Neutrinoproduction of low–lying baryon resonances

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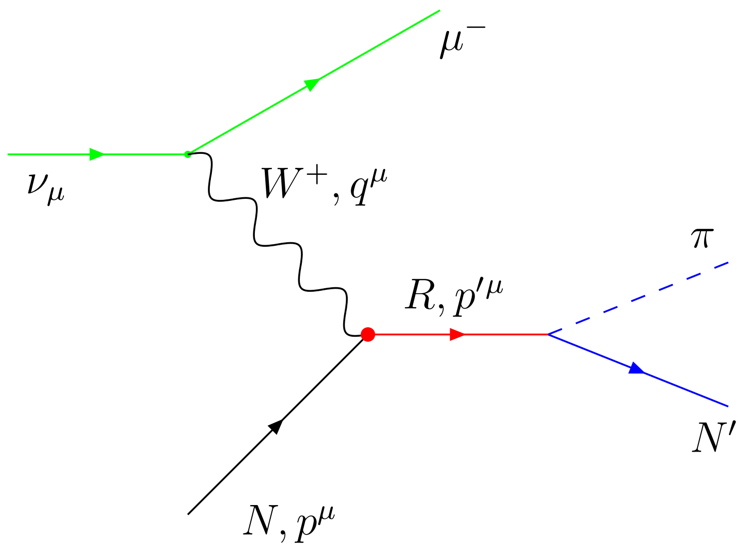
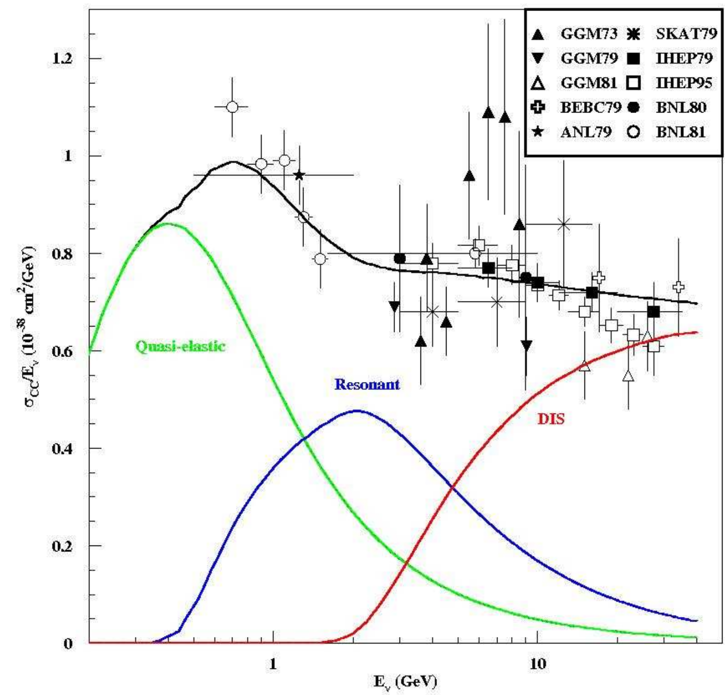
One-pion production



E.Christy, NuInt04



Threshold energy 0.3 GeV

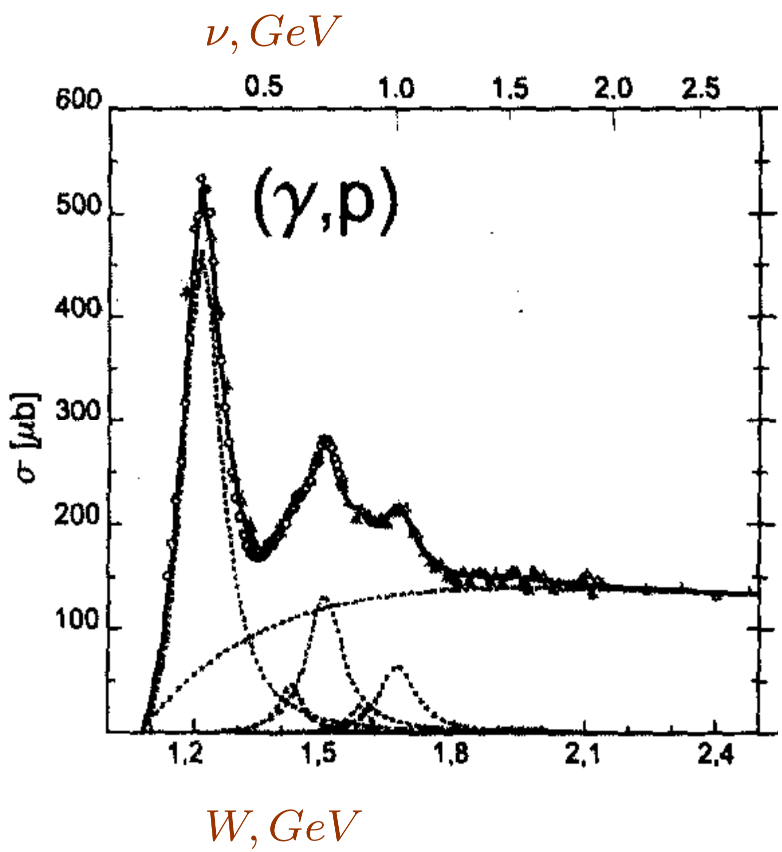
R	M_R, GeV	$\Gamma_{R(tot)}$, GeV	elasticity $\Gamma_R(R \to \pi N)/\Gamma_{R(tot)}$
$P_{33}(1232)(\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-})$	1.232	0.114	0.995
$P_{11}(1440)(P_{11}^{+}, P_{11}^{0})$	1.440	$0.350(250-450)$	$0.6(0.6-0.7)$
$D_{13}(1520)(D_{13}^{+}, D_{13}^{0})$	1.520	$0.125(110-135)$	$0.5(0.5-0.6)$
$S_{11}(1535)(S_{11}^{+}, S_{11}^{0})$	1.535	$0.150(100-250)$	$0.4(0.35-0.55)$

One-pion production

electroproduction

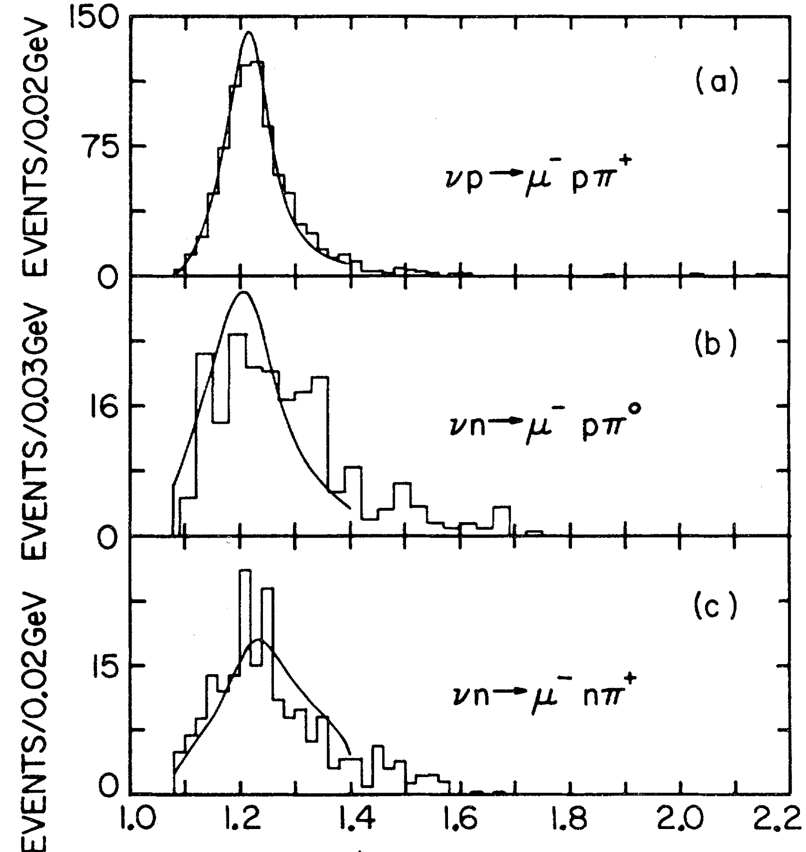
Krusche, Schadmand,

Progr. Part. Nucl. Phys. 51 (2003)



neutrinoproduction (ANL)

Radecky et al. PR D25 (1982)



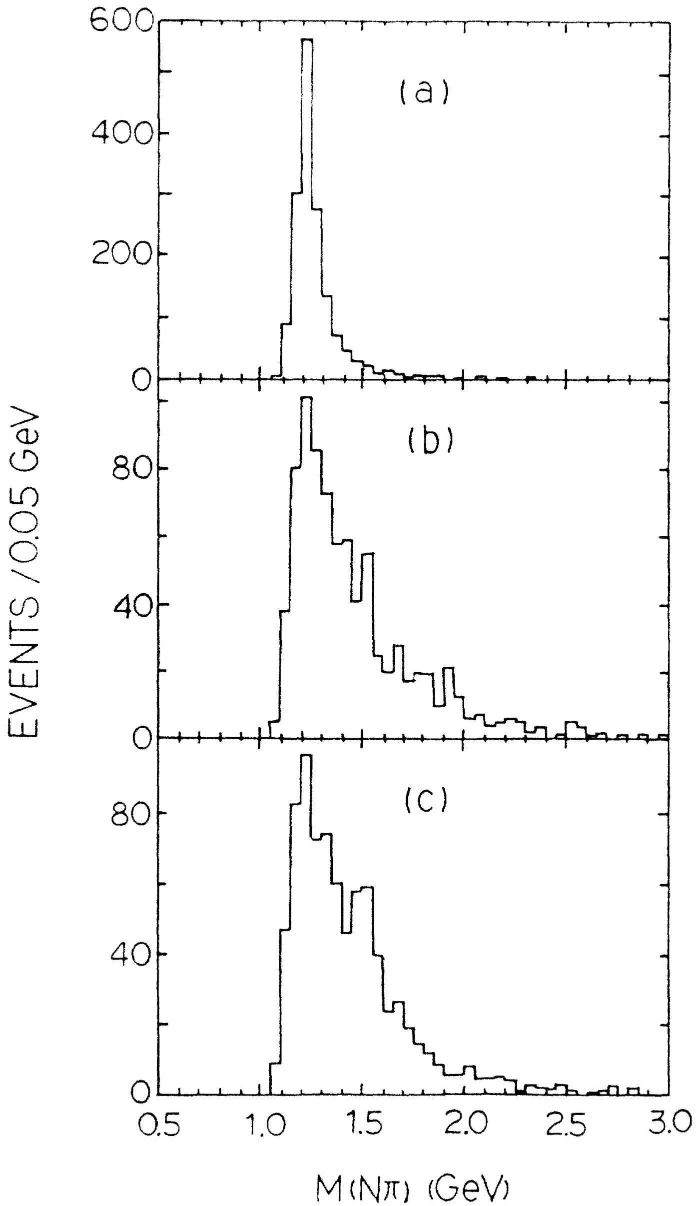


FIG. 4. The $N\pi$ invariant-mass $M(N\pi)$ distributions for the final states (a) $\mu^- p\pi^+$, (b) $\mu^- p\pi^0$, and (c) $\mu^- n\pi^+$.

neutrinoproduction (BNL)

Kitagaki et al, PR D34 (1986)

$$\nu p \to \mu^- \Delta^{++} \to \mu^- p\pi^+$$

$$\nu n \to \mu^- \Delta^+ \to \mu^- p\pi^0$$

$$\nu n \to \mu^- \Delta^+ \to \mu^- n\pi^+$$

Rein–Sehgal model Ann. Phys. 133 (1981)

- based on relativistic quark model Feynman, Kislinger, Ravndal, PR D3 (1971);
- complete set of formulas for cross sections, including pion angular distributions
- difficulty: not clear, what is the way to modify and ”fine tune” it
- recent updates from K.Kuzmin et al (Dubna), K.Hagiwara et al (KEK) (including effects from non-zero mass of the outgoing leptons), investigation of duality K.Graczyk et al (Wroclaw)

Δ–resonance

- Adler model Adler, Ann. Phys. 50 (1968) , based on multipole expansion
- isobar model introduces $\Delta - N$ transition form factors Albright,Liu, PRL 13,14 (1964); Llewellin Smith, Phys. Rep. 3 (1972), also earlier articles on electroproduction in these notation

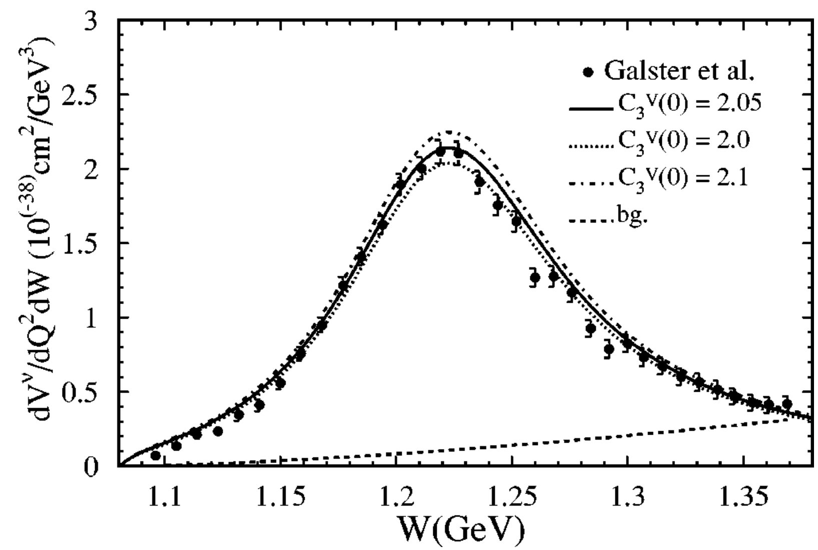
$$\langle\Delta|J^\nu|N\rangle = \bar\psi_\lambda^{(R)}\left[\frac{C_3^V}{m_N}(\not{q}g^{\lambda\nu} - q^\lambda\gamma^\nu) + \frac{C_4^V}{m_N^2}(q\cdot pg^{\lambda\nu} - q^\lambda p^\nu) + \frac{C_5^V}{m_N^2}(q\cdot p'g^{\lambda\nu} - q^\lambda p'^\nu)\right]\gamma_5 u_{(N)}$$

$$+\bar\psi_\lambda^{(R)}\left[\frac{C_3^A}{m_N}(\not{q}g^{\lambda\nu} - q^\lambda\gamma^\nu) + \frac{C_4^A}{m_N^2}(q\cdot pg^{\lambda\nu} - q^\lambda p^\nu) + C_5^A g^{\lambda\nu} + \frac{C_6^A}{m_N^2}q^\lambda q^\nu\right]u_{(N)}$$

- complete set of formulas for cross sections, including pion angular distributions Schreiner, von Hippel, Nucl Phys B58 (1973)

Vector $\Delta - N$ form factors

Paschos, Sakuda, Yu, PR D69 (2004)



magnetic dipole dominance predicted by quark model: $C_4^V = -\frac{m_N}{W} C_3^V, \quad C_5^V = 0$

comparison with electroproduction cross section (1968 - 1971) $C_3^V(0) = 2.05 \pm 0.04$

FF fall down with Q^2 faster than dipole $\dfrac{C_3^V(0)}{D_V} \dfrac{1}{1 + \frac{Q^2}{4{M_V}^2}}$ (PSY parametrization)

$D_V \equiv (1 + \frac{Q^2}{{M_V}^2})^2, \qquad M_V = 0.84 \text{ GeV}$

other parametrizations are possible, but describe qualitatively the same behaviour

Axial $\Delta - N$ form factors

PCAC $\quad i\overline{\Delta_\mu^+} q^\mu \left[C_5^A + \frac{C_6^A}{m_N^2} q^2 \right] u_N, = -i\sqrt{\frac{1}{3}} \frac{m_\pi^2 f_\pi}{q^2 - m_\pi^2} \overline{\Delta_\mu^+} g_\Delta q^\mu u_N.$

$$\Gamma(\Delta \to \pi N) = g_\Delta^2 \cdot kinematics$$

$$\Longrightarrow C_5^A(Q^2 = 0) = \frac{g_\Delta f_\pi}{\sqrt{3}} = 1.2 \qquad C_6^A = m_N^2 \frac{C_5^A}{m_\pi^2 + Q^2}$$

Comparison with neutrinoproduction cross section $C_4^A = -C_5^A/4, \quad C_3^A = 0$

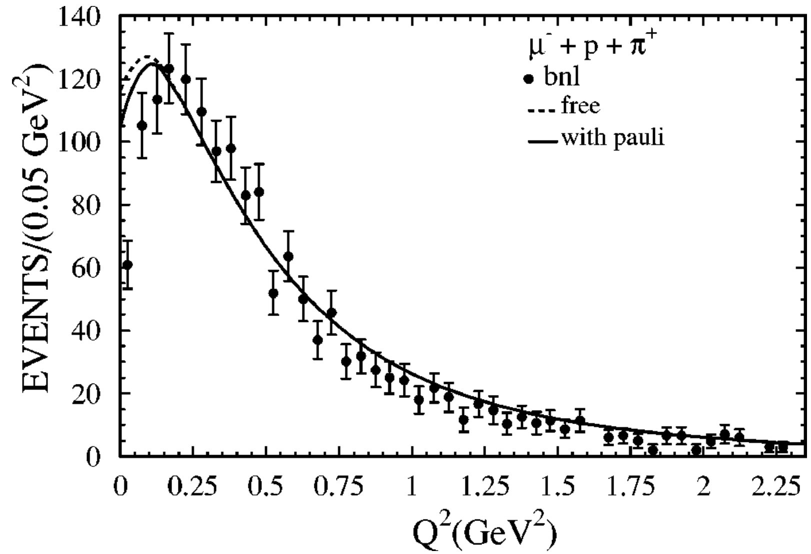
As Q^2 increases, axial form factors also fall down steeper than dipole.

$$C_5^A(Q^2) = \frac{C_5^A(Q^2)}{D_A} \cdot \frac{1}{1 + Q^2/3{M_A}^2} \quad \text{(PSY)}$$

$D_A \equiv \left(1 + \frac{Q^2}{{M_A}^2}\right)^2$ with $M_A \approx 1.05\ \text{GeV}$

other parametrizations are possible, but describe qualitatively the same behaviour

Paschos, Sakuda, Yu, PR D69 (2004)



Nuclear corrections

- are important for nuclear targets
- there are several models, which are mainly for QE scattering and Δ–production; use the isobar parametrization of the $\Delta - N$ vertex (within magnetic dominance approximation) or the Rein–Sehgal model as input

Why to try another approach, different from Schreiner–von Hippel formula?

- not clear, how to extend this approach for other resonances
- difficulty: the mass of the outgoing lepton was neglected; not clear, how to include it in calculations
- ? beauty: one cannot easily see, for example, that vector contribution vanishes for $Q^2 \to 0$

Statement of problem

To develop an approach to the resonance cross sections

- based on the isobar model and uses vector and axial form factors
- formulas for cross sections are as simple as possible; complete set of formulas can be presented in one paper
- to compare results with Schreiner–von Hippel cross section numerically and analytically for a simple case ($Q^2 \to 0$, $\Gamma_\Delta \to 0$);
- to calculate cross sections ($\equiv$ to express via form factors) for the second resonance region
- to use some ”reasonable” form factors and to estimate cross sections for different neutrino energies

Cross section via structure functions

Formulas from Paschos, O.L., PRD 71 (2005) give the cross section in a form close to DIS and include the terms with nonzero mass of the outgoing lepton (muon). The cross section in this form is the same for all the resonances

$$\frac{d\sigma}{dQ^2 dW} = \frac{G^2}{4\pi}\cos^2\theta_C \frac{W}{m_N E^2}\left\{\mathcal{W}_1(Q^2+m_\mu^2) + \frac{\mathcal{W}_2}{m_N^2}\left[2(pk)(pk') - \frac{1}{2}m_N^2(Q^2+m_\mu^2)\right] + \frac{\mathcal{W}_3}{m_N^2}\left[Q^2(pk) - \frac{1}{2}(pq)(Q^2+m_\mu^2)\right] + \frac{\mathcal{W}_4}{m_N^2}m_\mu^2\frac{(Q^2+m_\mu^2)}{2} - 2\frac{\mathcal{W}_5}{m_N^2}m_\mu^2(pk)\right\}$$

and the hadronic structure functions are defined as usual

$$\mathcal{W}^{\mu\nu} = -g^{\mu\nu}\,\mathcal{W}_1 + p^\mu p^\nu\,\frac{\mathcal{W}_2}{m_N^2} - i\varepsilon^{\mu\nu\sigma\lambda}p_\sigma q_\lambda\,\frac{\mathcal{W}_3}{2m_N^2} + \frac{\mathcal{W}_4}{m_N^2}q^\mu q^\nu + \frac{\mathcal{W}_5}{m_N^2}(p^\mu q^\nu + p^\nu q^\mu)$$

The functional dependence of the structure functions on the form factors vary with resonance.

- $\mathcal{W}_1$, $\mathcal{W}_2$, $\mathcal{W}_3$ give main contribution; $\mathcal{W}_i$ in terms of C_i are given in our paper
- $\mathcal{W}_3$ describe the vector-axial interference
- $\mathcal{W}_4$, $\mathcal{W}_5$ contribute to the Xsec proportional to the lepton mass

Structure functions

$$\mathcal{W}_i(Q^2,\nu) = \frac{1}{m_N} V_i(Q^2,\nu) R(W, M_R)$$

$$\begin{aligned} V_1 = {} & \frac{(g_1^V)^2}{\mu^4} Q^4 [(pq + m_N^2 \mp m_N M_R)] + \frac{(g_2^V)^2}{\mu^2} [2(pq)^2 \\ & + Q^2 (m_N^2 \pm m_N M_R - q \cdot p)] \\ & + \frac{g_1^V g_2^V}{\mu^3} 2Q^2 [(pq)(M_R \mp m_N) \pm m_N Q^2] \\ & + (g_1^A)^2 (m_N^2 \pm m_N M_R + q \cdot p) \end{aligned} \tag{4.25}$$

$$V_2 = 2m_N^2 \left[\frac{(g_1^V)^2}{\mu^4} Q^4 + \frac{(g_2^V)^2}{\mu^2} Q^2 + (g_1^A)^2 \right] \tag{4.26}$$

$$V_3 = 4m_N^2 \left[\frac{g_1^V g_1^A}{\mu^2} Q^2 + \frac{g_2^V g_1^A}{\mu} (M_R \pm m_N) \right] \tag{4.27}$$

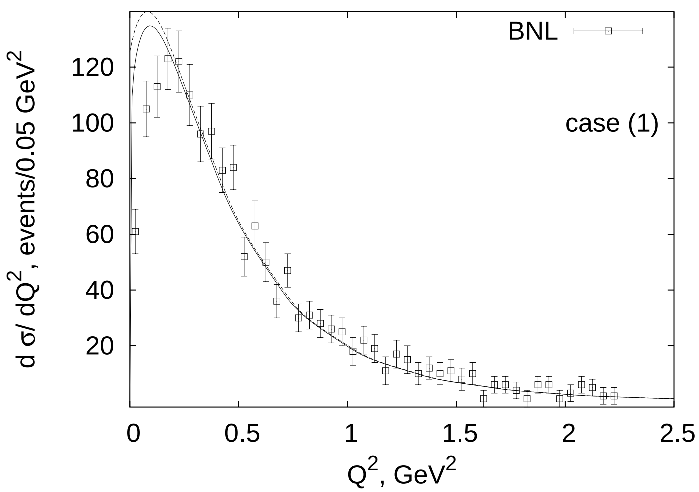
These are the formulas for the spin-1/2 resonances

$\pm$ signs here correspond to P_{11} and S_{11} resonances, respectively

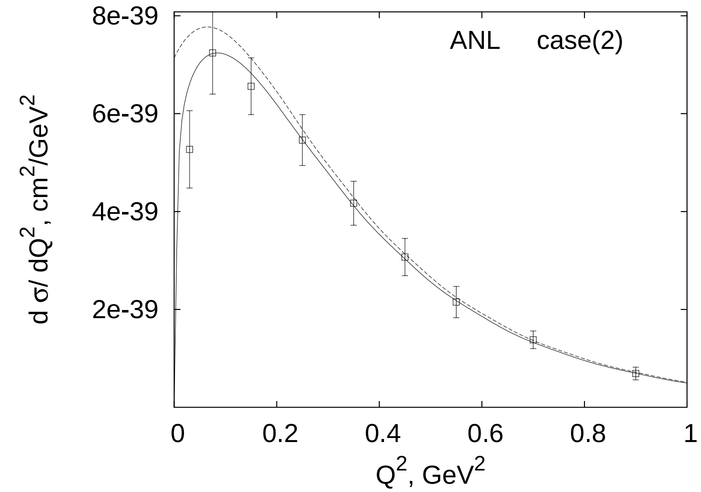
Formulas for spin-3/2 resonances are more cumbersome

OL, Paschos, Piranishvili, PRD74 (2006) 014009

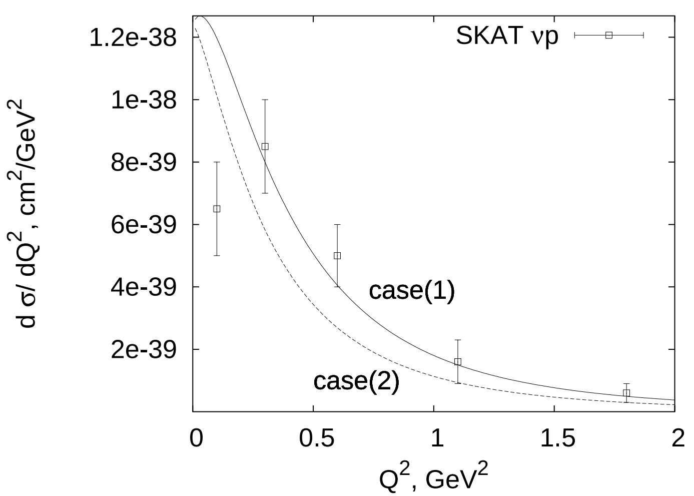
Cross section for $\nu_\mu p \to \mu^- \Delta^{++} \to \mu^- p \pi^+$



$\langle E_\nu \rangle \sim 1$ GeV



$\langle E_\nu \rangle \sim 1$ GeV



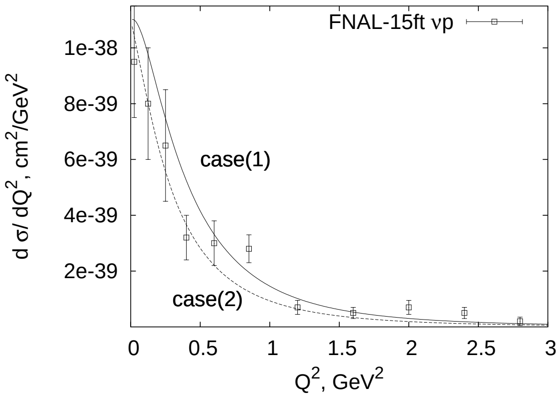
$\langle E_\nu \rangle \sim 7$ GeV

case (1): $C_5^A(Q^2) = \dfrac{C_5^A(0)}{D_A} \cdot \dfrac{1}{1 + \dfrac{Q^2}{\mathbf{3} {M_A}^2}}$

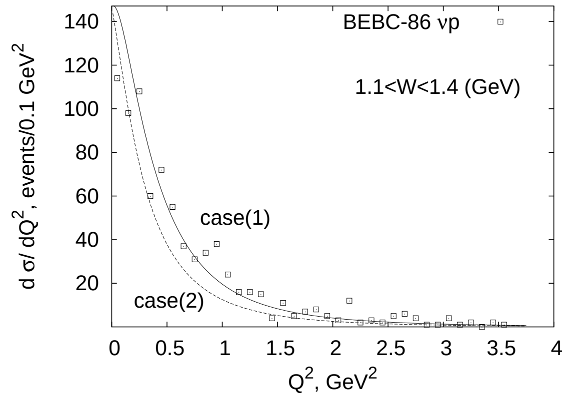
case (2): $C_5^A(Q^2) = \dfrac{C_5^A(0)}{D_A} \cdot \dfrac{1}{1 + \dfrac{\mathbf{2} Q^2}{{M_A}^2}}$

Paschos, OL PRD 71 (2005) 074003

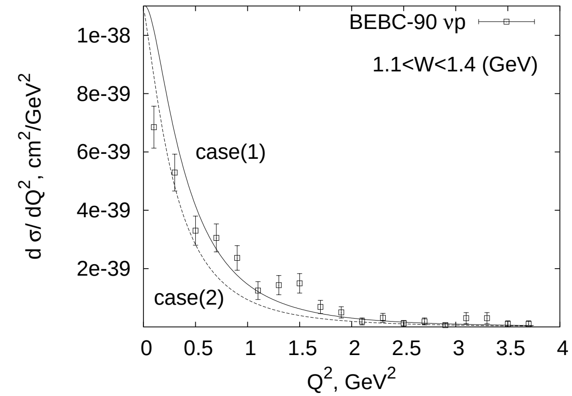
Cross section for $\nu_\mu p \to \mu^- \Delta^{++} \to \mu^- p \pi^+$



$E_\nu \sim 15 - 40$ GeV



$\langle E_\nu \rangle \sim 54$ GeV



$\langle E_\nu \rangle \sim 54$ GeV

The low Q^2 region still to be determined and understood precisely.

Factors, which decrease the cross section are: 1) Pauli blocking Paschos, Sakuda, Yu PRD 69 (2004) 014013 2) muon mass effects Paschos, O.L., PRD 71 (2005) 074003

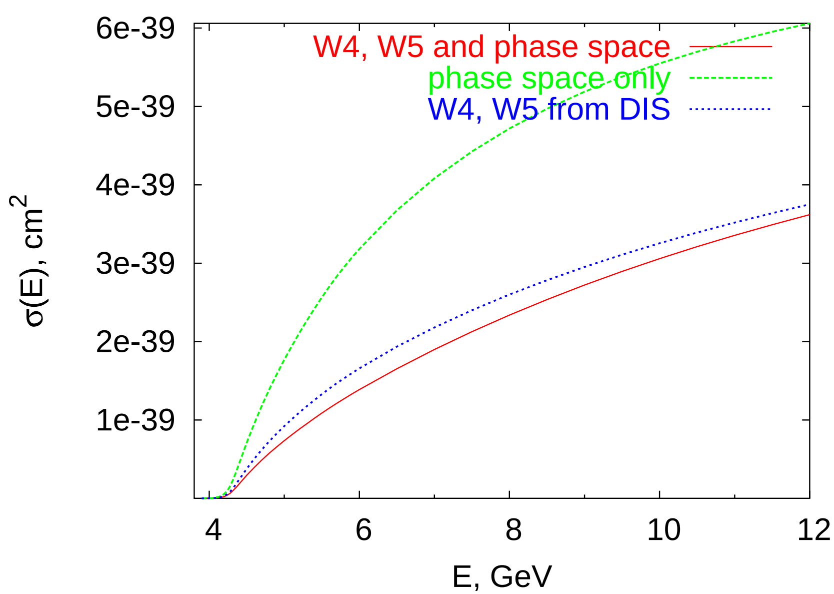
For the $d\sigma/dQ^2$ lepton mass effects are noticable at low Q^2 for small neutrino energies.

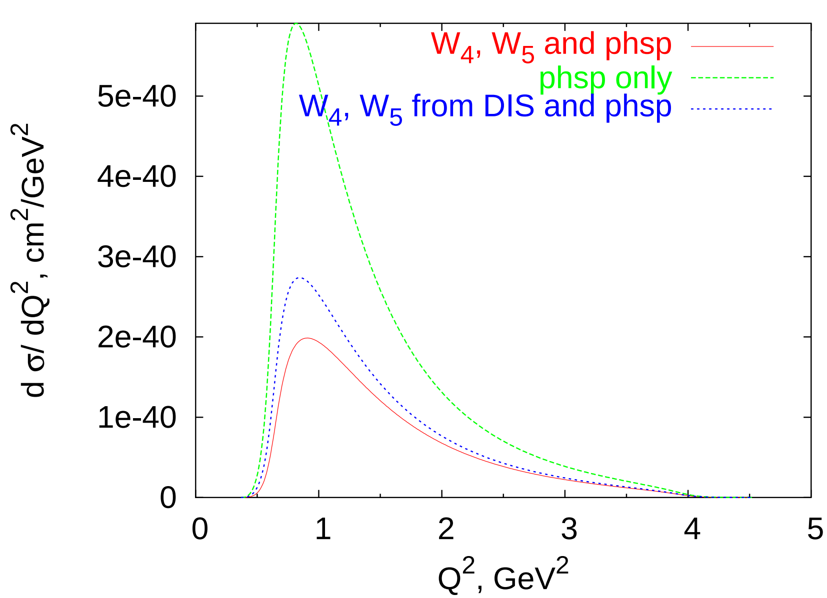
τ-production: $\nu_\tau p \longrightarrow \tau^- \Delta^{++} \longrightarrow \tau^- p \pi^+$

Taking into account nonzero mass if the fi nal lepton reduces the cross section in two ways:

1) due to the kinematical restrictions on the phase space available

2) due to the ”small” structure functions $\mathcal{W}_4$ and $\mathcal{W}_5$





red line: both effects are taken into account

green line: only the reduction of the phase space is taken into account; ($\mathcal{W}_4 = \mathcal{W}_5 = 0$)

blue line: structure functions in the DIS limit: $\mathcal{W}_4 = 0$, $\mathcal{W}_5 = \mathcal{W}_2/2x$

What are ”reasonable” form factors

The approach based on multipole expansion was further developed for electroproduction reactions, the results of resonance analysis are presented in the form of multipole amplitudes or helicity amplitudes

For $P_{33}(1232)$ the magnetic dipole dominance is used in all neutrinoproduction calculations, which appears to be a good approximation at low Q^2

2001: unambigious evidences from JLAB for the contribution of the electric $E2 \sim -2.5\%$, of scalar multipoles $S2 \sim -5\%$.

Idea to extract the form factors from the helicity amplitudes OL, Paschos, Piranishvilli, PRD74;

thus all multipoles are taken into account

Helicity amplitudes for $P_{33}(1232)$

Helicity amplitudes evaluated from the electroproduction data on proton at $W = M_R$ Tiator et al., EPJA 19 (2004); Burkert, Li, IJMP 13 (2004); Aznauryan (talk at NStar 2005) The relations to C_i^V are calculated by our group at arbitrary Q^2 and W

$$A_{3/2} = \sqrt{\frac{\pi\alpha_{em}}{m_N(W^2-m_N^2)}}\langle R,+\tfrac{3}{2}|J_{em}\cdot\varepsilon^{(R)}|N,+\tfrac{1}{2}\rangle =$$

$$= -\sqrt{N}\frac{|\vec{q}|}{p'^0+M_R}\left[\frac{C_3^{(em)}}{m_N}(m_N+M_R)+\frac{C_4^{(em)}}{m_N^2}(m_N\nu-Q^2)+\frac{C_5^{(em)}}{m_N^2}m_N\nu\right]$$

$$A_{1/2} = \sqrt{\frac{\pi\alpha_{em}}{m_N(W^2-m_N^2)}}\langle R,+\tfrac{1}{2}|J_{em}\cdot\varepsilon^{(R)}|N,-\tfrac{1}{2}\rangle$$

$$= \sqrt{\frac{N}{3}}\frac{|\vec{q}|}{p'^0+M_R}\left[\frac{C_3^{(em)}}{m_N}\left(m_N+M_R-2\frac{m_N}{M_R}(p'^0+M_R)\right)+\frac{C_4^{(em)}}{m_N^2}(m_N\nu-Q^2)+\frac{C_5^{(em)}}{m_N^2}\right.$$

$$S_{1/2} = \sqrt{\frac{\pi\alpha_{em}}{m_N(W^2-m_N^2)}}\frac{q_z}{\sqrt{Q^2}}\langle R,+\frac{1}{2}|J_{em}\cdot\varepsilon^{(S)}|N,+\frac{1}{2}\rangle =$$

$$= \sqrt{\frac{2N}{3}}\frac{|\vec{q}|^2}{M_R(p'^0+M_R)}\left[\frac{C_3^{(em)}}{m_N}M_R+\frac{C_4^{(em)}}{m_N^2}W^2+\frac{C_5^{(em)}}{m_N^2}m_N(m_N+\nu)\right]$$

$$N = \frac{\pi\alpha_{em}}{m_N(W^2-m_N^2)}2m_N(p'^0+M_R)$$

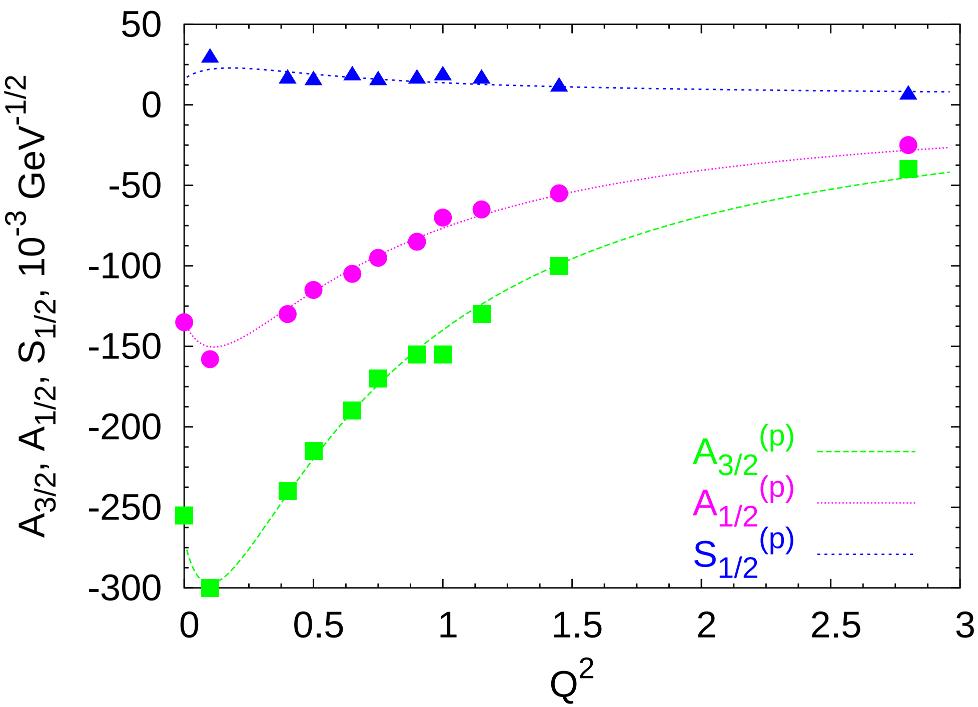
Beyond the magnetic dominance

Magnetic dominance for low Q^2: $A_{3/2} \approx \sqrt{3} A_{1/2}$

QCD asymptotics for $Q^2 \to \infty$:

$$A_{3/2} \sim \frac{1}{Q^5}, \quad A_{1/2} \sim \frac{1}{Q^3}, \quad S_{1/2} \sim \frac{1}{Q^4}$$

+ logarithmic corections



helicity amplitudes at $W = M_{P1232}$,

QCD asymptotics for $Q^2 \to \infty$:

$$C_3^V \sim \frac{1}{Q^6}, \quad C_4^V \sim \frac{1}{Q^8}, \quad C_5^V \lesssim \frac{1}{Q^8}$$

to be updated !

$$C_3^V = \frac{2.14/D_V}{1+Q^2/4M_V^2},$$

$$C_4^V = \frac{-1.56/D_V}{\left(1+Q^2/7.3M_V^2\right)^2},$$

$$C_5^V = \frac{-0.83/D_V}{\left(1+Q^2/0.95M_V^2\right)^2}$$

$$C_i^V = C_i^{(p)} = C_i^{(n)}$$

For $Q^2 < 3\,\mathrm{GeV}^2$ these form factors coincide with the "magnetic dominance" values with 4% accuracy

$D_{13}(1520)$: $J^P = 3/2^-, I = 1/2$

The formulas for this resonance are similar to that for P_{33} with the γ_5 changing the place: the are 3 independent vector form factors and 3 independent axial form factors

$$\langle D_{13}|V^\nu|N\rangle = \bar{\psi}_\lambda^{(R)} \left[\frac{C_3^V}{m_N}(\not{q} g^{\lambda\nu} - q^\lambda \gamma^\nu) + \frac{C_4^V}{m_N^2}(q \cdot p g^{\lambda\nu} - q^\lambda p^\nu) \right.$$

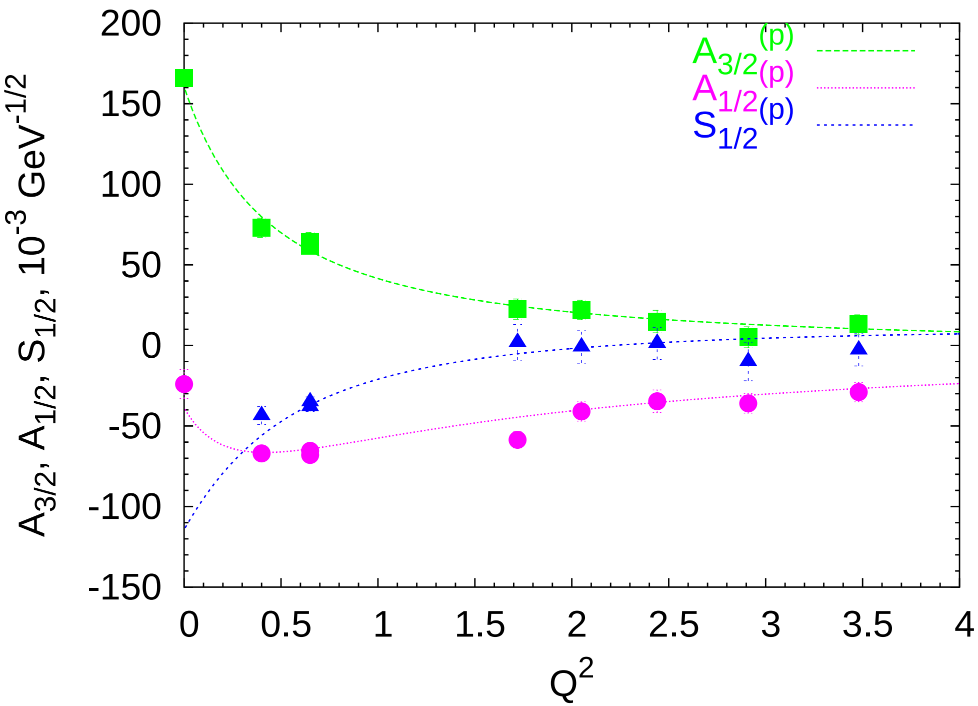
$$\left. + \frac{C_5^V}{m_N^2}(q \cdot p' g^{\lambda\nu} - q^\lambda p'^\nu) \right] u_{(N)}$$

$$\langle D_{13}|A^\nu|N\rangle = \bar{\psi}_\lambda \left[\frac{C_3^A}{m_N}(\not{q} g^{\lambda\nu} - q^\lambda \gamma^\nu) + \frac{C_4^A}{m_N^2}(q \cdot p g^{\lambda\nu} - q^\lambda p^\nu) + C_5^A g^{\lambda\nu} + \frac{C_6^A}{m_N^2} q^\lambda q^\nu \right] \gamma_5 u_{(}$$

$D_{13}(1520)$: Vector form factors

QCD asymptotics for $Q^2 \to \infty$:

$$A_{3/2} \sim \frac{1}{Q^5}, \quad A_{1/2} \sim \frac{1}{Q^3}, \quad S_{1/2} \sim \frac{1}{Q^4}$$



Helicity amplitudes for proton for $W = M_{D1520}$,

Data points from Aznauryan, talk at NStar 2005

are shown, updated points are coming ...

QCD asymptotics for $Q^2 \to \infty$:

$$C_3 \sim \frac{1}{Q^6}, \quad C_4 \sim \frac{1}{Q^8}, \quad C_5 \sim \frac{1}{Q^8}$$

to be updated !

$$C_3^{(p)} = \frac{2.98/D_V}{1+Q^2/8M_V^2}$$

$$C_4^{(p)} = \frac{-1.17/D_V}{\left(1+Q^2/17M_V^2\right)^2},$$

$$C_5^{(p)} = \frac{-0.49/D_V}{\left(1+Q^2/37M_V^2\right)^2},$$

$$C_3^{(n)} = \frac{-1.15/D_V}{1+Q^2/8M_V^2}$$

$$C_4^{(n)} = \frac{0.46/D_V}{\left(1+Q^2/17M_V^2\right)^2},$$

$$C_5^{(n)} = \frac{-0.18/D_V}{\left(1+Q^2/37M_V^2\right)^2},$$

where $D_V = (1+Q^2/M_V^2)^2$, $M_V^2 = 0.71\ \mathrm{GeV}^2$

$$C_i^V = C_i^{(n)} - C_i^{(p)}$$

$D_{13}(1520)$: Axial form factors

From PCAC $C_6^A(Q^2) = m_N^2 \frac{C_5^A(Q^2)}{m_\pi^2+Q^2}$, $\quad C_5^A(D_{13}) = -\sqrt{\frac{2}{3}} g_{\pi NR} f_{D13} = -2.1$

NO model for other axial form factors

The Q^2 dependence cannot be determined from experiment because of the lack of the data.

Instead, we consider two cases: (i) "fast fall-off", in which the Q^2 dependence is choosen the same as for the P_{33} resonance, and (ii) "slow fall-off", in which the Q^2 dependence is flatter and is taken to be the same as for the vector form factors for each resonace.

$$D_{13}(1520): \quad C_5^A = \frac{2.1/D_A}{1+Q^2/3M_A^2} \text{ ("fast fall-off")}$$

$$C_5^A = \frac{2.1/D_A}{1+Q^2/8M_A^2} \text{ ("slow fall-off")} .$$

with the axial mass common for all the resonances $M_A = 1.05 \text{ GeV}$

We take for simplicity $C_3^A(Q^2) = 0$, $C_4^A(Q^2) = 0$.

$P_{11}(1440)$, $J^P = \frac{1}{2}^+$ and $S_{11}(1535)$, $J^P = \frac{1}{2}^-$

For the spin-1/2 resonances all formulas are simpler

$$\langle P_{11}|J^\nu|N\rangle = \bar{u}(p')\left[\frac{g_1^V}{(m_N+M_R)^2}(Q^2\gamma^\nu + \not{q}q^\nu) + \frac{g_2^V}{m_N+M_R}i\sigma^{\nu\rho}q_\rho \right.$$
$$\left. -g_1^A\gamma^\nu\gamma_5 - \frac{g_3^A}{m_N}q^\nu\gamma_5\right]u(p),$$

where $\sigma^{\nu\rho} = \frac{i}{2}[\gamma^\nu, \gamma^\rho]$.

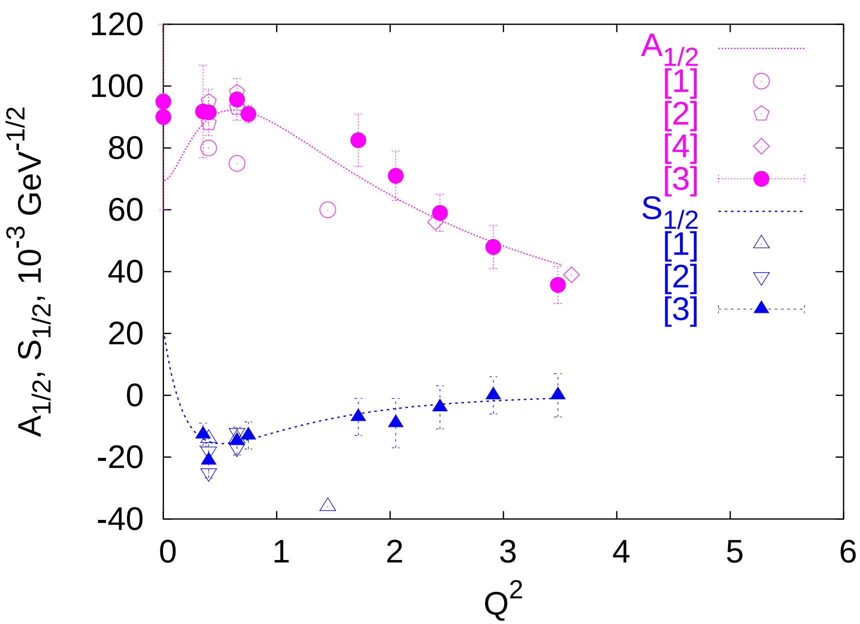
$$\langle S_{11}|J^\nu|N\rangle = \bar{u}(p')\left[\frac{g_1^V}{(m_N+M_R)^2}(Q^2\gamma^\nu + \not{q}q^\nu)\gamma_5 + \frac{g_2^V}{m_N+M_R}i\sigma^{\nu\rho}q_\rho\gamma_5 \right.$$
$$\left. -g_1^A\gamma^\nu - \frac{g_3^A}{m_N}q^\nu\right]u(p),$$

$S_{11}(1535)$: Vector form factors

QCD asymptotics for $Q^2 \to \infty$:

$$A_{1/2} \sim \frac{1}{Q^3}, \quad S_{1/2} \lesssim \frac{1}{Q^4}$$

+logarithmic corrections



Helicity amplit. for proton for $W = M_{S1535}$

[1] Tiator et al., EPJA 19 (2004);

[2] Burkert, Li, IJMP 13 (2004);

[3] Aznauryan (talk at NStar 2005)

[4] Armstrong et al., PRD 60 (2000)

QCD asymptotics for $Q^2 \to \infty$:

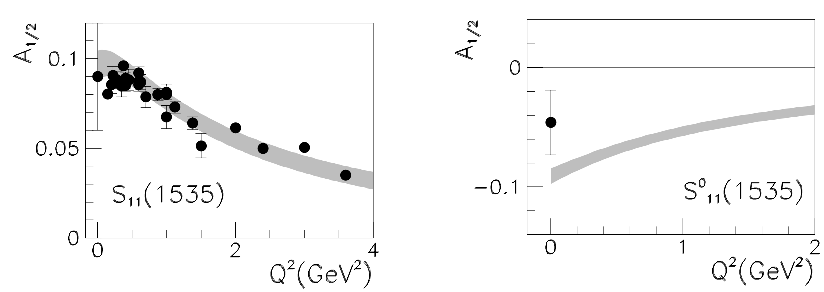
$$g_1^{(em)} \sim \frac{1}{Q^6}, \quad g_2^{(em)} \lesssim \frac{1}{Q^8}$$

to be updated!

$$g_1^{(p)} = \frac{1.87/D_V}{1+Q^2/1.2M_V^2}\left[1 + 7.07 \ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}\right.\right.$$

$$g_2^{(p)} = \frac{0.64/D_V}{(1+Q^2/17M_V^2)^2}\left[1 + 1.0 \ln\left(1 + \frac{Q}{1\ \mathrm{Ge}\right.\right.$$

neutron: $g_i^{(n)} = -g_i^{(p)}$ — neglecting isoscalar contribution (makes sense within the accuracy of the data available)

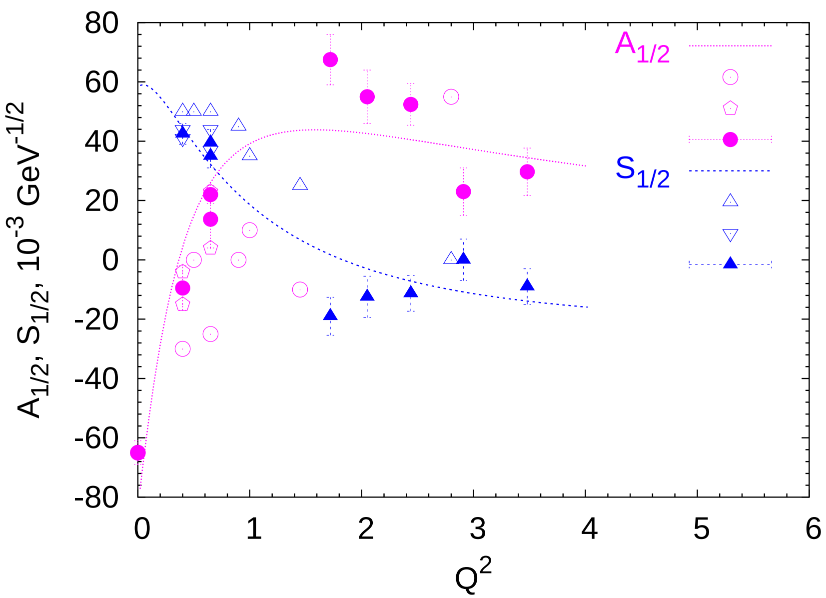


Burkert, Li, IJMP 13 (2004)

$P_{11}(1440)$: Vector form factors

QCD asymptotics for $Q^2 \to \infty$:

$$A_{1/2} \sim \frac{1}{Q^3}, \quad S_{1/2} \sim \frac{1}{Q^4}$$



Helicity amplit. for proton, $W = M_{P1440}$

[1] Tiator et al., EPJA 19 (2004);

[2] Burkert, Li, IJMP 13 (2004);

[3] Aznauryan (talk at NStar 2005)

QCD asymptotics for $Q^2 \to \infty$:

$$g_1^{(em)} \sim \frac{1}{Q^6}, \quad g_2^{em} \lesssim \frac{1}{Q^8},$$

to be updated!

$$g_1^{(p)} = \frac{2.2/D_V}{1 + Q^2/1.2M_V^2}\left[1 + 0.97\ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}}\right.\right.$$

$$g_2^{(p)} = \frac{-0.76/D_V}{\left(1 + Q^2/40M_V^2\right)^2}\left[1 - 2.8\ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}}\right.\right.$$

neutron: $g_1^{(n)} = -g_1^{(p)}$ — neglecting isoscalar contribution (which makes sense within the accuracy of the data available)

$$C_i^V = C_i^{(n)} - C_i^{(p)}$$

Axial form factors

Axial form factors are related by PCAC to the strong πNR couplings g_{P11} and g_{S11}, which in turn are determined from the elastic resonance width

$$P_{11}(1440): \quad g_3^A(Q^2) = \frac{m_N(M_R+m_N)}{Q^2+m_\pi^2} g_1^A(Q^2), \quad g_1^A(0) = -\sqrt{\frac{2}{3}} \frac{g_{P11} f_\pi}{M_R+m_N} = -0.51$$

$$S_{11}(1535): \quad g_3^A(Q^2) = \frac{m_N(M_R-m_N)}{Q^2+m_\pi^2} g_1^A(Q^2), \quad g_1^A(0) = -\sqrt{\frac{2}{3}} \frac{g_{S11} f_\pi}{M_R-m_N} = -0.21$$

The Q^2 dependence for g_1^A is not known, so we again consider "fast fall-off" and "slow fall-off" cases:

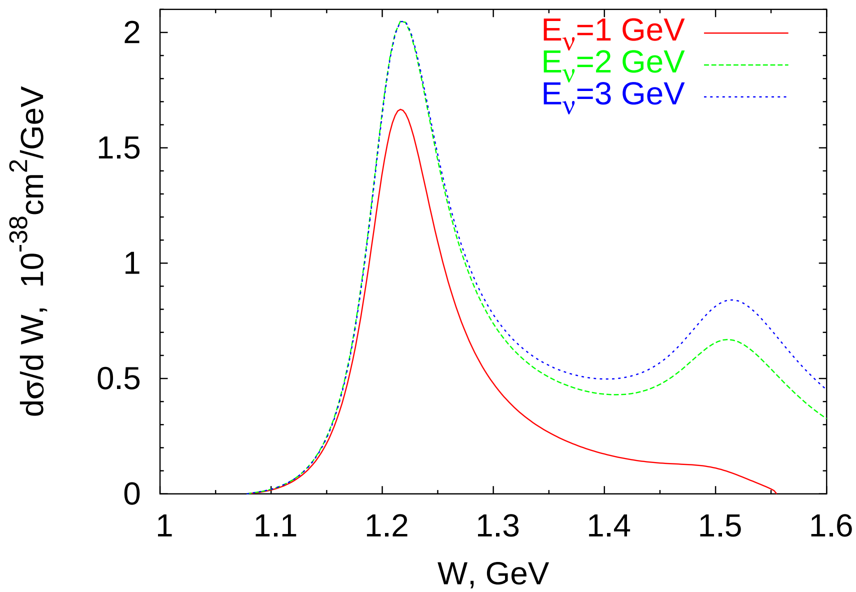
$$P_{11}(1440): \qquad g_1^A(Q^2) = \frac{-0.51/D_A}{1+Q^2/3M_A^2} \text{ ("fast fall-off")}$$

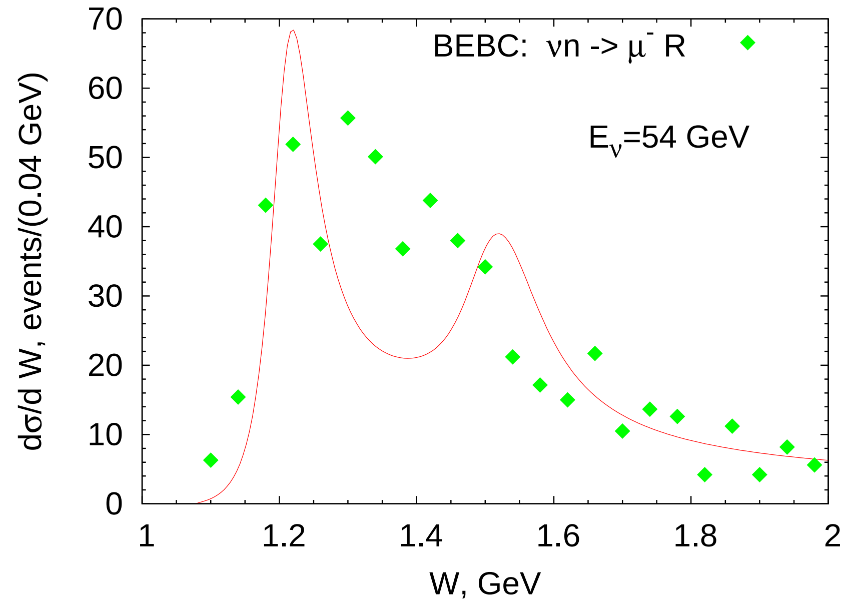
$$g_1^A(Q^2) = \frac{-0.51/D_A}{1+Q^2/4.3M_V^2} \text{ ("slow fall-off")},$$

$$S_{11}(1535): \qquad g_1^A(Q^2) = \frac{-0.21/D_A}{1+Q^2/3M_A^2} \text{ ("fast fall-off")}$$

$$g_1^A(Q^2) = \frac{-0.21/D_A}{1+Q^2/1.2M_A^2}\left[1+7.2\ln\left(1+\frac{Q^2}{1\ \mathrm{GeV}^2}\right)\right] \text{ ("slow fall-o}$$

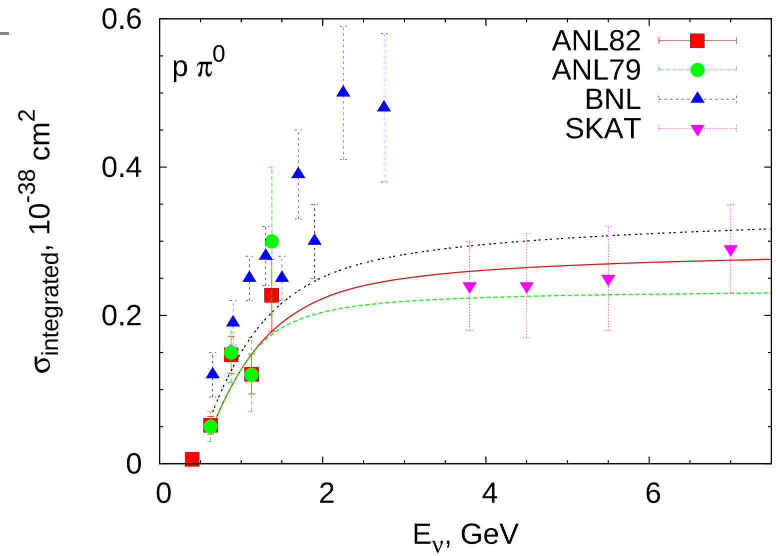
Neutrinoproduction at different E_ν

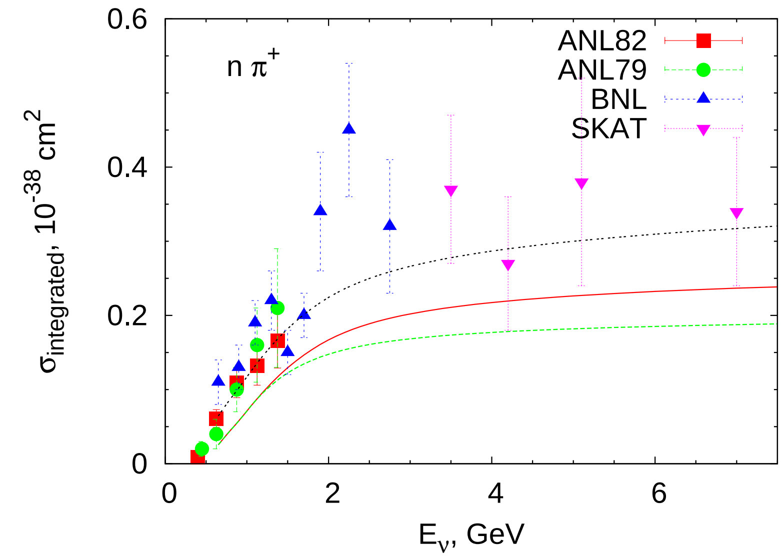




- At $E_\nu < 1\ \mathrm{GeV}$ the second resonance region is negligible in neutrino scattering. It will not be seen in K2K and MiniBOONE.
- At $E_\nu \sim 50\ \mathrm{GeV}$ the two peaks are clearly seen. However, BEBC experiment Allasia et al, NPB 343 (1990) 285 didn't resolve them.

$\nu n \to R^+ \to p\pi^0, \nu n \to R^+ \to n\pi^+$



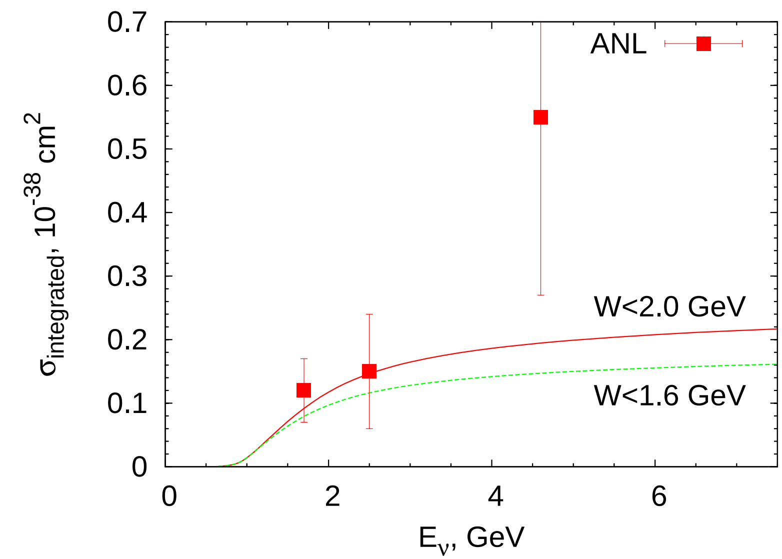


BNL data points are consistently higher than those of ANL and SKAT, errorbars are large

for $\pi^+ n$ channel our curve is a little low than experimental points: either contributions from higher resonances or a smooth isospin-1/2 incoherent background, for example

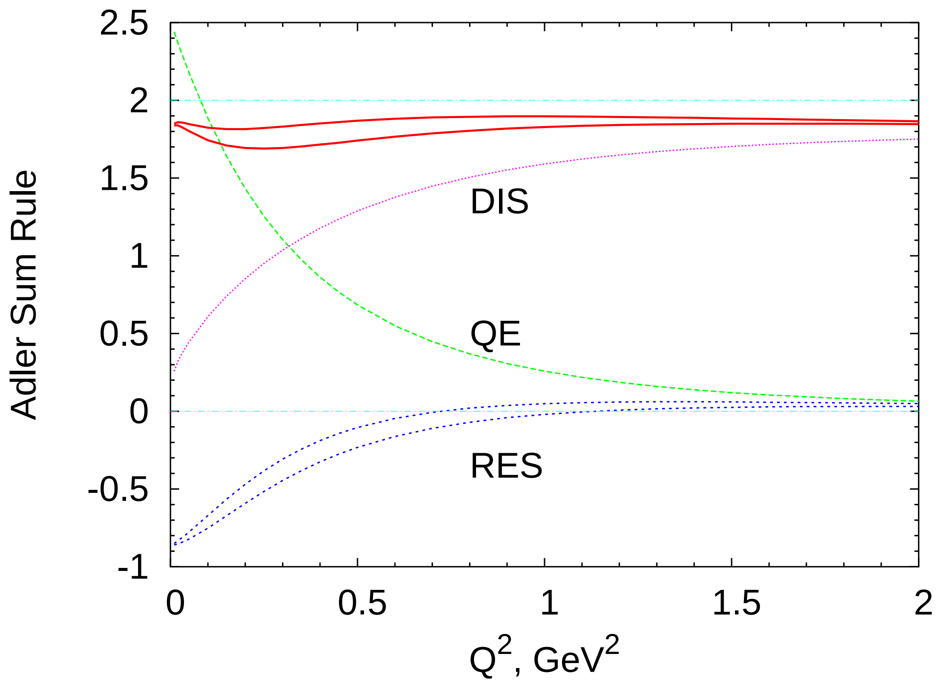
$$\sigma_{bgr}^{\pi^+ n} = 5 \cdot 10^{-40} \left(\frac{E_\nu}{1\ \mathrm{GeV}} - 0.28 \right)^{1/4}\ \mathrm{cm}^2,$$

$$\sigma_{bgr}^{\pi^0 p} = \frac{1}{2} \sigma_{bgr}^{\pi^+ n}$$



Adler sum rule

$$\left[g_{1V}^{(QE)}\right]^2 + \left[g_{1A}^{(QE)}\right]^2 + \left[g_{2V}^{(QE)}\right]^2 \frac{Q^2}{4m_N^2} + \int d\nu \left[W_2^{\nu n}(Q^2,\nu) - W_2^{\nu p}(Q^2,\nu)\right] = 2$$

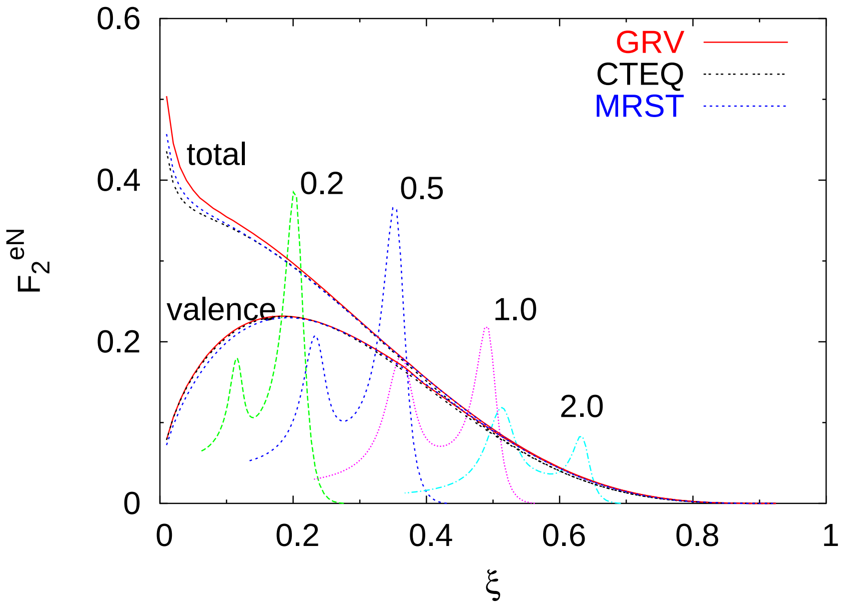


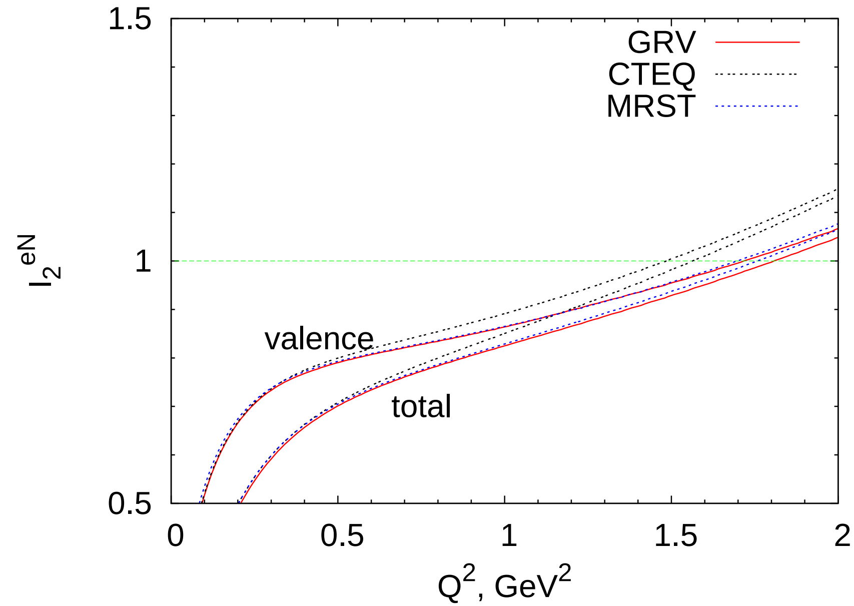
Adler sum Rule is satisfi ed with a 10% accuracy

Duality for electron scattering: F_2^{eN}

The use of the Nachtman scaling variable $\xi = \frac{2x}{1+(1+4x^2 m_N^2/Q^2)^{1/2}}$ includes some of the target mass corrections

As Q^2 increases, the resonance curves should slide along the DIS curve





OL, E.A. Paschos, W. Melnitchouk, hep-ph/0608058

Similar results for $P_{33}(1232)$ are in Matsui, Sato, Lee, PRC 72 (2005) 025204

$$I_2^{eN}(Q^2) = \frac{\int_{\xi_i}^{\xi_f} d\xi\ F_2^{eN\,(\mathrm{res})}(\xi,Q^2)}{\int_{\xi_i}^{\xi_f} d\xi\ F_2^{eN\,(\mathrm{LT})}(\xi,Q^2)},$$

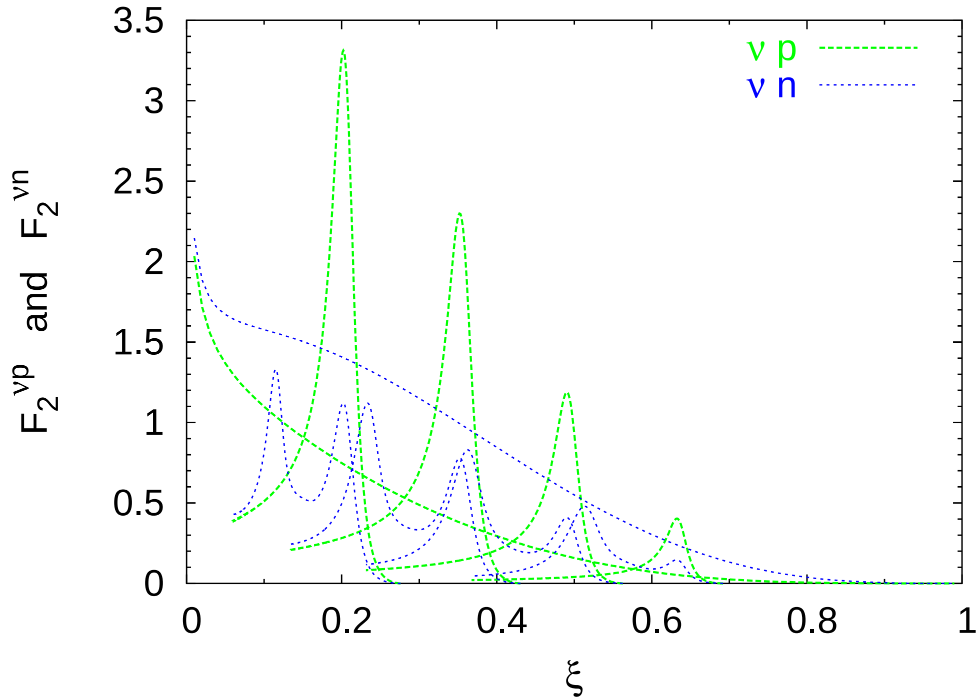
$$\xi_i = \xi(W = 1.6\,\mathrm{GeV},\ Q^2),$$

$$\xi_f = \xi(W = 1.1\,\mathrm{GeV},\ Q^2)$$

Duality neutrino Charge Current reaction

OL, E.A. Paschos, W. Melnitchouk, hep-ph/0608058

Duality does NOT work for protons and neutrons separately



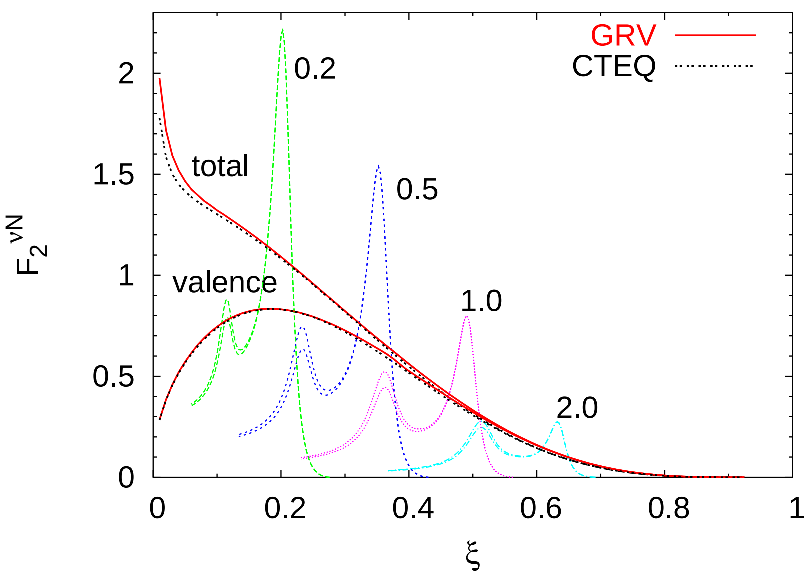
Resonances:

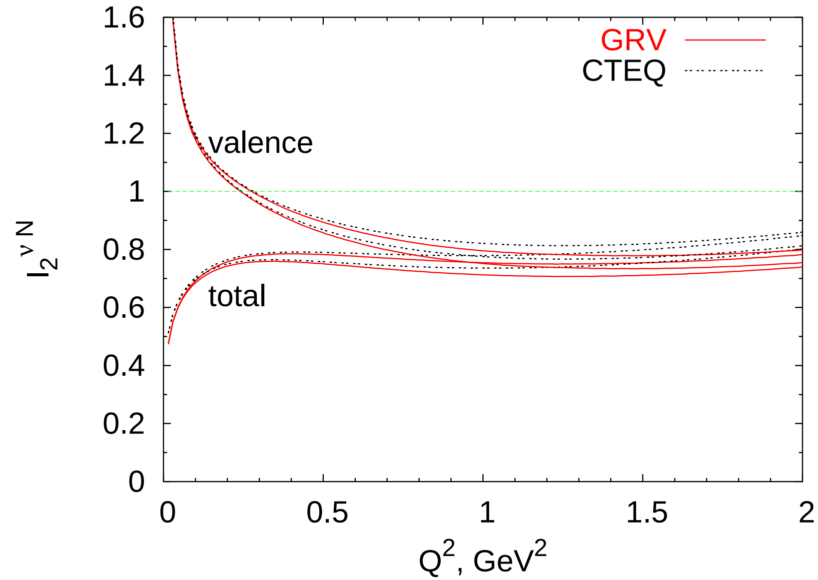
$F^{\nu p}_{i(RES)} > F^{\nu n}_{i(RES)}$

DIS:

$F^{\nu p}_{i(DIS)} < F^{\nu n}_{i(DIS)}$

Duality for CC neutrino scattering: $F_2^{\nu N}$





OL, E.A. Paschos, W. Melnitchouk, hep-ph/0608058

Similar results for $P_{33}(1232)$ are in Matsui, Sato, Lee, PRC 72 (2005) 025204

Analysis for Rein–Sehgal model is in Graczyk, Juszczak, Sobczyk, hep-ph/0512015

$$I_2(\mathrm{res/DIS}) = \frac{\int_{\xi_i}^{\xi_f} F_2^{\mathrm{res}}(\xi, Q^2) d\xi}{\int_{\xi_i}^{\xi_f} F_2^{\mathrm{DIS}}(\xi, Q^2) d\xi},$$

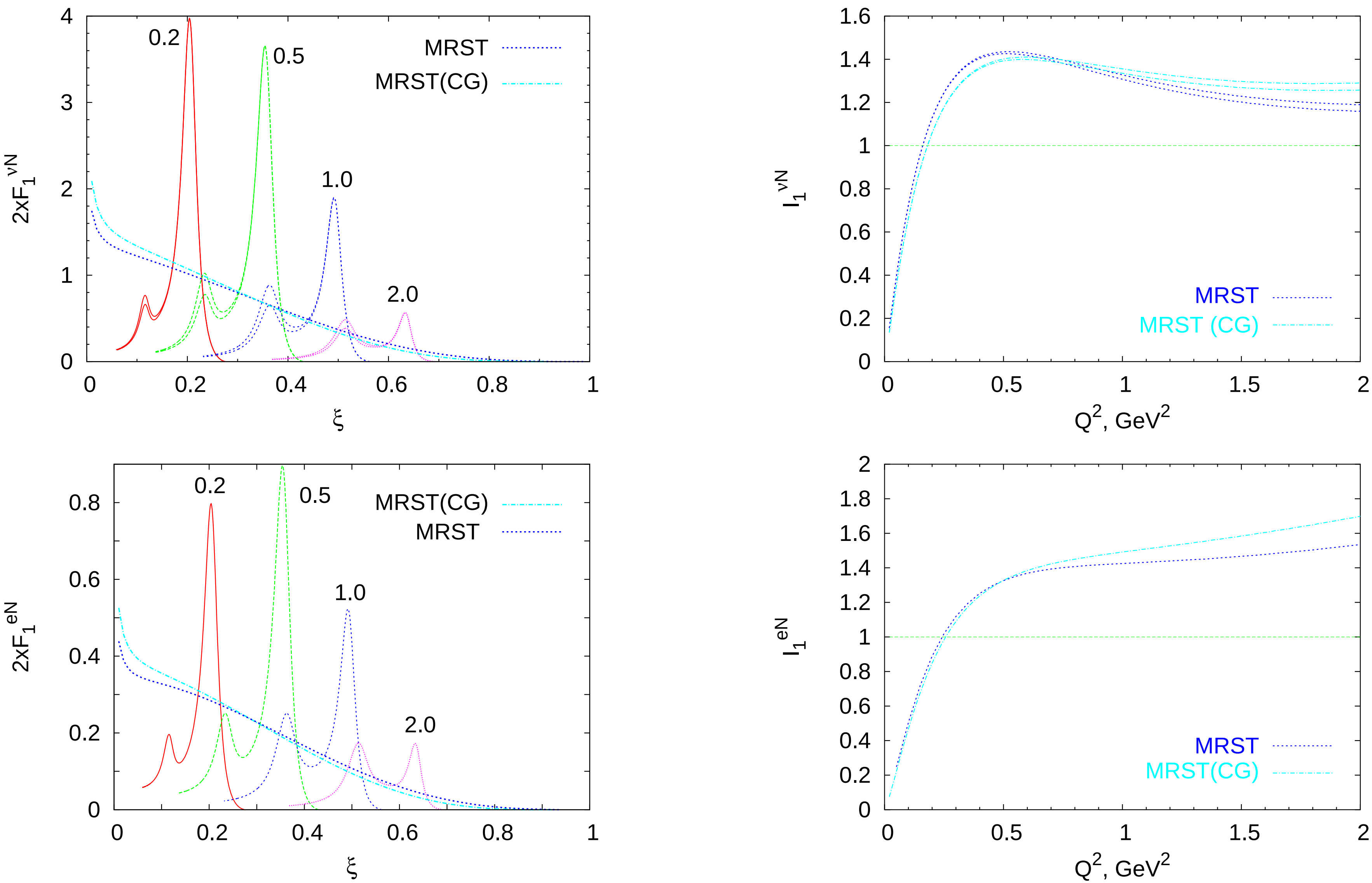
$\xi_i = \xi_i(W = 1.6\,\mathrm{GeV},\ Q^2)$,

$\xi_f = \xi_f(W = 1.1\,\mathrm{GeV},\ Q^2)$.

The discrepancy will be eleiminated by including other resonances, or background, or modifying the FF at large Q^2.

Duality for $2xF_1^{eN}$ and $2xF_1^{\nu N}$

DIS: $2xF_1 = \left(1 + \frac{4m_N^2 x^2}{Q^2}\right) F_2 - F_L$ DIS Callan-Gross: $2xF_1 = F_2$



target mass corrections are more significant in this case

Problems and further directions

- Background

  a) noninterfering — possibility to extract the background from F_2^{ep} structure function

  b) interfering with the resonant part

  — Benhar–Sakuda: electroproduction on carbon, background amplitude and phase shift are from MAID analysis (for Δ resonance)

  — Sato-Lee model with explicite calculation of background diagrams and meson "dressing" of resonance verticies (for Δ resonance)

- Resonance interference

  $$A(\nu n \to n\pi+) = \sqrt{\tfrac{1}{3}}A^{3/2} + \sqrt{\tfrac{2}{3}}A^{1/2}$$

  $$A(\nu n \to p\pi^0) = -\sqrt{\tfrac{2}{3}}A^{3/2} + \sqrt{\tfrac{1}{3}}A^{1/2}$$

- Angular distribution for pions

  a) following Rein–Sehgal approach ("factorized approximation")

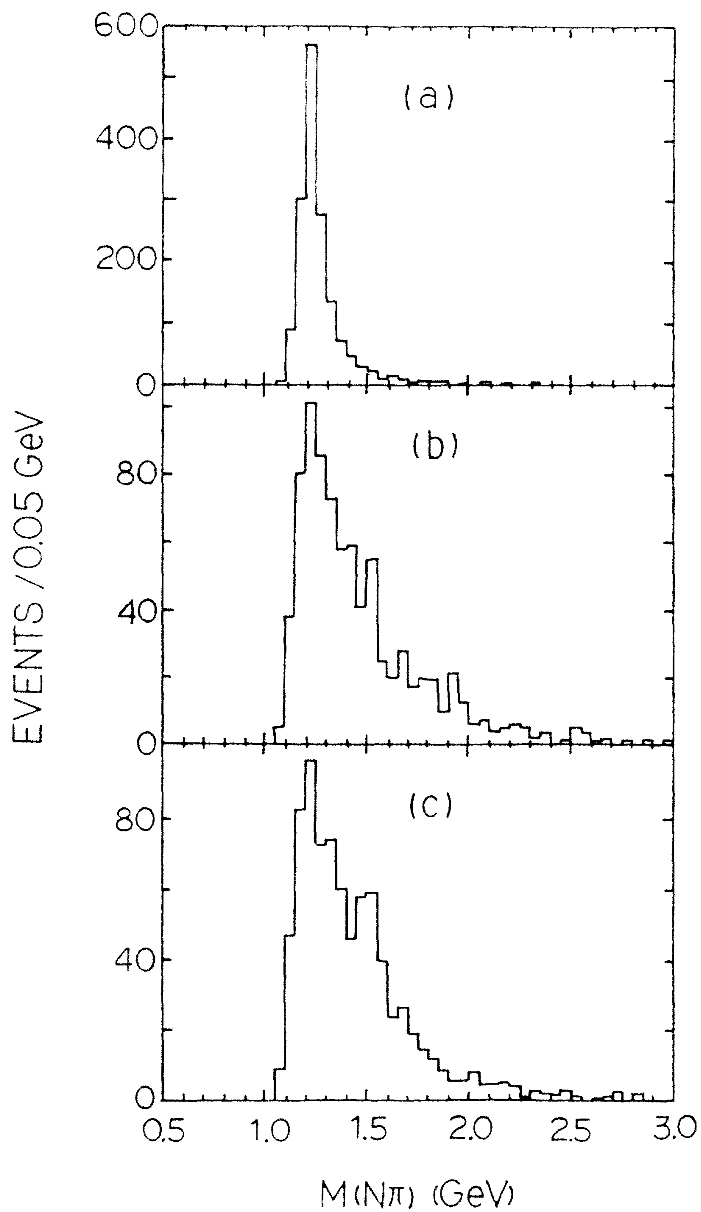
  $|A_{\nu N\to R}|^2 \cdot |A_{R\to\pi N}|^2$

  b) "full calculations" $|A_{\nu N\to R} \cdot A_{R\to\pi N}|^2$

Summary

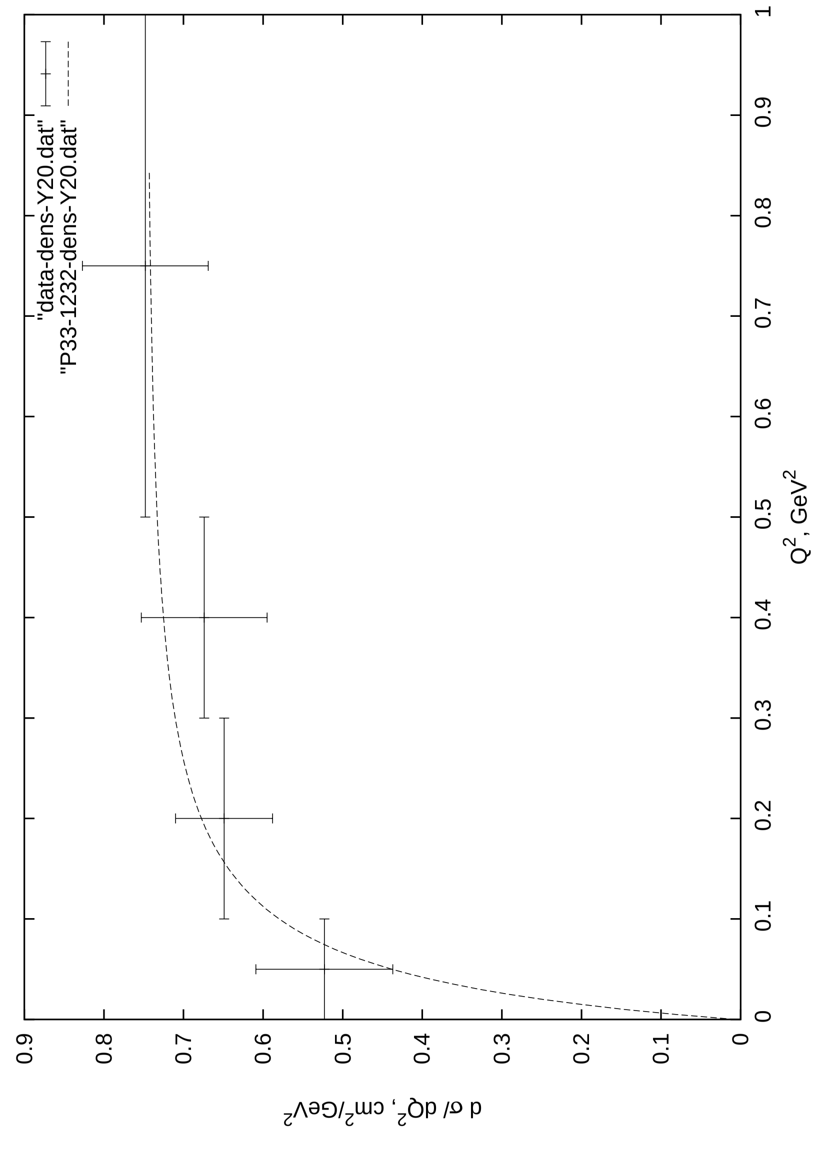
- Double differential cross section for the resonance neutrinoproduction can be written in a form close to DIS with the structure functions expressed via phenomenological form factors
- Vector form factors can be determined from electroproduction helicity amplitudes, some of the axial form factors — from PCAC
- Δ–resonance is understood better than others, but still requires investigation in low Q^2 region
- Second resonance region must be seen in neutrino experiments for $E_\nu > 2\ \mathrm{GeV}$
- More questions than answers for further development

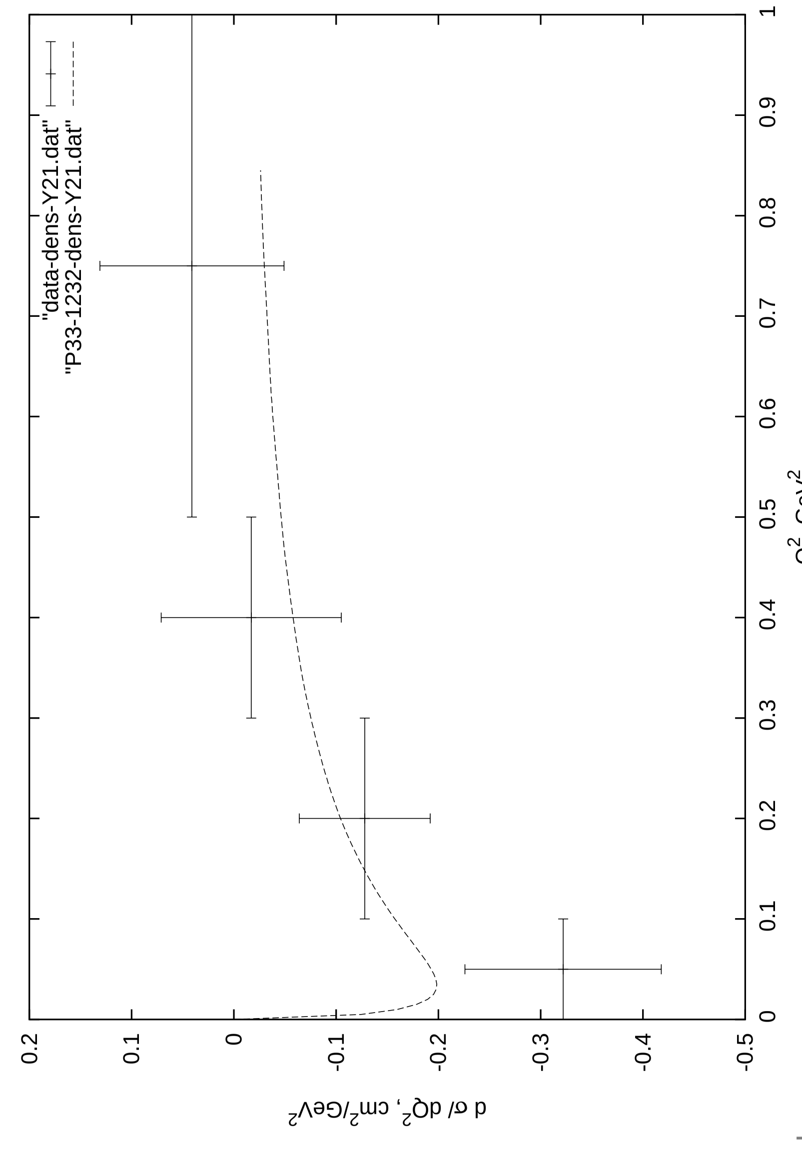
BNL

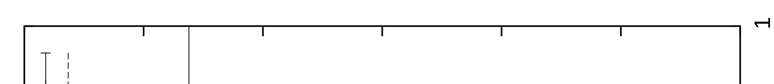


600
400
200
0
80
40
0
80
40
0
(a)
(b)
(c)
EVENTS / 0.05 GeV
0.5
1.0
1.5
2.0
2.5
3.0
M(Nπ) (GeV)

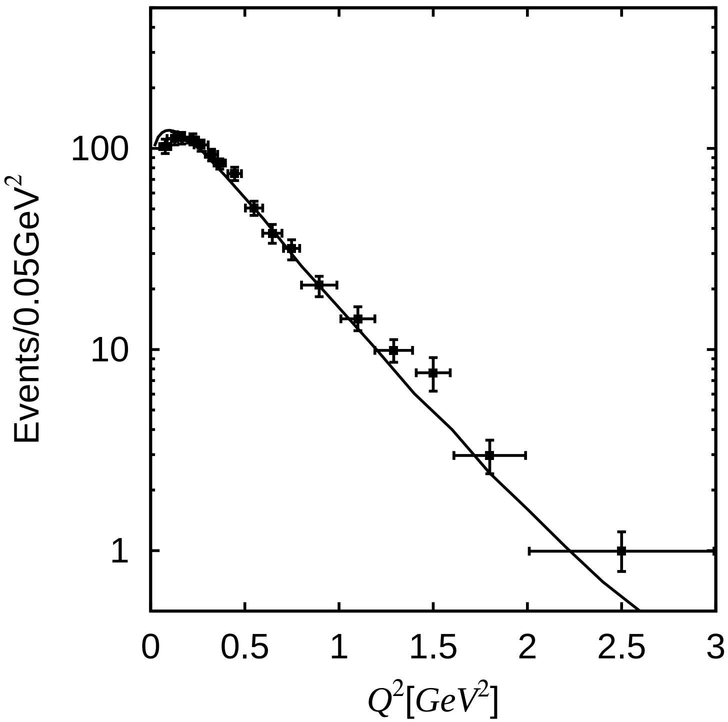
Angular distribution

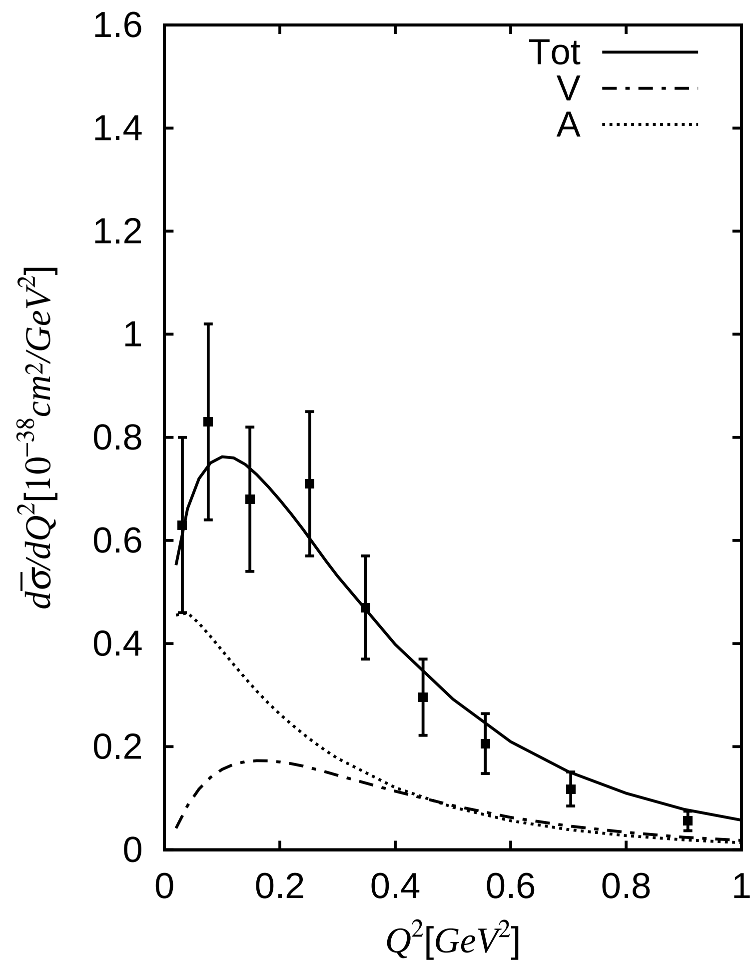






Cross section in Sato–Lee model





Sato–Lee: $C_5^A(Q^2) \quad = \quad \dfrac{C_5^A(0)}{D_A} \quad \cdot$

$$\left(1 + 0.154\frac{Q^2}{1\ \mathrm{GeV}^2}\exp(-0.166\frac{Q^2}{1\ \mathrm{GeV}^2})\right)$$

Sato, Lee PRC 67 (2003) 065201

Conclusion: ANL data show a steeper Q^2–dependence than BNL data

Possible techniques for calculations:

- ”multipole expansion” technique, which seems to give excellent results for electroproduction (Mainz and JLAb groups)
  — original paper Adler, Ann.Phys50 (1968), 123 pages: the most of formulas for both el-m and weak interactions;
  — further development for electroproduction and changing of notations
  — for electroproduction formulas are implemented and used by MAID group (Tiator) and JLab group(Burkert, Lee, Aznauryan); many years of experience
- using ”full calculation” — analytical formular for $d\sigma/dQ^2 dW d\Omega_\pi$ is cumbersome, done! C++ code, generated by MATHEMATICA for P_{33} for $m_\mu = 0$ is ∼5000 lines