QE neutrino CC scattering cross section on oxygen

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- Motivations.
- Formalism of quasi-elastic scattering.
- Nuclear model.
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Motivations

- New extremely intense neutrino beamlines are in operation or in being planed for the precision measurements of the mass splitting and mixing angles and detailed experimental study of neutrino mixing matrix.
- Neutrino beams cover the energy range from a few hundred MeV to several GeV. In this regime, the dominant contribution to neutrino nucleus cross section comes from quasi-elastic (QE) reactions and resonance production processes.
- The consideration of systematic uncertainties come to the first place. An important source of systematic uncertainties is related to nuclear effects in neutrino interactions. The cross section data in the relevant energy range are rather scarce and were taken on the targets, which are not used in the neutrino oscillation experiments.
- A variety of Monte Carlo codes developed to simulate neutrino detector response are based on the Relativistic Fermi Gas Model (RFGM). This model does not account a few important effects: nuclear shell structure, final state interaction between the outgoing nucleon and residual nucleus (FSI), and presence of strong nucleon-nucleon (NN) correlations.
- We study QE neutrino charged-current interactions within model:

($\star$) Relativistic Fermi Gas Model (RFGM)
($\star$) Plane-Wave Impulse Approximation (PWIA) - no final state interaction (FSI)
($\star$) FSI effects within Relativistic Distorted-Wave Impulse Approximation (RDWIA)

Formalism of the quasi-elastic scattering

We consider lepton charged-current (CC) QE exclusive

$$l(k_i) + A(p_A) \longrightarrow l'(k_f) + N(p_x) + B(p_B),$$

and inclusive

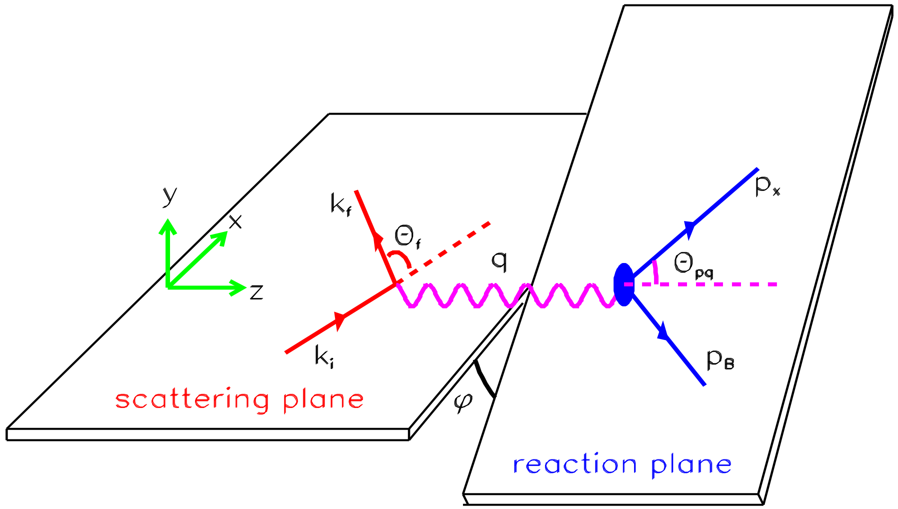
$$l(k_i) + A(p_A) \longrightarrow l'(k_f) + X$$

scattering off nuclei, where

- ⋆ l is incident lepton (e/ν_μ), l' is scattered lepton (e/μ), $k_i = (\varepsilon_i, \boldsymbol{k}_i)$ and $k_f = (\varepsilon_f, \boldsymbol{k}_f)$ are initial and final lepton momenta
- ⋆ $p_A = (\varepsilon_A, \boldsymbol{p}_A)$, and $p_B = (\varepsilon_B, \boldsymbol{p}_B)$ are the initial and final target momenta, $p_x = (\varepsilon_x, \boldsymbol{p}_x)$ is ejectile nucleon momentum
- ⋆ $q = (\omega, \boldsymbol{q})$ is momentum transfer carried by the virtual photon (W-boson), and $Q^2 = -q^2 = \boldsymbol{q}^2 - \omega^2$ is photon (W-boson) virtuality
- ⋆ m, m_A and m_B are masses of nucleon, target and recoil nucleus, respectively. The missing energy and momentum are defined by $\boldsymbol{p}_m = \boldsymbol{p}_x - \boldsymbol{q}, \quad \varepsilon_m = m + m_B - m_A$

A. QE lepton-nucleus cross section

A1. Exclusive reactions



Kinematic definitions for A$(l, l'N)$B reactions.

In the lab frame the differential cross section for exclusive electron (σ^{el}) and (anti)neutrino (σ^{cc}) CC scattering can be written as

$$\frac{d^6\sigma^{el}}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{|\boldsymbol{p}_x|\varepsilon_x}{(2\pi)^3}\frac{\varepsilon_f}{\varepsilon_i}\frac{\alpha^2}{Q^4}L^{(el)}_{\mu\nu}\mathcal{W}^{\mu\nu(el)}$$

$$\frac{d^6\sigma^{cc}}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{|\boldsymbol{p}_x|\varepsilon_x}{(2\pi)^5}\frac{|\boldsymbol{k}_f|}{\varepsilon_i}\frac{G^2\cos^2\theta_c}{2}L^{(cc)}_{\mu\nu}\mathcal{W}^{\mu\nu(cc)},$$

where Ω_f is the solid angle for the lepton momentum, Ω_x is the solid angle for the ejectile nucleon momentum, $\alpha \simeq 1/137$ is the fine-structure constant, $G \simeq 1.16639 \times 10^{-11}$ MeV^{-2} is the Fermi constant, θ_C is the Cabbibo angle ($\cos\theta_C \approx$ 0.9749).
The lepton tensor can be written as the sum of symmetric $L_S^{\mu\nu}$ and antisymmetric $L_A^{\mu\nu}$ tensors

$$L^{\mu\nu} = L_S^{\mu\nu} + L_A^{\mu\nu}$$

$$L_S^{\mu\nu} = 2\left(k_i^\mu k_f^\nu + k_i^\nu k_f^\mu - g^{\mu\nu}k_i k_f\right)$$

$$L_A^{\mu\nu} = h2i\epsilon^{\mu\nu\alpha\beta}(k_i)_\alpha(k_f)_\beta,$$

where h is $+1$ for positive lepton helicity and -1 for negative lepton helicity, $\epsilon^{\mu\nu\alpha\beta}$ is the antisymmetric tensor For the scattering of unpolarized incident electrons $L^{\mu\nu(el)}$ only has symmetric part and (anti)neutrino tensor $L^{\mu\nu(cc)}$ involves both, symmetric and antisymmetric parts.

The electromagnetic and the weak CC hadronic tensors, $\mathcal{W}_{\mu\nu}^{(el)}$ and $\mathcal{W}_{\mu\nu}^{(cc)}$, are given by bilinear products of the transition matrix elements of the nuclear electromagnetic or CC operator $J_{\mu}^{(el)(cc)}$ between the initial nucleus state $|A\rangle$ and the final state $|B_f\rangle$ as

$$\mathcal{W}_{\mu\nu}^{(el)(cc)} = \sum_f \langle B_f, p_x | J_{\mu}^{(el)(cc)} | A\rangle \langle A | J_{\nu}^{(el)(cc)\dagger} | B_f, p_x\rangle \delta(\varepsilon_A + \omega - \varepsilon_x - \varepsilon_{B_f}),$$

where the sum is taken over undetected states.

If only a single discrete state or narrow resonance of the target is excited it is possible to integrate over the peak in missing energy and obtain a fivefold differential cross section of the form

$$\frac{d^5\sigma^{el}}{d\varepsilon_f d\Omega_f d\Omega_x} = R\frac{|\boldsymbol{p}_x|\tilde{\varepsilon}_x}{(2\pi)^3}\frac{\varepsilon_f}{\varepsilon_i}\frac{\alpha^2}{Q^4}L_{\mu\nu}^{(el)}W^{\mu\nu(el)}$$

$$\frac{d^5\sigma^{cc}}{d\varepsilon_f d\Omega_f d\Omega_x} = R\frac{|\boldsymbol{p}_x|\tilde{\varepsilon}_x}{(2\pi)^5}\frac{|\boldsymbol{k}_f|}{\varepsilon_i}\frac{G^2\cos^2\theta_c}{2}L_{\mu\nu}^{(cc)}W^{\mu\nu(cc)},$$

where R is a recoil factor

$$R = \int d\varepsilon_x \delta(\varepsilon_x + \varepsilon_B - \omega - m_A) = \left| 1 - \frac{\tilde{\varepsilon}_x}{\varepsilon_B} \frac{\boldsymbol{p}_x \cdot \boldsymbol{p}_B}{\boldsymbol{p}_x \cdot \boldsymbol{p}_x} \right|^{-1},$$

$\tilde{\varepsilon}_x$ is solution to equation $\varepsilon_x + \varepsilon_B - m_A - \omega = 0$, where $\varepsilon_B = \sqrt{m_B^2 + \boldsymbol{p}_B^2}$, $\boldsymbol{p}_B = \boldsymbol{q} - \boldsymbol{p}_x$.

It is also useful to define a reduced cross section

$$\sigma_{red} = \frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} / K\sigma_{lN},$$

where $K^{el} = Rp_x\varepsilon_x/(2\pi)^3$ and $K^{cc} = Rp_x\varepsilon_x/(2\pi)^5$ are phase-space factors for electron and neutrino scattering, and σ_{lN} is corresponding elementary cross section for the lepton scattering from moving free nucleon.

A2. Hadronic tensor for exclusive reaction

- A general structure of the hadronic tensor can be established basing on requirements of Lorentz invariance, parity and time reversal symmetries and current conservation.
- Lorentz invariance $=$ this tensor must be constructed from three linearly independed four-vectors q, p_x, and p_A, the scalars that can be constructed from them, the second-rank metric tensor $g_{\mu\nu}$, and completely antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$.
- Symmetries: generally, due to the final state interaction effects the time reversal symmetry does not constraint the form of the nuclear tensor for the exclusive reactions [A.Picklesimer et al. (1985) and (1987)],

For electron scattering [A.Picklesimer at al. (1985), T.W.Donnelly et al. (1986)] (taking into account current and parity conservation)

$$W^{\mu\nu(el)} = W_S^{\mu\nu(el)} + W_A^{\mu\nu(el)},$$

$$W_S^{\mu\nu(el)} = W_1^{(el)}\tilde{g}^{\mu\nu} + W_2^{(el)}\tilde{p}_x^{\mu}\tilde{p}_x^{\nu} + W_3^{(el)}\tilde{p}_A^{\mu}\tilde{p}_A^{\nu} + W_4^{(el)}(\tilde{p}_x^{\mu}\tilde{p}_A^{\nu} + \tilde{p}_x^{\nu}\tilde{p}_A^{\mu}),$$

$$W_A^{\mu\nu(el)} = W_5^{(el)}(\tilde{p}_x^{\mu}\tilde{p}_A^{\nu} - \tilde{p}_x^{\nu}\tilde{p}_A^{\mu}),$$

where

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{Q^2},$$

$$\tilde{p}_x^{\mu} = p_x^{\mu} + \frac{p_x \cdot q}{Q^2}q^{\mu},$$

$$\tilde{p}_A^{\mu} = p_A^{\mu} + \frac{p_A \cdot q}{Q^2}q^{\mu}.$$

In the chosen coordinate system the result of contraction of the electron and nuclear response tensors reduces to the form

$$L_{\mu\nu}^{(el)}W_S^{\mu\nu(el)} = 4\varepsilon_i\varepsilon_f\cos^2\frac{\theta}{2}(V_L R_L^{(el)} + V_T R_T^{(el)} + V_{LT} R_{LT}^{(el)}\cos\phi + V_{TT} R_{TT}^{(el)}\cos 2\phi)$$

where

$$
\begin{aligned}
V_L &= Q^4/\boldsymbol{q}^4, \\
V_T &= \frac{Q^2}{2\boldsymbol{q}^2} + \tan^2\frac{\theta}{2}, \\
V_{LT} &= \frac{Q^2}{\boldsymbol{q}^2}\left(\frac{Q^2}{\boldsymbol{q}^2} + \tan^2\frac{\theta}{2}\right), \\
V_{TT} &= \frac{Q^2}{2\boldsymbol{q}^2}
\end{aligned}
$$

are the electron coupling coefficients and

$$
\begin{aligned}
R_L^{(el)} &= W^{00(el)}, \\
R_T^{(el)} &= W^{xx(el)} + W^{yy(el)}, \\
R_{LT}^{(el)}\cos\phi &= -\left(W^{0x(el)} + W^{x0(el)}\right), \\
R_{TT}^{(el)}\cos 2\phi &= W^{xx(el)} - W^{yy(el)}
\end{aligned}
$$

are four independ nuclear response functions $R = f(Q^2, \omega, \boldsymbol{p}_x, \theta_{pq})$.

In weak interactions the weak current and parity are not conserved. Therefore, a general nuclear tensor can be written as

$$
\begin{aligned}
W^{\mu\nu(cc)} &= W_S^{\mu\nu} + W_A^{\mu\nu}, \\
W_S^{\mu\nu} &= W_1 g^{\mu\nu} + W_2 q^\mu q^\nu + W_3 p_x^\mu p_x^\nu + W_4 p_A^\mu p_A^\nu + W_5 (p_x^\mu q^\nu + p_x^\nu q^\mu) \\
&+ W_6 (p_A^\mu q^\nu + p_A^\nu q^\mu) + W_7 (p_x^\mu p_A^\nu + p_x^\nu p_A^\mu), \\
W_A^{\mu\nu} &= W_8 (p_x^\mu q^\nu - p_x^\nu q^\mu) + W_9 (p_A^\mu q^\nu - p_A^\nu q^\mu) + W_{10} (p_x^\mu p_A^\nu - p_x^\nu p_A^\mu) \\
&+ W_{11} \epsilon^{\mu\nu\tau\rho} q_\tau p_{x\rho} + W_{12} \epsilon^{\mu\nu\tau\rho} q_\tau p_{A\rho} + W_{13} \epsilon^{\mu\nu\tau\rho} p_{x\tau} p_{A\rho}.
\end{aligned}
$$

The result of contraction of the lepton and nuclear tensors can be written as

$$
\begin{aligned}
L_{\mu\nu}^{(cc)} W^{\mu\nu(cc)} &= L_{\mu\nu}^S W_S^{\mu\nu} + L_{\mu\nu}^A W_A^{\mu\nu} = 2\varepsilon_i \varepsilon_f \{ v_0 R_0 + v_T R_T + v_{TT} R_{TT} \cos 2\phi + v_{zz} R_{zz} \\
&+ (v_{xz} R_{xz} - v_{0x} R_{0x}) \cos\phi - v_{0z} R_{0z} + h[v_{yz}(R'_{yz} \sin\phi + R_{yz} \cos\phi) \\
&- v_{0y}(R'_{0y} \sin\phi + R_{0y} \cos\phi) - v_{xy} R_{xy}] \},
\end{aligned}
$$

where

$$v_0 = 1 + \beta \cos\theta,$$

$$v_T = 1 - \beta \cos\theta + \frac{\varepsilon_i \beta |\boldsymbol{k}_f| \sin^2\theta}{q^2},$$

$$v_{TT} = \frac{\varepsilon_i \beta |\boldsymbol{k}_f| \sin^2\theta}{\boldsymbol{q}^2},$$

$$v_{0z} = \frac{\omega}{|\boldsymbol{q}|}(1 + \beta \cos\theta) + \frac{m_l^2}{|\boldsymbol{q}|\varepsilon_f},$$

$$v_{zz} = 1 + \beta \cos\theta - 2\frac{\varepsilon_i |\boldsymbol{k}_f| \beta}{\boldsymbol{q}^2} \sin^2\theta,$$

$$v_{0x} = (\varepsilon_i + \varepsilon_f)\frac{\beta \sin\theta}{|\boldsymbol{q}|},$$

$$v_{xz} = \frac{\beta}{\boldsymbol{q}^2} \sin\theta \left[(\varepsilon_i + \varepsilon_f)\omega + m_l^2)\right],$$

$$v_{xy} = \frac{\varepsilon_i + \varepsilon_f}{|\boldsymbol{q}|}(1 - \beta \cos\theta) - \frac{m_l^2}{|\boldsymbol{q}|\varepsilon_f},$$

$$v_{yz} = \beta \frac{\omega}{|\boldsymbol{q}|} \sin\theta,$$

$$v_{0y} = \beta \sin\theta, \quad \beta = |\boldsymbol{k}_f|/\varepsilon_f$$

are neutrino coupling coefficients and

$$
\begin{aligned}
R_0 &= W_S^{00}, \\
R_T &= W_S^{xx} + W_S^{yy}, \\
R_{TT}\cos 2\phi &= W_S^{xx} - W_S^{yy}, \\
R_{0z} &= W_S^{0z} + W_S^{z0}, \\
R_{zz} &= W_S^{zz}, \\
R_{0x}\cos\phi &= W_S^{0x} + W_S^{x0}, \\
R_{xz}\cos\phi &= W_S^{xz} + W_S^{zx}, \\
R_{xy} &= i\left(W_A^{xy} - W_A^{yx}\right), \\
R'_{yz}\sin\phi + R_{yz}\cos\phi &= i\left(W_A^{yz} - W_A^{zy}\right), \\
R'_{0y}\sin\phi + R_{0y}\cos\phi &= i\left(W_A^{0y} - W_A^{y0}\right)
\end{aligned}
$$

are ten independ response functions which describe the weak properties of hadronic system.

In the absence of FSI effect (plane-wave limit) the time reversal symmetry of operators and states provides an additional constraint on the Lorenz form of the antisymmetric part of nuclear tensor. Then we have

$$W_A^{\mu\nu} \quad = \quad W_{11}\varepsilon^{\mu\nu\tau\rho}q_\tau p_{x\rho} + W_{12}\epsilon^{\mu\nu\tau\rho}q_\tau p_{A\rho} + W_{13}\epsilon^{\mu\nu\tau\rho}p_{x\tau}p_{A\rho}$$

and

$$\begin{aligned} L_{\mu\nu}^{(cc)}W^{\mu\nu(cc)} \quad &= 2\varepsilon_i\varepsilon_f\{v_0R_0 + v_TR_T + v_{TT}R_{TT}\cos 2\phi + v_{zz}R_{zz} + (v_{xz}R_{xz} - v_{0x}R_{0x})\cos\phi \\ &\qquad - v_{0z}R_{0z} + h(v_{yz}R_{yz}\cos\phi - v_{0y}R_{0y}\cos\phi - v_{xy}R_{xy})\}, \end{aligned}$$

where

$$R_{yz}\cos\phi = i\,(W_A^{yz} - W_A^{zy})\,,$$

$$R_{0y}\cos\phi = i\left(W_A^{0y} - W_A^{y0}\right).$$

Note that asymmetry, which is measured at azimuthal angles $\phi = \pi/2$ and $\phi = -\pi/2$, vanishes in the absence of the FSI.

The exclusive lepton scattering cross sections can be written in terms of response functions as

$$\frac{d^5\boldsymbol{\sigma}^{el}}{d\varepsilon_f d\Omega_f d\Omega_x} = \frac{|\boldsymbol{p}_x|\tilde{\varepsilon}_x}{(2\pi)^3}\sigma_M R\big(V_L R_L^{(el)} + V_T R_T^{(el)} + V_{LT} R_{LT}^{(el)} \cos\phi + V_{TT} R_{TT}^{(el)} \cos 2\phi\big),$$

$$\begin{aligned}\frac{d^5\boldsymbol{\sigma}^{cc}}{d\varepsilon_f d\Omega_f d\Omega_x} = \frac{|\boldsymbol{p}_x|\tilde{\varepsilon}_x}{(2\pi)^5} & G^2 \cos^2\theta_c \varepsilon_f |\boldsymbol{k}_f| R\big\{v_0 R_0 + v_T R_T + v_{TT} R_{TT}\cos 2\phi + v_{zz} R_{zz} \\ & + (v_{xz} R_{xz} - v_{0x} R_{0x})\cos\phi - v_{0z} R_{0z} + h\big[v_{yz}(R'_{yz}\sin\phi + R_{yz}\cos\phi) \\ & - v_{0y}(R'_{0y}\sin\phi + R_{0y}\cos\phi) - v_{xy} R_{xy}\big]\big\},\end{aligned}$$

where

$$\sigma_M = \frac{\alpha^2\cos^2\theta/2}{4\varepsilon_i^2\sin^4\theta/2}$$

is the Mott cross section. The response functions R_i depend on the variables $Q^2, \omega, |\boldsymbol{p}_x|$, and θ_{pq}.

A3. Inclusive reactions

In the inclusive reactions only the outgoing lepton is detected and the differential cross sections can be written as

$$\frac{d^2\sigma^{el}}{d\varepsilon_f d\Omega_f} = \frac{\varepsilon_f}{\varepsilon_i}\frac{\alpha^2}{Q^4} L^{(el)}_{\mu\nu}\overline{W}^{\mu\nu(el)},$$
$$\frac{d^2\sigma^{cc}}{d\varepsilon_f d\Omega_f} = \frac{1}{(2\pi)^2}\frac{|\boldsymbol{k}_f|}{\varepsilon_i}\frac{G^2\cos^2\theta_c}{2} L^{(cc)}_{\mu\nu}\overline{W}^{\mu\nu(cc)},$$

where $\overline{W}^{\mu\nu}$ is inclusive hadronic tensor.

For inclusive lepton scattering there are only two linearly independ four vectors q and p_A. The time reversal symmetry reduces the number of Lorenz structures in hadronic tensor and we have

$$\overline{W}^{\mu\nu(el)}_S = \overline{W}^{(el)}_1 \tilde{g}^{\mu\nu} + \overline{W}^{(el)}_2 p^\mu_A p^\nu_A + \overline{W}^{(el)}_3 (q^\mu p^\nu_A + q^\nu p^\mu_A).$$

The result of contraction of the electron and nuclear tensors is given by

$$L^{(el)}_{\mu\nu}\overline{W}^{\mu\nu(el)} = 4\varepsilon_i\varepsilon_f\cos^2\frac{\theta}{2}(V_L R^{(el)}_L + V_T R^{(el)}_T).$$

The charged-current weak nuclear tensor for inclusive neutrino scattering can be written as the sum

$$\begin{aligned}\overline{W}^{\mu\nu(cc)} &= \overline{W}_S^{\mu\nu} + \overline{W}_A^{\mu\nu},\\ \overline{W}_S^{\mu\nu} &= \overline{W}_1 g^{\mu\nu} + \overline{W}_2 q^\mu q^\nu + \overline{W}_3 p_A^\mu p_A^\nu + \overline{W}_4 (p_A^\mu q^\nu + p_A^\nu q^\mu),\\ \overline{W}_A^{\mu\nu} &= \overline{W}_5 \epsilon^{\mu\nu\tau\rho} q_\tau p_{A\rho}\end{aligned}$$

and the result of contraction of the neutrino and nuclear tensors is given by

$$L_{\mu\nu}^{(cc)}\overline{W}^{\mu\nu(cc)} = 2\varepsilon_i\varepsilon_f(v_0R_0 + v_TR_T + v_{zz}R_{zz} - v_{0z}R_{0z} - hv_{xy}R_{xy}).$$

Than the inclusive lepton scattering cross sections reduce to

$$\begin{aligned}\frac{d^2\sigma^{el}}{d\varepsilon_f d\Omega_f} &= \sigma_M\left(V_LR_L^{(el)} + V_TR_T^{(el)}\right),\\ \frac{d^2\sigma^{cc}}{d\varepsilon_f d\Omega_f} &= \frac{G^2\cos^2\theta_c}{(2\pi)^2}\varepsilon_f|\boldsymbol{k}_f|\left(v_0R_0 + v_TR_T + v_{zz}R_{zz} - v_{0z}R_{0z} - hv_{xy}R_{xy}\right),\end{aligned}$$

where the response functions now depend only on Q^2 and ω.

B. Nuclear current

- The electromagnetic vertex function for a free nucleon can be represented by any of three operators [T. de Forest (1983), C.R. Chinn et al (1992)]

$$\Gamma^\mu = G_M(Q^2)\gamma^\mu - \frac{P^\mu}{2m}F_M^{(el)}(Q^2) \qquad (CC1),$$

$$\Gamma^\mu = F_V^{(el)}(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_M^{(el)}(Q^2) \qquad (CC2),$$

$$\Gamma^\mu = \frac{P^\mu}{2m}F_V^{(el)}(Q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}G_M(Q^2) \qquad (CC3),$$

  which are related by the Gordon identity. Here $\sigma^{\mu\nu} = i[\gamma^\mu\gamma^\nu]/2,\ P = p_m + p_x,\ F_V^{(el)}$ and $F_M^{(el)}$ are the Dirac and Pauli nucleon form factors.

- For electron scattering off nuclei, CC1,CC2 and CC3 representations of the electromagnetic vertex function are not equivalent because the bound nucleons are off shell ($p_m^2 \neq 0$). Theoretical uncertainties:

  ⋆ What expression for the Γ^μ should be used?
  Most calculations use the CC2 electromagnetic vertex function.

⋆ The vertex Γ^μ should be extrapolated to off-shell region. How ?
To this end we employ de Forest prescription

$$\tilde{\Gamma}^\mu = F_V^{(el)}(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{\tilde{q}_\nu}{2m}F_M^{(el)}(Q^2),$$

where $\tilde{q} = (\varepsilon_x - \tilde{E}, \boldsymbol{q})$ and the nucleon energy $\tilde{E} = \sqrt{m^2 + (\boldsymbol{p}_x - \boldsymbol{q})^2}$ is placed on shell. We use the MMD approximation [P.Mergell et al (1996)] of the nucleon form factors. The Coulomb gauge is assumed for the single-nucleon current, $i.e.$ $J_3 = (\omega/\boldsymbol{q})J_0$.

- The single-nucleon charged current has $V - A$ structure $J^{\mu(cc)} = J_V^\mu + J_A^\mu$. For a free nucleon vertex function $\Gamma^{\mu(cc)} = \Gamma_V^\mu + \Gamma_A^\mu$ we use CC2 vector current vertex function

$$\Gamma_V^\mu = F_V(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_M(Q^2)$$

and the axial current vertex function

$$\Gamma_A^\mu = F_A(Q^2)\gamma^\mu\gamma_5 + F_P(Q^2)q^\mu\gamma_5.$$

Weak vector form factors F_V and F_M are related to corresponding electromagnetic ones for proton $F_{i,p}^{(el)}$ and neutron $F_{i,n}^{(el)}$ by the hypothesis of conserved vector current (CVC)

$$F_i = F_{i,p}^{(el)} - F_{i,n}^{(el)}.$$

- We use de Forest prescription for off-shell extrapolation of $\Gamma^{\mu(cc)}$. Similar to electromagnetic current, Coulomb gauge is applied for the vector current J_V.

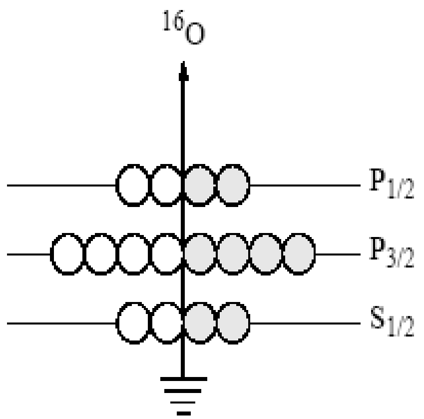
- The axial F_A and psevdoscalar F_P form factors in the dipole approximation are parameterized as

$$F_A(Q^2) = \frac{F_A(0)}{(1+Q^2/M_A^2)^2}, \quad F_P(Q^2) = \frac{2mF_A(Q^2)}{m_\pi^2 + Q^2},$$

where $F_A(0) = 1.267$, m_π is the pion mass, and $M_A \simeq 1.032$ GeV is the axial mass.

Model

In Independ Particle Shell Model (IPSM) the model space for ^{16}O$(e, e'N)$ consists of $1s_{1/2}$, $1p_{3/2}$, and $1p_{1/2}$ nucleon-hole states in ^{15}N and ^{15}O nuclei, for total of 6 states. The $1s_{1/2}$ state is regarded as a discrete state.



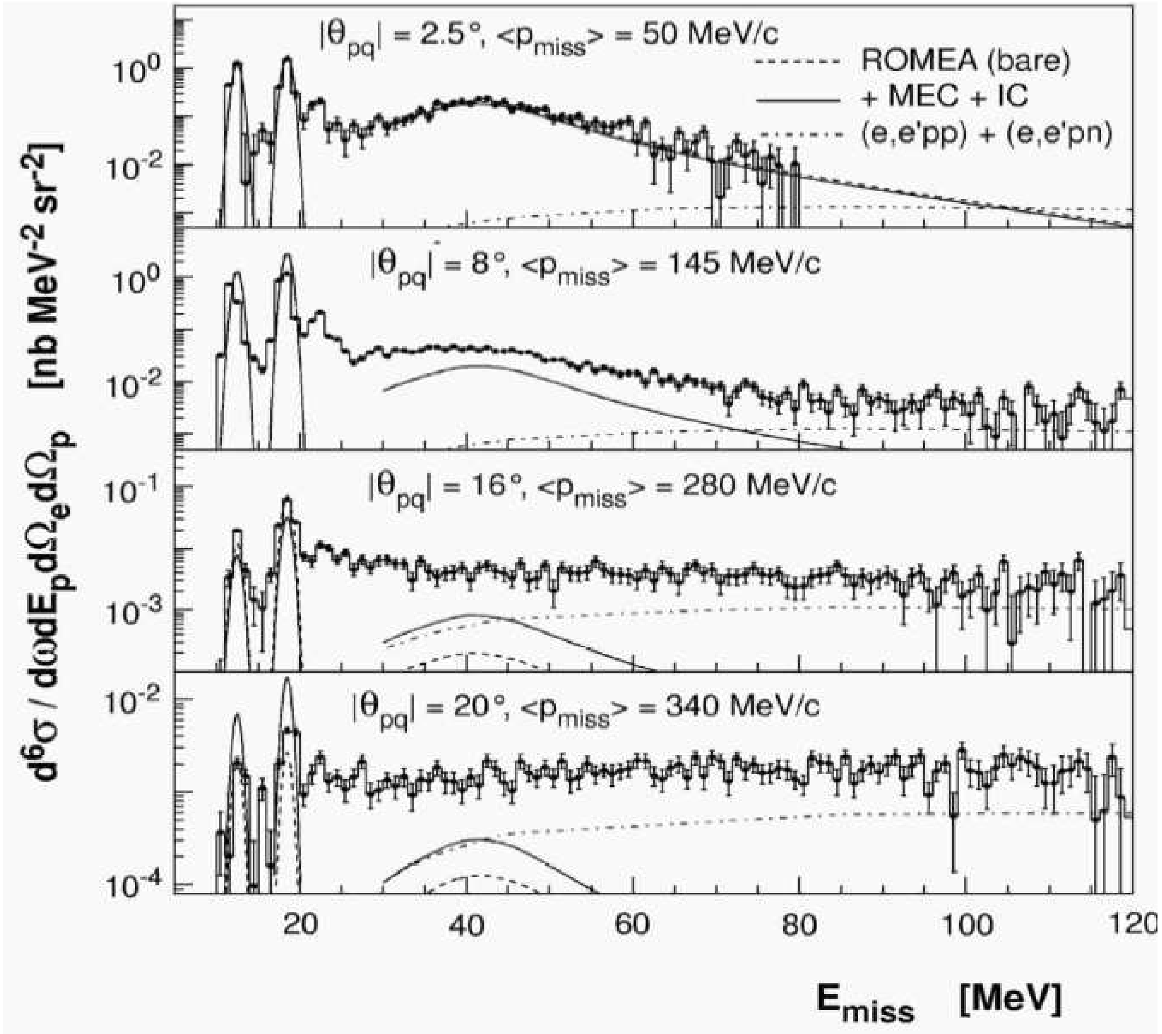
Shell occupancy:

S($P_{1/2}$)=0.7
S($P_{3/2}$)=0.66
S($S_{1/2}$)=1

Average occupancy of nuclear shells $\overline{S}$=0.75
(supported by recent JLab measurement)

Missing Energy

Neutron: $E_m(1p_{1/2})$=15.7 MeV, $E_m(1p_{3/2})$=21.2 MeV, $E_m(1s_{1/2})$=42.9 MeV
Proton: $E_m(1p_{1/2})$=12.1 MeV, $E_m(1p_{3/2})$=18.4 MeV, $E_m(1s_{1/2})$=40.1 MeV

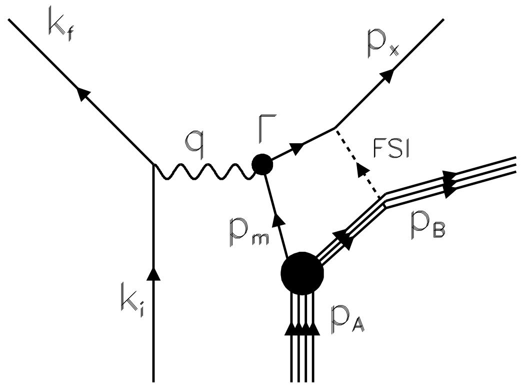


$|\theta_{pq}|$ = 2.5°, <p_{miss}> = 50 MeV/c
ROMEA (bare)
+ MEC + IC
(e,e'pp) + (e,e'pn)
$|\theta_{pq}|$ = 8°, <p_{miss}> = 145 MeV/c
$|\theta_{pq}|$ = 16°, <p_{miss}> = 280 MeV/c
$|\theta_{pq}|$ = 20°, <p_{miss}> = 340 MeV/c
$d^6\sigma / d\omega dE_p d\Omega_e d\Omega_p$ [nb MeV^{-2} sr^{-2}]
E_{miss} [MeV]

In our calculation of the hadron tensor we use relativistic distorted-wave impulse approximation (RDWIA), plane-wave impulse approximation (PWIA) and the Fermi gas model.

A. RDWIA

We can write the matrix elements of the current operator for single-nucleon knockout leaving the residual nucleus in asymptotic channel α as follows



$$\langle p, B_\alpha | J^\mu | A\rangle = \sum_{\beta\gamma m_b m_b'} c_{\beta\gamma} \int d^3 r \exp(i\boldsymbol{t}\cdot\boldsymbol{r}) \langle \bar{\psi}^{(-)}_{\alpha\beta} | \boldsymbol{r} m_b\rangle \times \langle \boldsymbol{r} m_b | \tilde{\Gamma}^\mu | \boldsymbol{r} m_b'\rangle \langle \boldsymbol{r} m_b' | \phi_{\beta\gamma}\rangle,$$

where $c_{\beta\gamma}$ is a parentage coefficient,

$\langle \boldsymbol{r} m_b | \tilde{\Gamma}^{\mu}_{\lambda\lambda'} | \boldsymbol{r} m'_b \rangle$ is a 2×2 spin matrix,

$$\langle \boldsymbol{r} m'_b | \phi_{\beta\gamma} \rangle = \begin{pmatrix} F_{\beta\gamma m'_b}(\boldsymbol{r}) \\ iG_{\beta\gamma m'_b}(\boldsymbol{r}) \end{pmatrix}$$

is the bound state overlap wave function and

$$\langle \bar{\psi}^{(-)}_{\alpha\beta} | \boldsymbol{r} m_b \rangle = N_\alpha \begin{pmatrix} \chi^{(-)*}_{\alpha\beta m_b}(\boldsymbol{r}) \\ -i\zeta^{(-)*}_{\alpha\beta m_b}(\boldsymbol{r}) \end{pmatrix}$$

is the Dirac adjoint of time-reversed distorted waves.

To describe the bound nucleon states we use relativistic shell model waves functions, obtained as the self-consistent (Hartree) solutions of a Dirac equation, derived within a relativistic mean field approach, from Lagrangian containing σ, ω and ρ mesons.

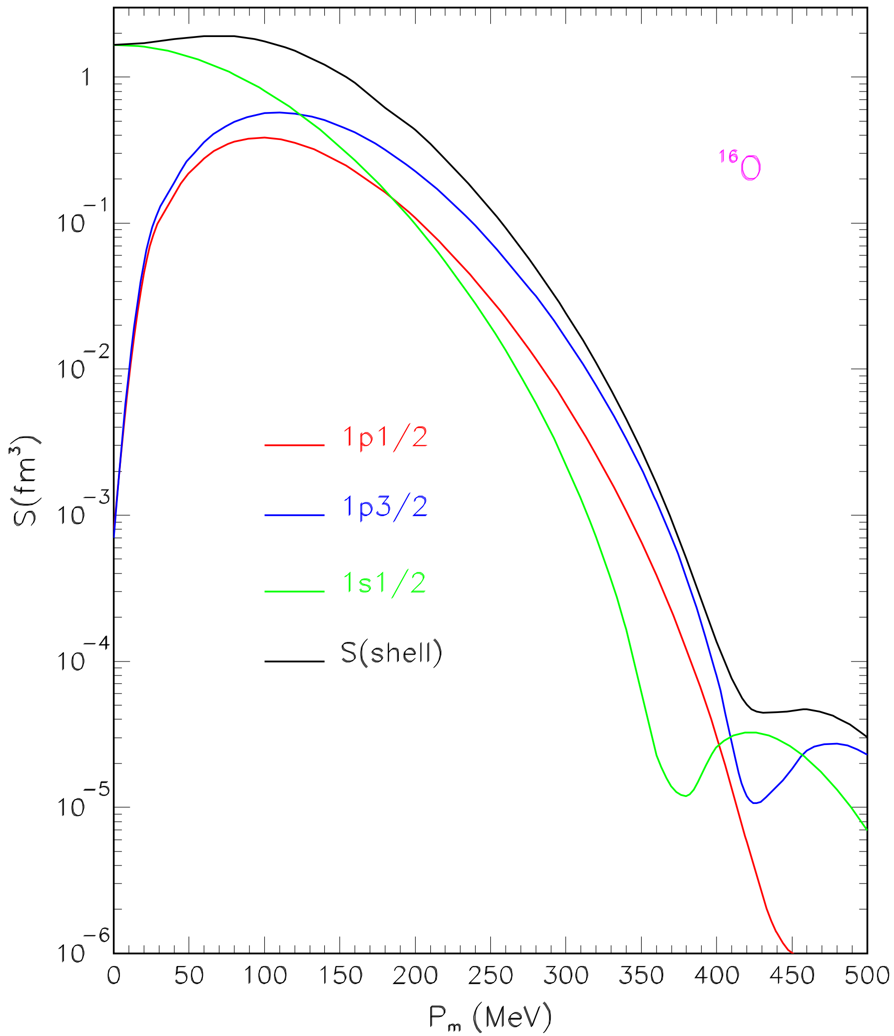
The upper and lower radial wave functions in partial-wave expansion for bound-state wave functions satisfy the usual coupled differential equations

$$\left(\frac{d}{dr}+\frac{\kappa_\gamma+1}{r}\right)F_{\beta\gamma}(r)=\left[E_\gamma+m+S_\gamma(r)-V_\gamma(r)\right]G_{\beta\gamma}(r),$$
$$\left(\frac{d}{dr}-\frac{\kappa_\gamma+1}{r}\right)G_{\beta\gamma}(r)=\left[-E_\gamma+m+S_\gamma(r)+V_\gamma(r)\right]F_{\beta\gamma}(r),$$

where S_γ and V_γ are spherical scalar and vector potentials, $j_\gamma=|\kappa_\gamma|-1/2$ is the total angular momentum. The missing momentum distribution is determined by the wave functions in momentum space

$$P_\beta(p_m)=\frac{|c_\beta|^2}{2\pi^2}\left(|\tilde{F}_\beta(p_m)|^2+|\tilde{G}_\beta(p_m)|^2\right).$$

In this work NLSH bound-nucleon wave function [M.M. Sharma et al (1993)] are used in numerical analysis with normalization factors $S_\alpha=|c_\alpha|^2$ relative to full occupancy of ^{16}O: $S(1p_{3/2})=0.66$, $S(1p_{1/2})=0.7$ [K.G. Fissum et al (2004) (JLab)] and $S(1s_{1/2})=1$.



Momentum distribution for the NLSH models. The NLSH wave functions predict binding energies, single-particle energies, and a charge radius for ^{16}O which are all in good agreement with the data.

The distorted wave functions are evaluated using a relativized Schrödinger equation for upper components of Dirac wave functions. For simplicity let consider a single-channel Dirac equation

$$[\alpha \cdot \boldsymbol{p} + \beta(m+S)]\,\psi = (E-V)\psi,$$

where

$$\psi(\boldsymbol{r}) = \begin{pmatrix} \psi_+(\boldsymbol{r}) \\ \psi_-(\boldsymbol{r}) \end{pmatrix}$$

is the four-component Dirac spinor. Using the direct Pauli reduction method [J.Udias et al (1995), M.Hedayati-Poor et al (1995)] the system of two coupled first-order radial Dirac equations can be reduced to a single second-order equation

$$\left[\nabla^2 + k^2 - 2\mu\left(U^C + U^{LS}\mathbf{L}\cdot\sigma\right)\right]\xi = 0,$$

where ξ is a two-component Pauli spinor. Here k is the relativistic wave number, μ is the reduced mass of the scattering state, and

$$U^C = \frac{E}{\mu}\left[V + \frac{m}{E}S + \frac{S^2 - V^2}{2E}\right] + U^D,$$

$$U^D = \frac{1}{2\mu}\left[-\frac{1}{2r^2 D}\frac{d}{dr}(r^2 D') + \frac{3}{4}\left(\frac{D'}{D}\right)^2\right],$$

$$U^{LS} = -\frac{1}{2\mu r}\frac{D'}{D},$$

$$D = 1 + \frac{S - V}{E + m}.$$

$D(r)$ is known as the Darwin nonlocality factor and U^C and U^{LS} are the central and spin-orbit potentials. The upper and lower components of the Dirac wave functions are then obtained using

$$\psi_+ = D^{1/2}\xi,$$

$$\psi_- = \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E + m + S - V}\,\psi_+$$

We use the LEA program [J.J. Kelly (1995)] for numerical calculation of the distorted wave functions with EDAD1 SV relativistic optical potential [E. Cooper (1993)].

B. PWIA

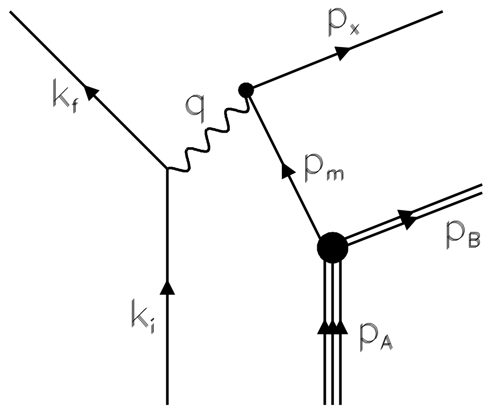
In the PWIA the final state interaction between the outgoing nucleon and the residual nucleus is neglected. In nonrelativistic PWIA the knockout cross section has a factorized form

$$\frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} = K\sigma_{ex}P(E,\boldsymbol{p})$$

where

$$K^{el} = R\frac{p_x\varepsilon_x}{(2\pi)^3}, \quad K^{cc} = R\frac{p_x\varepsilon_x}{(2\pi)^5}$$

are the phase-space factors, R is recoil factor, σ_{ex} is the half-off-shell cross section for scattering of a lepton by moving nucleon.



The nuclear spectral function P can be written as

$$P(E, \boldsymbol{p}) = \sum_f \left|\langle B_f|a(\boldsymbol{p})|A\rangle\right|^2 \delta(E - \varepsilon_m).$$

and represents the probability of removing a nucleon with momentum $\boldsymbol{p}$ and energy E from the nuclear target A and leaving the residual nucleus B in the state B_f, $\varepsilon_m = m + m_{B_f} - m_A$, where m_{B_f} and m_A are the nuclear masses in the corresponding states.
In the nuclear shell model the momentum distribution $P_\beta(\boldsymbol{p})$ for an orbit β is related to the upper component of the corresponded bound-state wave function as

$$P_\beta(\boldsymbol{p}) = \frac{c_\beta^2}{2\pi^2}\left|\tilde{F}_\beta(\boldsymbol{p})\right|^2$$

and normalized to an occupancy number of the orbit, such that

$$\int d^3p P_\beta(\boldsymbol{p}) = |c_\beta|^2.$$

C. Fermi gas model

In the RFGM the nucleons are described as a system of quasi-free nucleons. This model takes into account the Fermi motion of bound nucleon, Pauli blocking factor and relativistic kinematics. The Fermi gas model provides a simplest form of the spectral function which is given by

$$P_{FG}(E, |\boldsymbol{p}|) = \frac{3}{4\pi p_F^3}\Theta(p_F - |\boldsymbol{p}|)\Theta(|\boldsymbol{p}+\boldsymbol{q}| - p_F) \times \delta[(\boldsymbol{p}^2 + m^2)^{1/2} - \varepsilon - E],$$

where p_F is the Fermi momentum and ε is effective binding energy, introduced to account of nuclear binding. For oxygen we use p_F=250 MeV/c and ε=27 MeV.

The RFGM does not account nuclear shell structure, FSI effect, and the presence of NN-correlations.

D. Inclusive and total cross sections

In order to calculate inclusive and total cross sections, we use the approach, in which only the real part of the optical potential EDAD1 is included because a complex optical potential produces adsorbtion of flux. Then the contribution of the $1p$- and $1s$-states to the inclusive cross section can be obtained as follows:

$$\left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{RDWIA} = \int_0^{2\pi} d\phi \int_{p_{min}}^{p_{max}} dp_m \frac{p_m}{p_x|\boldsymbol{q}|} R_c \times \left(\frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x}\right)_{RDWIA},$$

where $p_m = |\boldsymbol{p}_m|,\ p_x = |\boldsymbol{p}_x|,\ \boldsymbol{p}_m = \boldsymbol{p}_x - \boldsymbol{q}$, and

$$\cos\theta_{pq} = \frac{\boldsymbol{p}_x^2 + \boldsymbol{q}^2 - \boldsymbol{p}_m^2}{2p_x|\boldsymbol{q}|},$$

$$R_c = 1 + \frac{\varepsilon_x}{2p_x^2\varepsilon_B}(\boldsymbol{p}_x^2 + \boldsymbol{q}^2 - \boldsymbol{p}_m^2).$$

The effect of the FSI on the inclusive cross section can be evaluated using the ratio

$$\Lambda(\varepsilon_f, \Omega_f) = \left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{RDWIA} \Big/ \left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{PWIA},$$

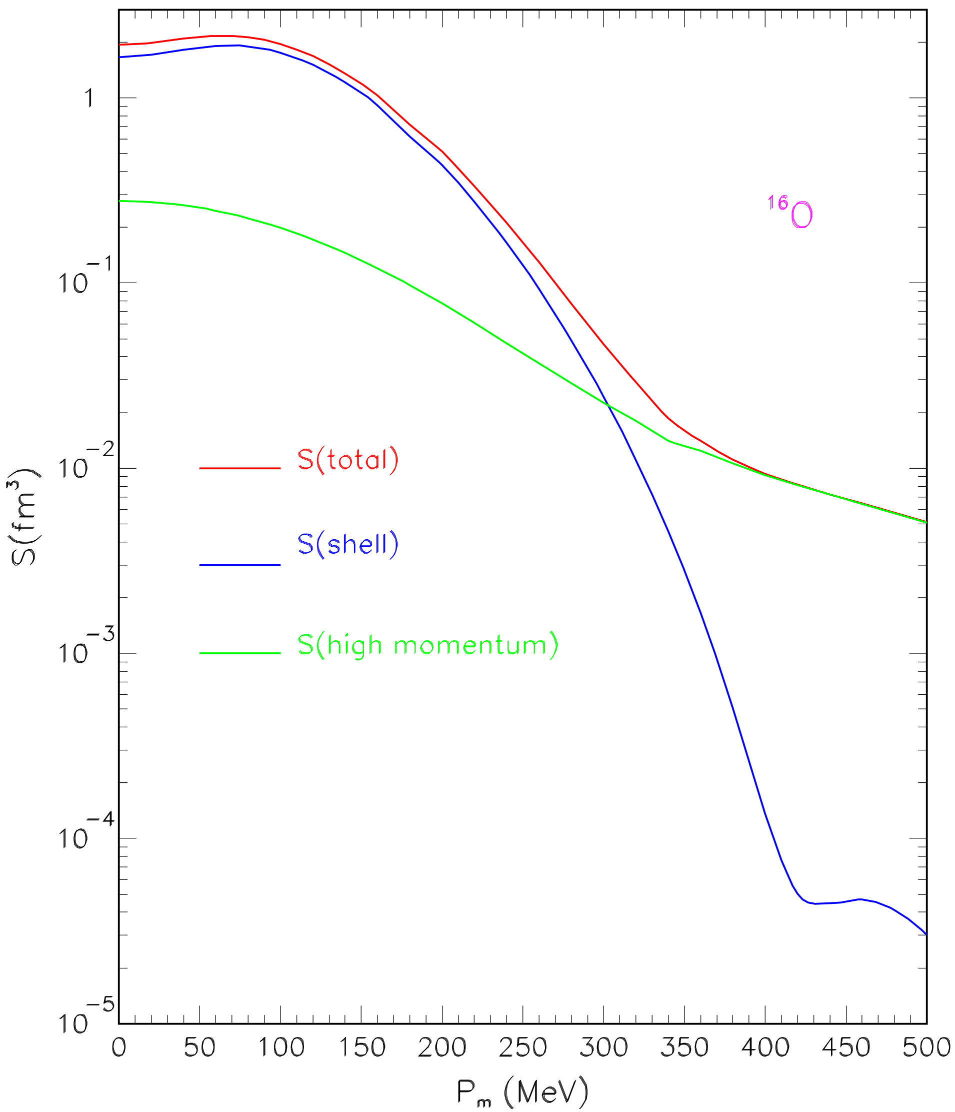
where $\left(d^3\sigma/d\varepsilon_f d\Omega_f\right)_{PWIA}$ is the result obtained in the PWIA.

D1. NN-correlations contribution

- According to JLab data [K. Fissum (2004)] the occupancy of the IPSM orbitals of ^{16}O is approximately 75% on average. We assume that the missing strength can be attributed to the short-range NN-correlations in the ground state.

- We consider a phenomenological model which incorporates high-energy and high-momentum component P_{HM} due to NN-correlations [C.Ciofi degli Atti et al (1996), S.Kulagin et al (2006)].

- In our calculations the spectral function P_{HM} incorporates 25% of the total normalization of the spectral function.



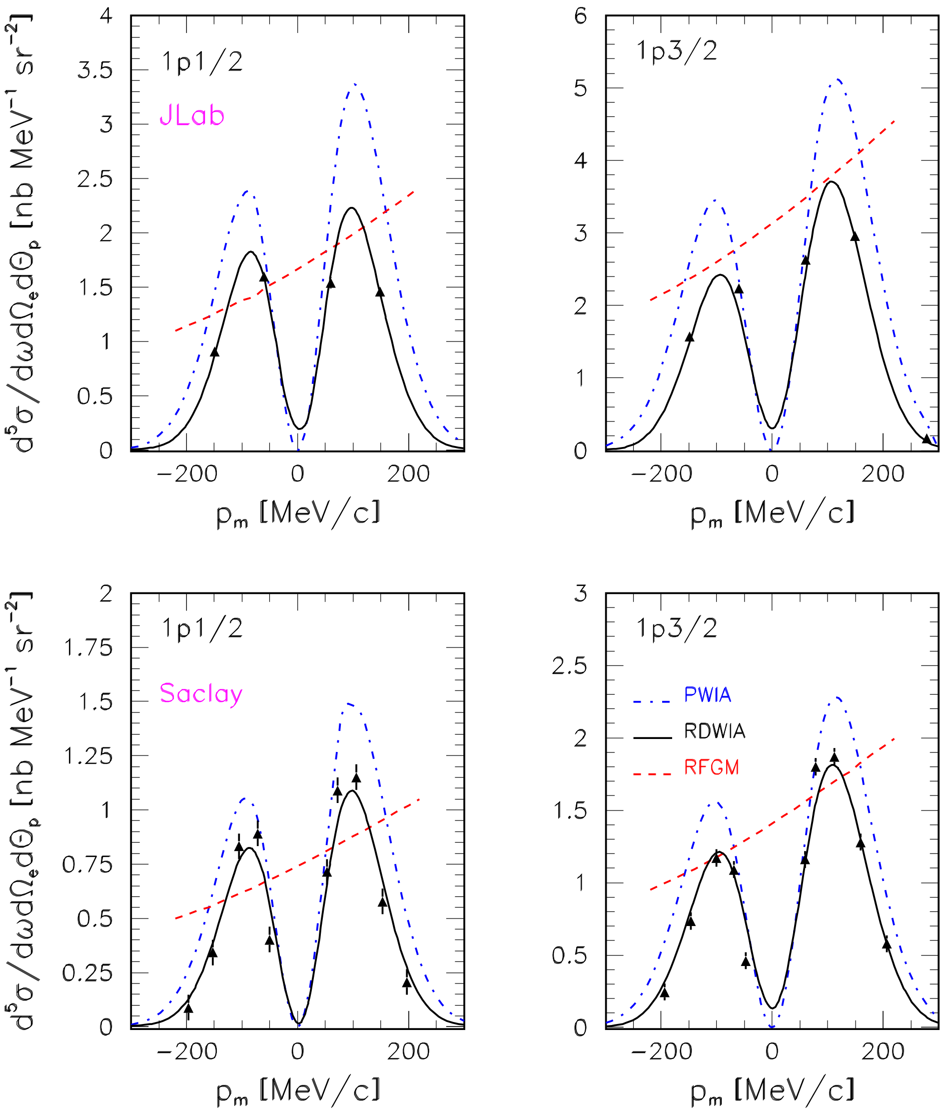
Momentum distribution for the NLSH models and hight-momentum component of the NN-corelations. The occupancy of the orbitals of ^{16}O is 75% on average and the spectral function P_{HM} incorporates 25% of the total normalization of the spectral function.

- The FSI effect for the high-momentum component is estimated by scaling the PWIA result $(d^3\sigma/d\varepsilon_f d\Omega_f)_{HM}$ with $\Lambda(\varepsilon_f,\Omega_f)$ function. Then the total inclusive cross section can be written as

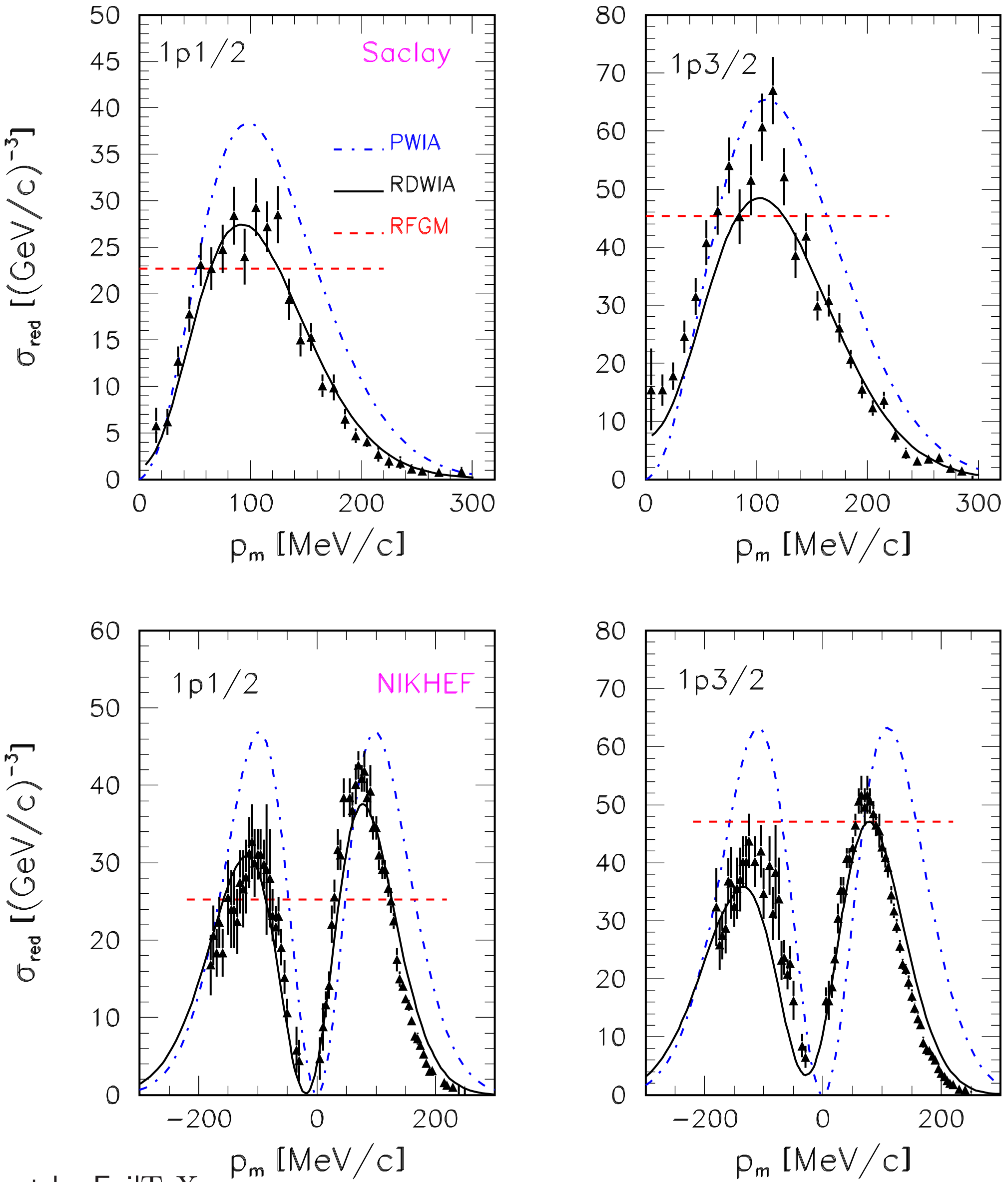
$$\frac{d^3\sigma}{d\varepsilon_f d\Omega_f} = \left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{RDWIA} + \Lambda(\varepsilon_f,\Omega_f)\left(\frac{d^3\sigma}{d\varepsilon_f d\Omega_f}\right)_{HM}.$$

- The LEA code for nucleon knockout by electron scattering was adopted in this work for neutrino interactions. This code was successfully tested against $A(e,e'p)$ data [J.J. Kelly(2005), J.Gao(200), K. Fissum (2004)]

Results

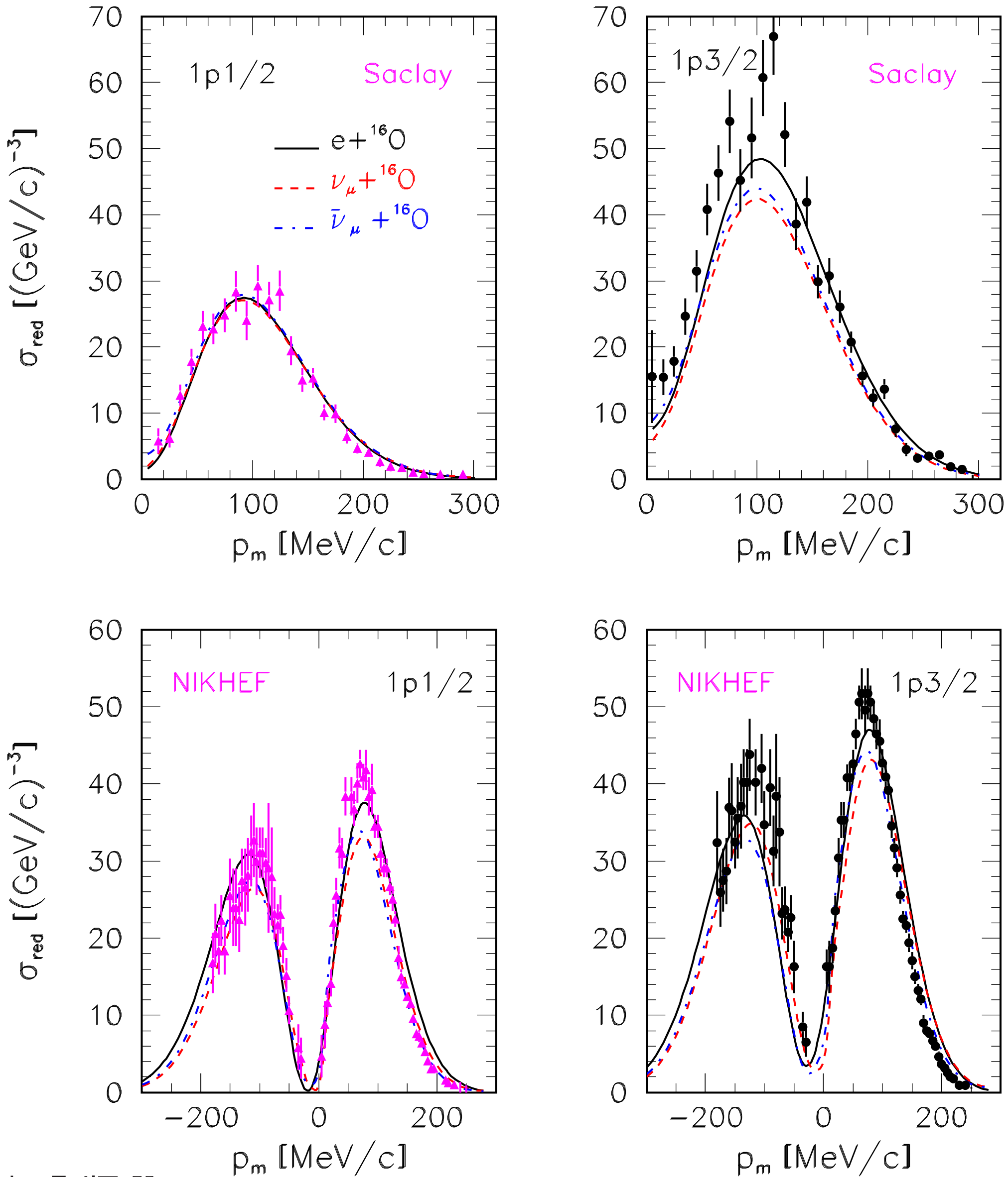


Measured differential exclusive cross-section data for the removal of protons from 1p-shell of ^{16}O as a function of missing momentum . The upper panels show JLab data for electron beam energy E_{beam}=2.442 GeV, proton kinetic energy T_p=427 MeV, and Q^2=0.8 GeV^2. The lower panels show Saclay data for E_{beam}=580 MeV, T_p=160 MeV, and Q^2=0.3 GeV^2. The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations. Negative values of p_m correspond to $\phi = \pi$ and positive ones to $\phi = 0$.



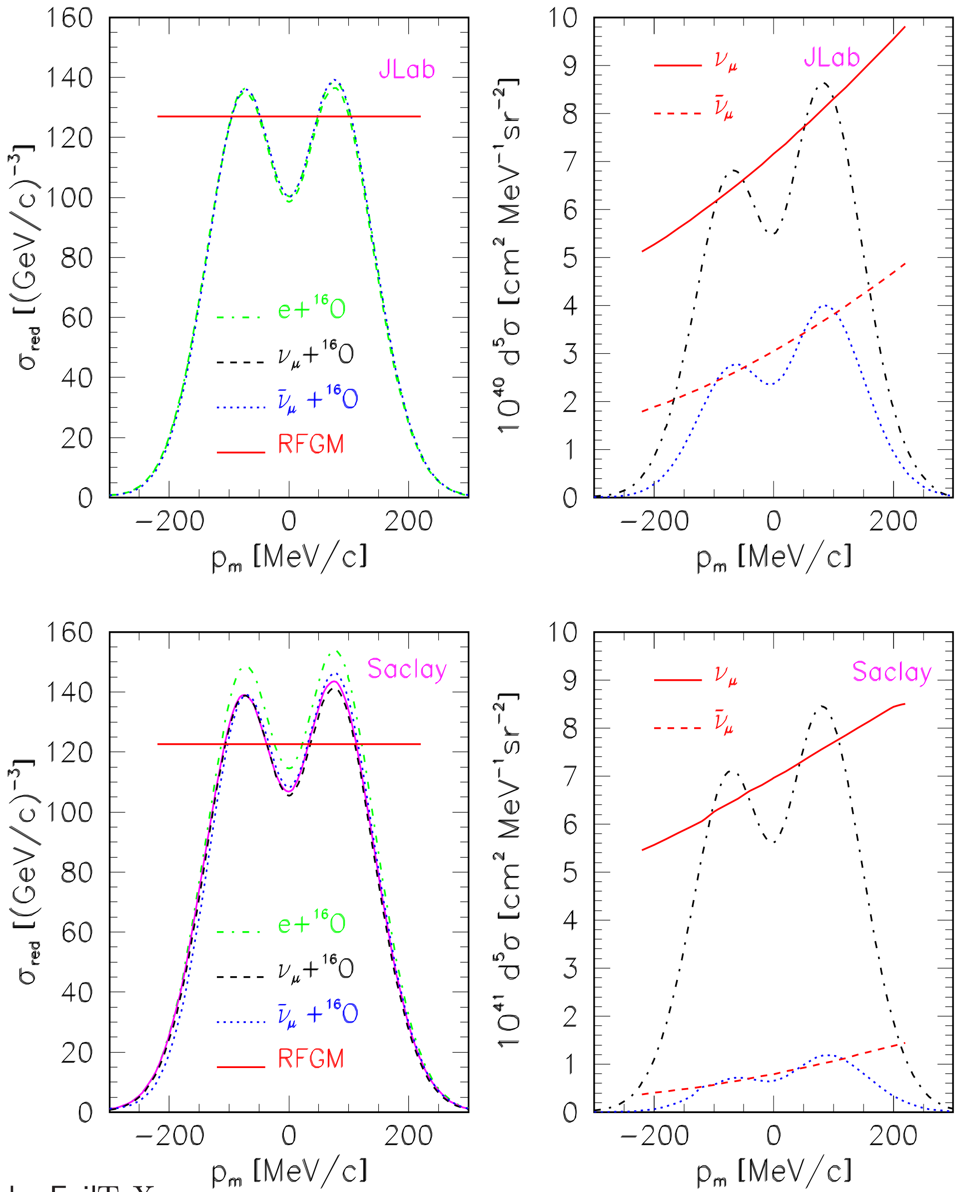
Measured reduced exclusive cross-section data for the removal of protons from 1p-shell of ^{16}O as a function of missing momentum. The upper panels show Saclay data for electron beam energy E_{beam}=500 MeV, proton kinetic energy T_p=100 MeV, and Q^2=0.3 GeV^2. The lower panels show NIKHEF data for E_{beam}=521 MeV, T_p=96 MeV, Q^2 is varied. The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations.

The RFGM predictions are completely off of the exclusive data.

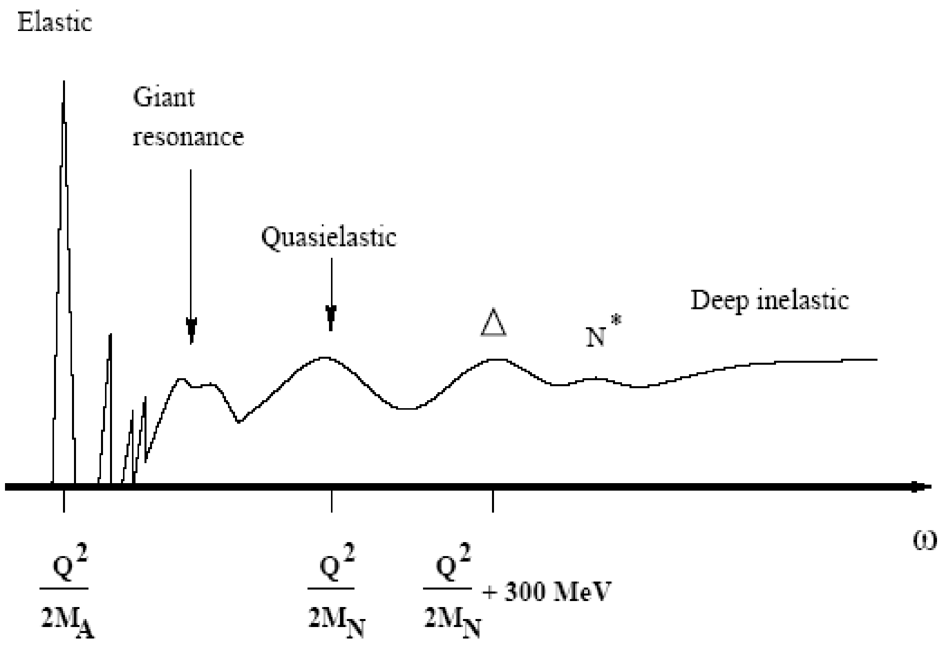


Comparison of the RDWIA electron, neutrino and antineutrino reduced cross sections for the removal of nucleons from 1p-shell of ^{16}O for Saclay (upper panels) and NIKHEF (lower panels) kinematic as functions of p_m. The solid line is electron while the dashed and dashed-dotted lines are respectively neutrino and antineutrino cross sections.

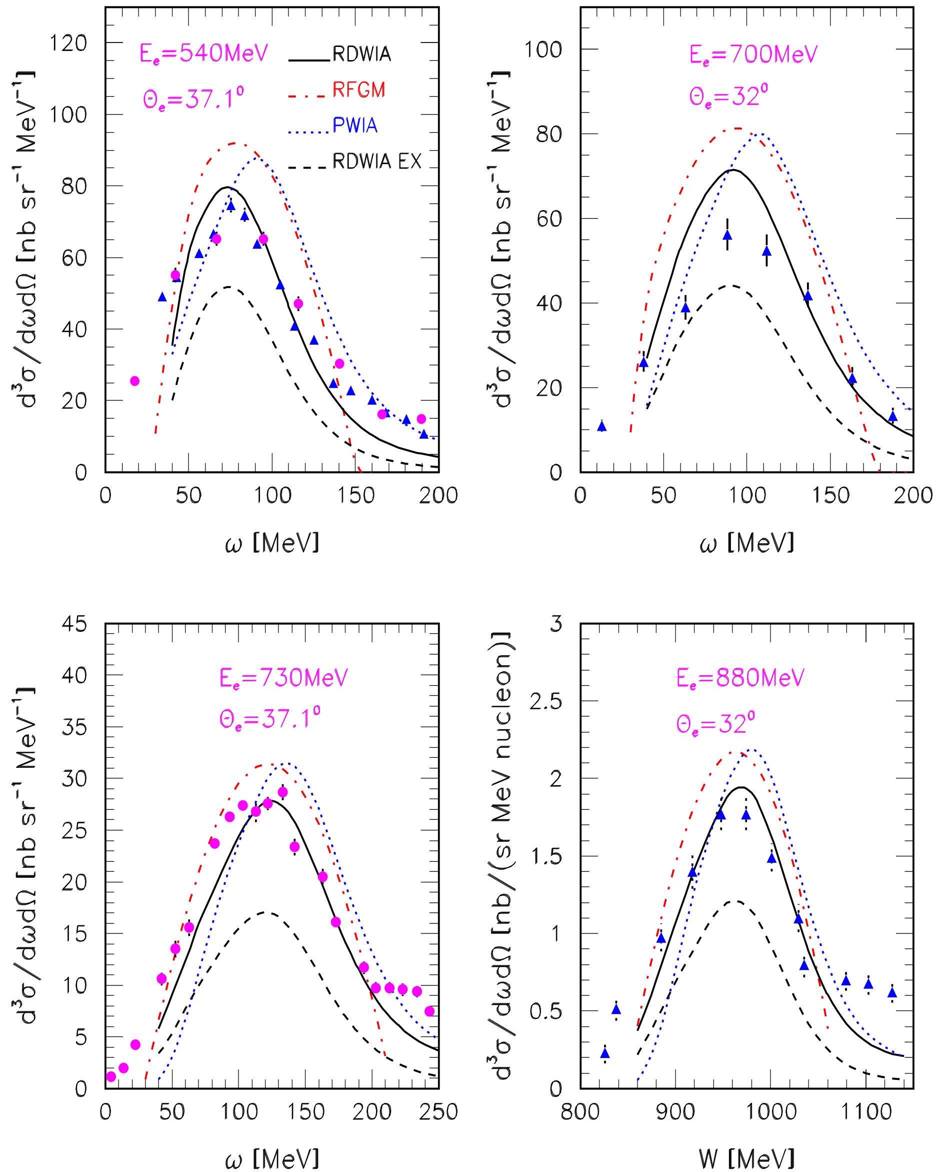
At the maximum electron cross section are higher (less than 10%) than (anti)neutrino ones.



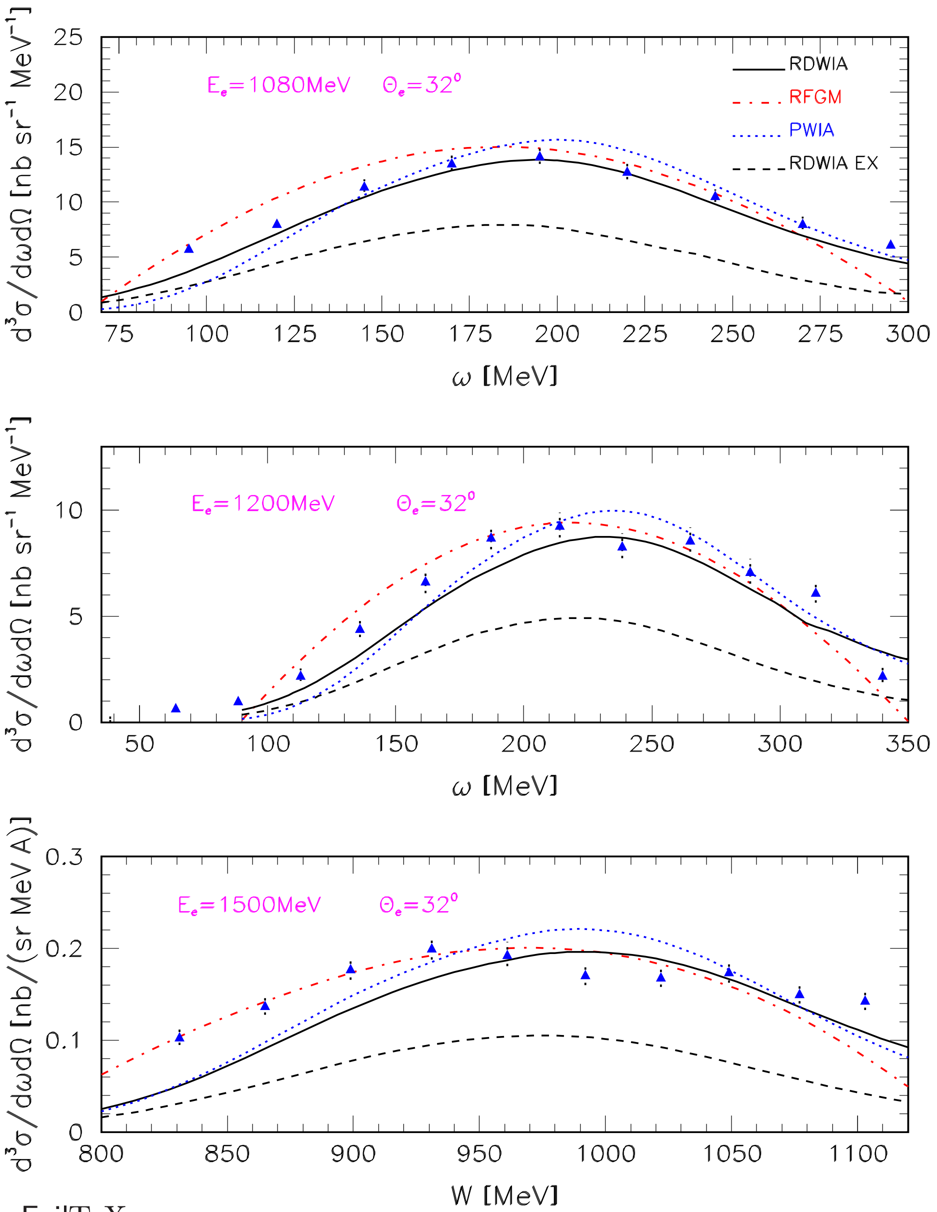
Comparison of the RDWIA and the RFGM calculations for electron, neutrino and antineutrino reduced and differential results for the removal of nucleons from 1p- and 1s-shells of ^{16}O. The dashed-dotted line is the RDWIA calculation for electron scattering while the dashed and dotted lines are respectively for neutrino and antineutrino scattering. The solid line on the left panels shows the RFGM result while the solid and dashed lines on the right panels are respectively neutrino and antineutrino cross sections calculated in the Fermi gas model. The dashed-dotted and dotted lines on right panels are respectively neutrino and antineutrino cross sections calculated in the RDWIA.



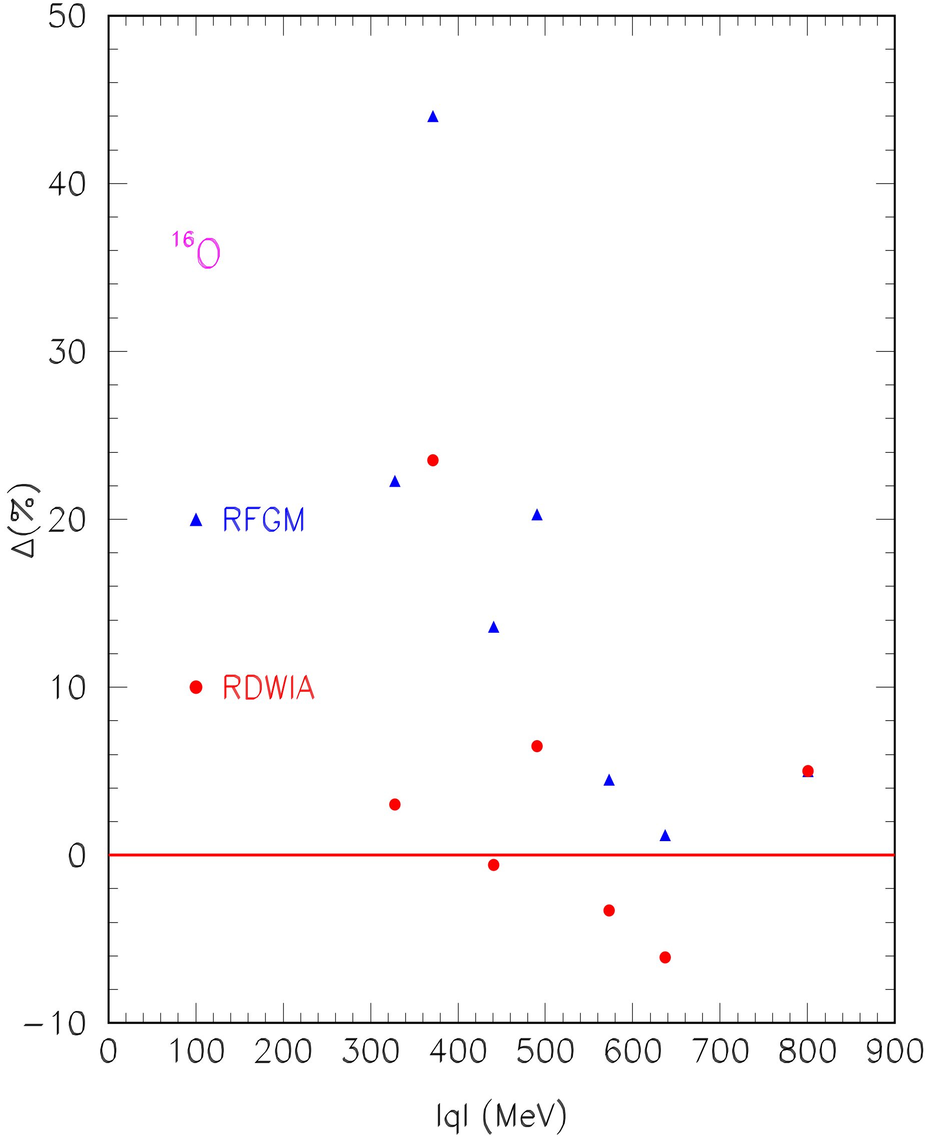
General inclusive spectrum at fixed Q^2.



Inclusive cross section versus the energy transfer ω or invariant mass W (lower-right panel) for electron scattering on ^{16}O. The data are from SLAC (filled circles) and Frascati filled triangles). SLAC data are for electron beam energy E_e=540, 730 MeV and scattering angle θ=37.1°. Frascati data are for E_e=540 MeV and θ=37.1°, E_e=700, 880 MeV and θ=32°. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.

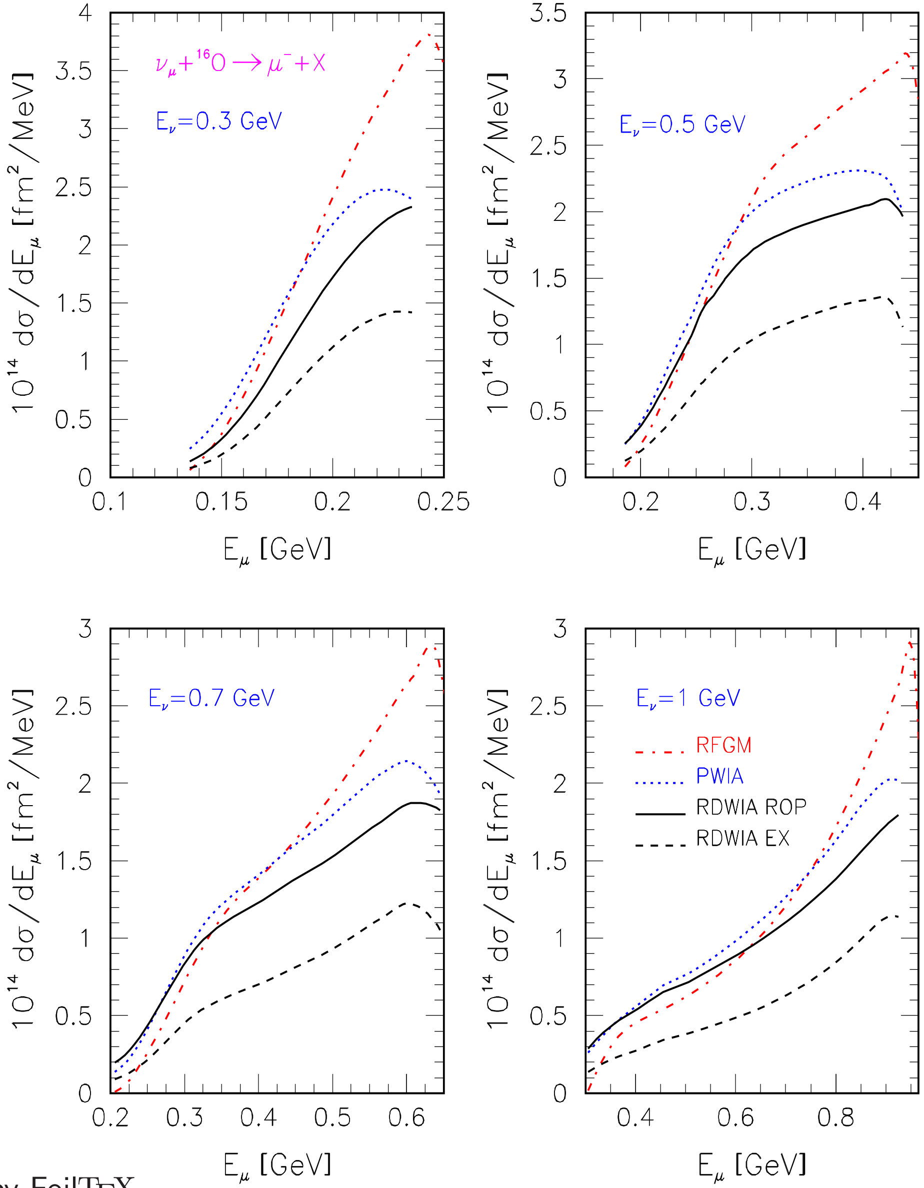


Inclusive cross section versus the energy transfer ω or invariant mass W (lower panel) for electron scattering on ^{16}O. The data are from Frascati for electron beam energy E_e=1080, 1200, and 1500 MeV and scattering angle θ=32.°. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with a complex optical potential.

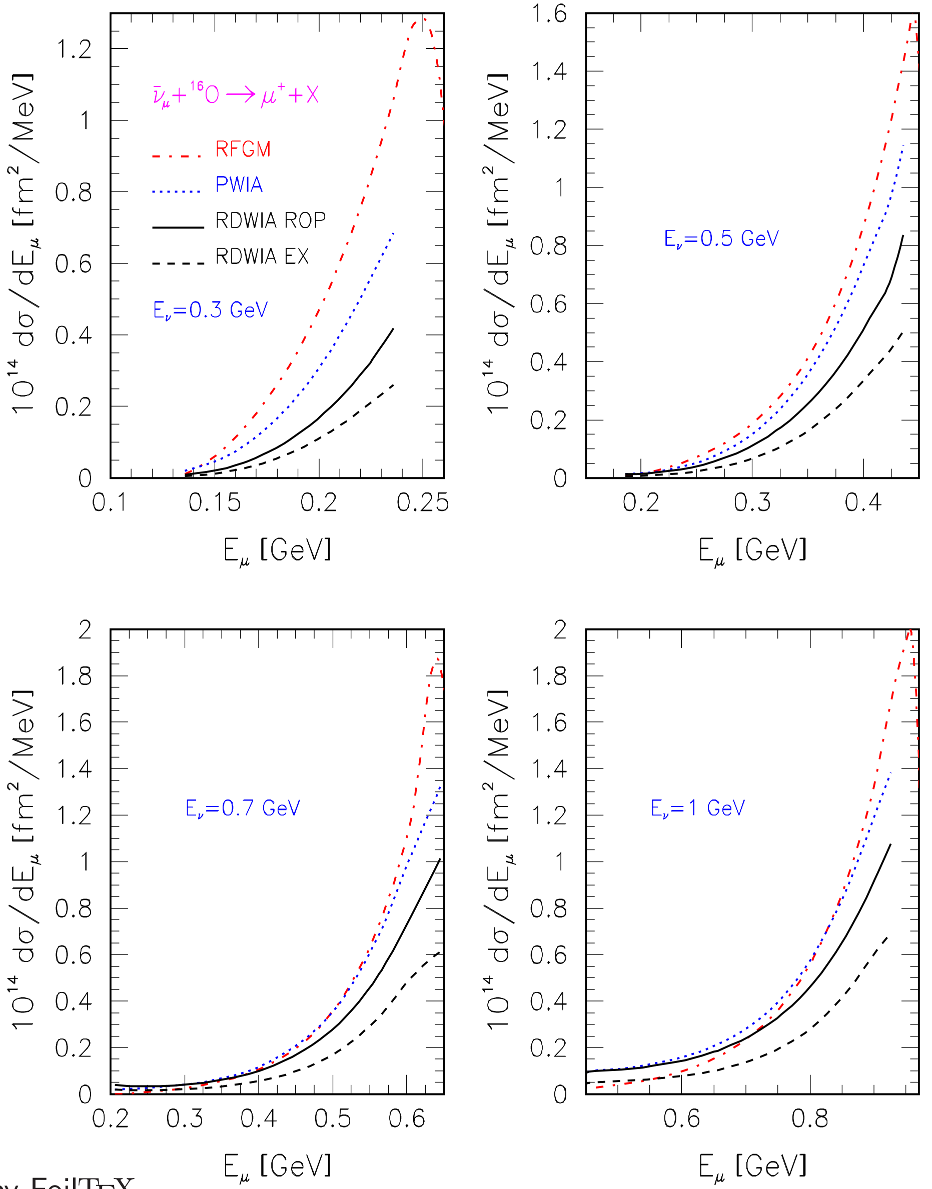


Differences between calculated and measured values of the inclusive cross sections at maximum as a function of three-momentum transfer.

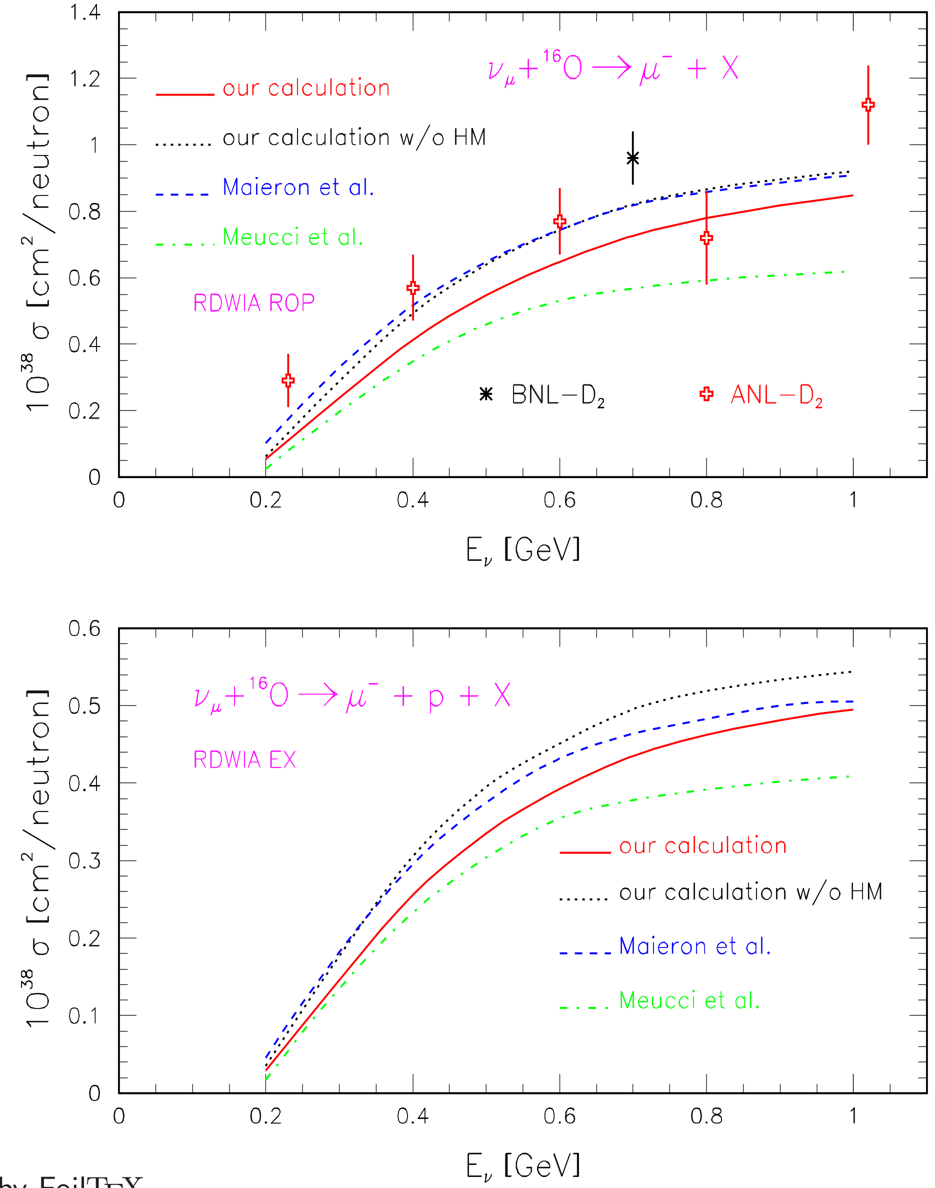
Note that: $Q^2 = \boldsymbol{q}^2 - \omega^2$



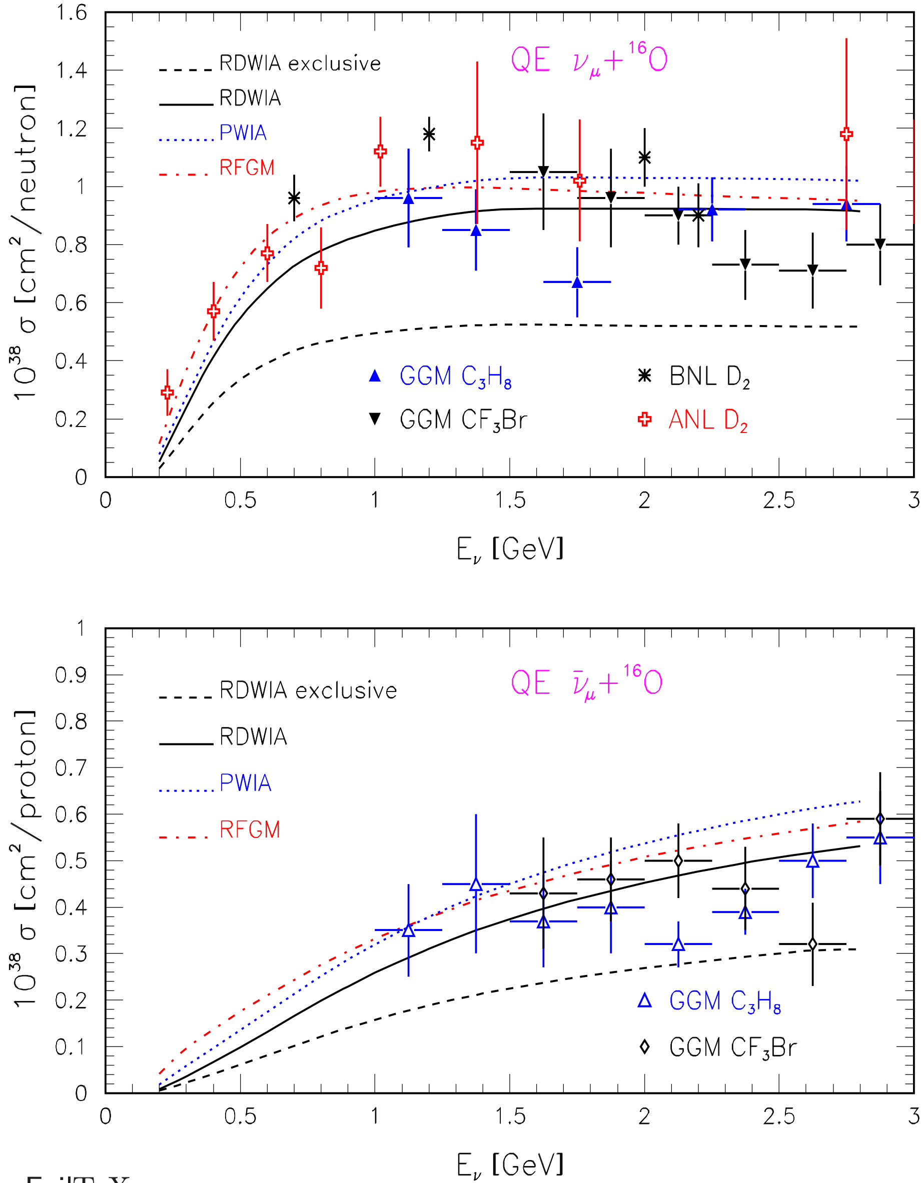
Inclusive cross section versus the muon energy for neutrino scattering on ^{16}O and for the four values of incoming neutrino energy: E_ν=0.3, 0.5, 0.7 and 1 GeV. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.



Inclusive cross section versus the muon energy for antineutrino scattering on ^{16}O and for the four values of incoming neutrino energy: E_ν=0.3, 0.5, 0.7 and 1 GeV. The solid line is the RDWIA calculation while the dotted and dashed lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.



Total cross section for the CC QE scattering of muon neutrino on ^{16}O as a function of the incoming neutrino energy. The RDWIA results with the real part of optical potential (upper panel) and complex optical potential (lower panel) are shown together with calculations from [C.Maieron et al (2003)] (dashed-dotted line) and [A.Meucci et al (2003)] (dashed line). The solid and dotted lines are respectively results obtained in this work with and without contribution of the high-momentum component. For comparison, also shown are the data from ANL and BNL for the D_2 target.



Total cross section for CC QE scattering of muon neutrino (upper panel) and antineutrino (lower panel) on ^{16}O as a function of incoming (anti)neutrino energy. The solid and dashed lines are respectively the RDWIA results with the real and complex optical potential. The dashed-dotted and dotted lines are respectively the RFGM and PWIA results. Data points for different targets are from ANL, BNL, and GGM

Summary

QE CC $\nu(\bar{\nu})^{16}$O cross sections were studied in different approaches.

- In RDWIA the reduced exclusive cross sections for $\nu(\bar{\nu})$ scattering are similar to those of electron scattering and in a good agreement with data.
- The inclusive and total cross sections were calculated neglecting the imaginary part of relativistic optical potential and taking into account the effect of NN-correlations in the target ground state and tested against ^{16}O(e,e') scattering data.
- FSI effect reduces the total cross section for about 30% for $E_\nu = 200$ MeV compared to PWIA and decreases with neutrino energy down to 10% at 1 GeV.
- Effect of NN-correlations futher reduces the total cross section for about 15% for $E_\nu = 200$ MeV. This effect also decreases with neutrino energy, down to 8% at 1 GeV.

- The Fermi gas model was tested against e^{16}O data:
  - (⋆) In the peak region RFGM overestimates the value of inclusive cross section at low momentum transfer ($|\boldsymbol{q}| < 500$ MeV/c). The discrepancy with data is about 20% at $|\boldsymbol{q}| = 300$ MeV/c and decreases as momentum transfer increases.
  - (⋆) RFGM fails completely when compared to exclusive cross section data.
- For total neutrino cross sections RFGM result is about 15% higher than the RDWIA predictions at $E_\nu \sim 1$ GeV.
- Our results show that nuclear-model dependence of the inclusive and total cross sections weakens with neutrino energy but still remains significant for energy $E_\nu \lesssim 1$ GeV.