7$%(4.3%) for energy 0.3 GeV and $\Delta_{fr}(\Delta_{me}) = -0.7$%(3.4%) for $\varepsilon_\nu = 2.5$ GeV.

- Apparently the accuracy of the kinematical reconstruction neutrino energy for one-track events depends on the nuclear models of QE neutrino CC interaction with nuclei and on the neutrino energy reconstruction methods.

- We can estimate the systematic uncertainties of this approach by comparing Δ_{FG} and δ_{FG} calculated in the RFGM with Δ_R and δ_R evaluated in the RWDIA approach using the mean energy method. It is clear that uncertainties depend on neutrino energy:
$\Delta_{FG}(\Delta_R) \approx -4.7\%(-4\%)$ and $\delta_{FG}(\delta_R) \approx 5.4\%(13.7\%)$ for $\varepsilon_\nu = 0.3$ GeV
$\Delta_{FG}(\Delta_R) \approx -2.3\%(4.1\%)$ and $\delta_{FG}(\delta_R) \approx 10.6\%(16.3\%)$ for $\varepsilon_\nu = 0.8$ GeV
$\Delta_{FG}(\Delta_R) \approx -0.7\%(4.5\%)$ and $\delta_{FG}(\delta_R) \approx 12\%(11.5\%)$ for $\varepsilon_\nu = 2.5$ GeV.

- So, the bias uncertainty increases with energy from $(\Delta_R - \Delta_{FG}) \approx 0.7\%$ for $\varepsilon_\nu = 0.3$ GeV up to 5.2% for $\varepsilon_\nu = 2.5$ GeV and the energy resolution uncertainty decreases with increasing energy from $\delta_R - \delta_{FG} \approx 8.3\%$ to 0.5% in this energy range.

- We note that these estimations may depend on the values of $(k_f)_{cut}$, $(\cos\theta)_{cut}$ and E_{max}.

For the two-track events the moments of the $\varepsilon_r(k_f, \cos\theta)$ distribution can be written as

$$\langle \varepsilon_r^n(\varepsilon_i)\rangle = \sum_\alpha w_\alpha \langle \varepsilon_r^n(\varepsilon_i)\rangle_\alpha,$$

where

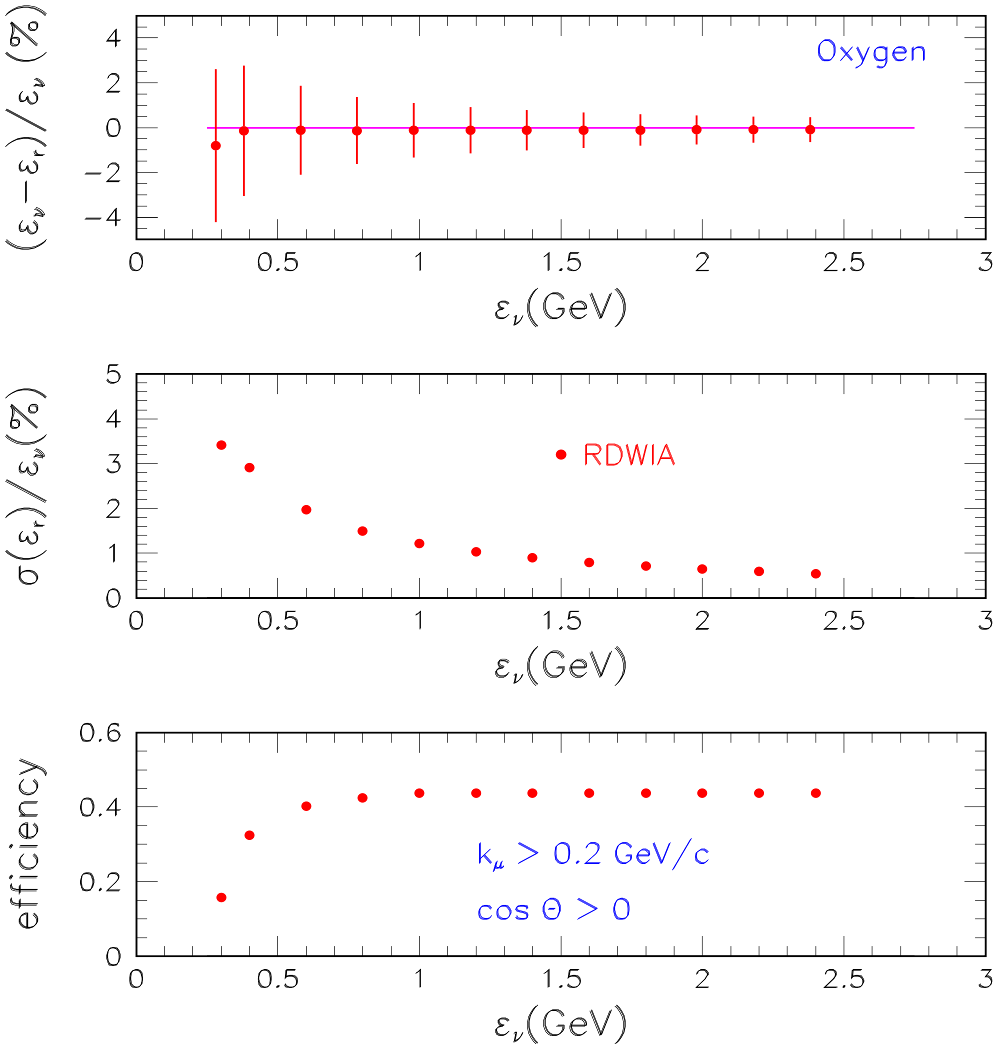
$$\langle \varepsilon_r^n(\varepsilon_i)\rangle_\alpha = \int dk_f \int d\cos\theta \int_0^{2\pi} d\phi \int_{p_{min}}^{p_{max}} [\varepsilon_f + T_p + \langle \varepsilon_m\rangle]^n W_\alpha(k_f, \cos\theta, \phi, p_m) dp_m,$$

$$W_\alpha = \frac{1}{\sigma_\alpha^{ex}} \Big[\frac{d^5\sigma}{dk_f d\cos\theta d\phi dp_m}\Big]_\alpha$$

$$\sigma_\alpha^{ex} = \int dk_f \int d\cos\theta \int_0^{2\pi} d\phi \int_{p_{min}}^{p_{max}} \Big[\frac{d^5\sigma}{dk_f d\cos\theta d\phi dp_m}\Big]_\alpha dp_m,$$

$$w_\alpha = \sigma_\alpha^{ex} / \sum_\alpha \sigma_\alpha^{ex}$$

and $d^5\sigma/dk_f d\cos\theta d\phi dp_m$ is the QE neutrino CC scattering exclusive cross section.



In the calorimetric reconstruction ε_r is formed as the sum of muon energy ε_f, kinematic proton energy T_p and mean missing energy $\langle\varepsilon_m\rangle$
$\varepsilon = \varepsilon_f + T_p + \langle\varepsilon_m\rangle$.
Bias (top panel), variance (middle panel) of the reconstructed neutrino energy, and efficiency (bottom panel) of the two-tracks events detection with $k_f \geq$0.2 (GeV/c) and $\cos\theta \geq$0 and without any cuts for proton. At $\varepsilon_\nu > 0.3$ (GeV) $\Delta = -0.1\%$. The energy resolution: 3.4% at $\varepsilon_\nu = 0.3$ (GeV) and 0.5% at $\varepsilon_\nu = 2.5$ (GeV). The challenge is identifying proton track and reconstructing its kinetic energy with reliable accuracy at low threshold energy for proton detection.

Summary

QE CC $\nu(\bar{\nu})^{16}$O cross sections were studied in different approaches.

- The calculated within the RDWIA inclusive $d^2\sigma/dQ^2$ and $d\sigma/d\cos\theta$ cross sections and measured Q^2, $\cos\theta$-distributions of CCQE events exhibit similar feature as compared to the Fermi gas model.
- We showed that the efficiency and purity of the CCQE two-track events selection are nuclear model dependent and the difference decreases with increasing energy transfer and neutrino energy.
- We studied the nuclear-model dependence of the energy reconstruction accuracy, neglecting by systematics related to event selection and resolution. We found that the accuracy of the kinematic reconstruction for one-track events depends on the nuclear model of CCQE neutrino interaction and neutrino energy reconstruction method.
- In the case of two-track events accuracy may be higher and does not depend on nuclear models of CCQE neutrino-nucleus interaction.